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EXTRAGALACTIC PLUS GALACTIC MODEL FOR ICECUBE NEUTRINO EVENTS

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ABSTRACT

The hypothesis that high-energy cosmic neutrinos are power law distributed is critically analyzed. We propose a model with two components that better explains the observations. The extragalactic component of the high-energy neutrino flux has a canonical E_ν^{-2} spectrum while the galactic component has a $E_\nu^{-2.7}$ spectrum; both of them are significant. This model has several implications, which can be tested by IceCube and ANTARES over the next several years. Moreover, the existence of a diffuse component, close to the Galactic plane and that yields (20–30)% of IceCube's events, is interesting for the future km^3 neutrino telescopes located in the Northern Hemisphere and for gamma-ray telescopes aiming to measure events up to a few 100 TeV from the southern sky.

Key words: neutrinos

1. INTRODUCTION

Before IceCube, the most popular expectations regarding ultra-high-energy neutrinos, typically adopted in sensitivity studies, were that (1) most of them have an extragalactic origin, as motivated by the existence of many extreme astrophysical objects and of extragalactic cosmic rays, and (2) their spectrum is distributed as E_ν^{-2}, as motivated by the expectations for cosmic rays at the sources and by Fermi's acceleration mechanism. A particular very well-known implementation was the one developed in Waxman & Bahcall (1999).

After IceCube, this hypothesis has been minimally modified in order to take into account that the *observed* spectrum does not agree well with E_ν^{-2}. The minimal modification consists of using a single power law $E^{-\alpha}$, isotropic and with $\alpha \sim 2.5$, a value obtained by fitting the observations of IceCube. In this work, pluses and the minuses of this position are summarized, motivating an alternative and more satisfactory hypothesis to account for the observations of IceCube and to forecast future findings. If the alternative hypothesis is correct, it is premature to dismiss the position just described above. Moreover, as we will discuss below, there are a lot of interesting consequences for the astronomy of high-energy neutrinos.

Before proceeding, two important clarifications are in order:

(1) The aim of the present paper is not to prove wrong the assumption of a single power law $E^{-\alpha}$, isotropically distributed, which is most commonly adopted for analyses of IceCube data. This is still viable at present, if one adopts very conservative criteria (i.e., one discards some data set or information and/or if one accepts various discrepancies). Our aims are to illustrate the reasoning behind another hypothesis capable of interpreting the IceCube data, illustrating its advantages and its physical interest, deriving its implications, and showing that it can be tested with future data.

(2) Also, the issue of minimality requires additional discussion. The "single power law" hypothesis has two free parameters, the normalization and the slope, adding only one parameter to the previously adopted hypothesis (the normalization of the assumed E_ν^{-2} flux). However, in view of the fact that, to date, we lack a firm theoretical bases for the interpretation of IceCube observations, we should not exaggerate the heuristic power of mathematical criteria. Moreover, in the simplest version of our model, which we adopt, the spectrum has two components with fixed slopes: one of them behaves as E_ν^{-2}, the other one as $E_\nu^{-2.7}$. Thus the free parameters for the fit are again two, the normalizations of the fluxes.

The structure of this paper is as follows. In Section 2 we examine the single-component hypothesis for high-energy neutrinos, stressing its shortcoming. In Section 3 we motivate and present the new hypothesis that includes two components: one due to extragalactic neutrinos and the other one due to galactic neutrinos. In that section, we also quantify the relative intensity and discuss the spectra. In Section 4 we illustrate the predictions of our model for existing and future neutrino telescopes and the implications for gamma astronomy at the highest energy. A brief summary is contained in Section 5.

2. CRITICAL DISCUSSION OF THE SINGLE-POWER-LAW MODEL

In this section we proceed to a critical assessment of the single-power-law model for high-energy neutrino emission, which, in view of its popularity and of the almost general acceptance at present, can be considered as the *null hypothesis* adopted for data analysis.

2.1. Angular Anisotropy

The list of the candidate extragalactic sources of ultra-high-energy neutrinos comprises several disparate astrophysical objects, including various classes of active galactic nuclei (AGNs; Stecker 2005, 2013) such as BL Lacertae objects (Padovani & Resconi 2014; Tavecchio et al. 2014; Padovani et al. 2016); of peculiar galaxies, such as starburst-galaxies (Loeb & Waxman 2006; Stecker 2007; Tamborra et al. 2014); of extreme stellar objects such as gamma-ray bursts (GRB; Becker et al. 2006; Asano & Mészáros 2014), etc. They differ greatly in physical and observable characteristics. However, a common feature of these objects is to have an almost isotropic angular distribution in the sky, making a possible exception for the brightest among them, which could stand out. This is the main reason why it is assumed that, in the first approximation, the (supposedly extragalactic) high-energy neutrinos seen by IceCube should be isotropically distributed (but note incidentally that also certain hypothetical sources in the Galaxy, such as the halo, might lead to a similar distribution).

In Neronov & Semikoz (2016), the angular distribution of the events observed by IceCube was analyzed, showing a significant excess of events on the galactic plane. The analysis is repeated with different angular bins and the significance is never less than 3σ. Similar work is performed by Troitsky (2015), where the arrival directions of high-energy neutrinos are discussed, focusing on the possibility that neutrinos are not only extragalactic but they also can be produced in the halo or in the disk of our Galaxy. The result of Troitsky (2015) is that the present data are compatible with a purely extragalactic component, but also that a mixed flux (extragalactic plus galactic neutrinos) is well compatible with the data. On the contrary, a purely galactic component is disfavored at about 2σ, as can be seen in Figure 2 of that paper. Later in this paper, Section 3.2.2, we perform a new independent analysis of the angular distribution, obtaining results consistent with those described here.

The angular anisotropy represents the first hint of reconsidering the null hypothesis.

2.2. Spectral Distribution

In this section, we discuss the fact that different data sets collected by IceCube suggest different power-law distributions. We begin with an introduction to the relevant data sets, where we indicate how they correspond to the neutrino fluxes (see Section 2.2.1). Then, we will compare the power-law distributions, obtained from different data sets, in Section 2.2.2.

2.2.1. Different Data Sets and Their Summaries as Power-law Distributions

IceCube has observed two main classes of events. The high-energy starting events (HESE), whose vertex is contained in the detector, and the passing muons (a.k.a. through-going muons, a.k.a. tracks) that are the traditional signal of neutrinos. The HESE are divided into two classes: (1) *direction*, those from the northern sky and those from the southern sky, and (2) *topology*, those of shower type and those of track type. The passing muons instead are all from the northern sky and all of track type. Note that the experimental sample of HESE north events is not very large, and the subset of HESE north tracks is still smaller; the experimental sample of passing muon events, collected in IceCube, is much larger than the sample of HESE north muons. (We discuss below in detail which are the specific sets that we use in the analysis.)

The above considerations concern the experimental classification of the data. Passing now to the physical interpretation of these data, in terms of neutrino fluxes, it is essential to remark that the passing muon events, measured in IceCube, give us information on the flux of muon neutrinos and antineutrinos coming from the northern sky, just as the set of HESE north muon events.

Let us recall the well-known fact that neutrino oscillations connect tightly (see, e.g., Palladino & Vissani 2015) the electron, the muon, and the tau neutrino fluxes, and in the particular and natural case when the neutrinos derive from pion decays, these fluxes are approximately the same. Let us finally recall that the inclusion of passing muon data has been important to verify the consistency of IceCube observations with three-flavor neutrino oscillations in the recent past (Palladino et al. 2015b). All these considerations point to the importance of including the passing muons in the analysis.

2.2.2. Comparison of the Power-Law Distributions Obtained from Different Data Sets

Let us begin discussing the HESE events. The all-flavor extraterrestrial component of the HESE observed by IceCube, after four years of data collection, is fitted by a single-power-law spectrum, without an energy cutoff, obtaining

$$\frac{d\phi}{dE_\nu} = F \cdot 10^{-18} \frac{1}{\text{GeV cm}^2 \text{ s sr}} \left(\frac{E_\nu}{100 \text{ TeV}}\right)^{-\alpha} \quad (1)$$

with $F = 6.7^{+1.1}_{-1.2}$ and $\alpha = 2.50 \pm 0.09$ (Aartsen et al. 2015b). However, an attentive analysis of the most recent IceCube data reveals a difference between the flux that comes from the southern sky and the one that comes from the northern sky. Indeed, these two subsets of data give the following best-fit fluxes: $F_\text{N} = 2.1^{+2.9}_{-1.6}$, $\alpha_\text{N} = 2.0^{+0.3}_{-0.4}$, $F_\text{S} = 6.8^{+1.6}_{-1.5}$, and $\alpha_\text{S} = 2.56^{+0.11}_{-0.12}$. In order to quantify the difference between north and south HESE power-law fluxes we analyze the distribution of their spectral indices, described approximately by two Gaussian functions $g_\text{N}(\alpha)$ and $g_\text{S}(\alpha)$. Following the procedure reported in the Appendix, a mild tension of 1.3σ between the data sets of HESE north and HESE south, $\delta\alpha = 0.56 \pm 0.42$, is found.

Let us discuss now the passing muons. Since the IceCube experiment is located on the South Pole, the passing muons provide information on the muon neutrino flux coming from the northern sky. This information must be consistent with the information from the HESE north data set, which is also generated by neutrinos that come from the northern sky. Moreover, as already recalled, the passing muon data set contains a larger number of events than the HESE north data set alone, which allows us to obtain stronger inferences. More precisely, comparing the flux obtained fitting the passing muons with the flux obtained fitting the HESE seen from the southern sky results in a much greater tension. In Aartsen et al. (2015c) the flux of diffuse astrophysical muons is analyzed and it is found that $\alpha_p = 1.91 \pm 0.20$ for the spectral index. This flux is obtained using events in the energy range between 170 TeV and 3.8 PeV, but, if the single-power-law hypothesis is the true one, this flux *must be valid* also at lower energies. On the other side, the data set of HESE south gives a preference for a softer spectrum, i.e., $\alpha_s \simeq 2.56 \pm 0.12$.

Using the procedure described in the Appendix, it is possible to compare the fluxes that describe the various data sets, just as we did above with the fluxes obtained only from HESE data. The results are as follows.

1. A discrepancy equal to 2.8σ when we compare the fluxes obtained from the passing muons with the one obtained from the HESE southern sky, namely, $\delta\alpha = 0.65 \pm 0.23$.
2. A similar tension of 2.7σ between the power-law distribution that describes the passing muon data set and the global best fit of HESE, namely, $\delta\alpha = 0.59 \pm 0.22$.
3. Compatibility between HESE north and passing muons, namely, $\delta\alpha = 0.09 \pm 0.45$.

In a recent talk, presented by Schoenen (IceCube Collaboration) at "TeV Particle Astrophysics 2015, Kashiwa, Japan" on the October 29, a similar result is obtained: the disagreement between the tracks detected in six years and the combined analysis of four years is claimed to have a significance of 3.6σ that is a bit stronger and thus even more significant than the results we have just illustrated.

Since the tracks are generated by muon neutrinos[3] and the showers by all-flavor neutrinos, the discrepancy may be explained in two major ways.

1. The flux of ν_μ has a different behavior with respect to the flux of ν_e and ν_τ, but there are no theoretical reasons to sustain this point.
2. The northern sky and the southern sky are detecting two different populations of neutrinos: an almost purely extragalactic component from the northern sky and a mixed component (galactic plus extragalactic) from the southern sky.

In conclusion, analyzing the spectral distribution of the fluxes derived by the different data sets, it is possible to conclude that there is a difference in the shape between the neutrino flux observed in the southern sky and the one observed in the northern sky, with a significance of at least 2.7σ. Combining the 1.3σ (HESE north–HESE south) and 2.7σ (Passing muons–HESE south) hints, the overall significance becomes 3.0σ, which means a p-value of less than 0.3%.

This significance represents the second hint for going beyond the single-power-law model.

2.3. Extragalactic Power Law Neutrinos, Protons, and γ-rays

In this section we analyze the connections between cosmic neutrino and other radiations, i.e., cosmic rays and gamma-rays. Let us consider a source transparent to protons, as happens in the model proposed by Waxman & Bahcall (1999). This model predicts an upper bound for the cosmic neutrino flux, connected to the amount of energy of cosmic rays, in the energy interval between 10^{19} and 10^{21} eV. Their argument goes as follows: High-energy protons, produced by the extragalactic sources, lose a fraction ϵ of their energy through photo-meson production of pions before escaping the source, and the resulting present-day energy density of all-flavor neutrinos will be given by

$$E_\nu^2 \frac{d\phi}{dE_\nu} \simeq \frac{3}{2}\, \epsilon\, I_{\max} \tag{2}$$

where $I_{\max}$ is given by

$$\begin{aligned} I_{\max} &= 0.25 \xi_Z t_H \frac{c}{4\pi} E_{\rm CR}^2 \frac{d\dot{N}_{\rm CR}}{dE_{\rm CR}} \\ &\simeq 1.5 \times 10^{-8} \xi_Z \text{ GeV cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1} \end{aligned} \tag{3}$$

where ξ_Z is connected with cosmology and takes into account the energy loss by neutrinos at redshift different from 0 and the upper bound is obtained for $\epsilon = 1$, i.e., when all the energy of protons is transferred to pions. The numerical value of $I_{\max}$ is obtained using $E_{\rm CR}^2 \frac{d\dot{N}_{\rm CR}}{dE_{\rm CR}} = 10^{44}$ erg Mpc^{-3} yr^{-1}. Comparing this model with the fluxes that have been derived by IceCube using the four-year data set, the results illustrated in Figure 1 are obtained. The bands related to the fluxes are due to the uncertainties on the normalization and on the shape. The upper limit is given by

$$\begin{aligned} \phi + \Delta\phi &= \phi \\ &+ \sqrt{[\phi(F, \alpha) - \phi(F^+, \alpha)]^2 + [\phi(F, \alpha) - \phi(F, \alpha^+)]^2} \end{aligned} \tag{4}$$

where ϕ is the flux at best fit, $F^+ = F + \Delta F$, and $\alpha^+ = \alpha + \Delta\alpha$ if $E_\nu < 100$ TeV or $\alpha^+ = \alpha - \Delta\alpha$ if $E_\nu > 100$ TeV. With the same procedure, the lower limit $\phi - \Delta\phi$ is obtained. The values of F, α, and their uncertainties are reported in Section 2.2.

The two fluxes derived from HESE northern sky and from the passing muons data set are compatible with the Waxman–Bahcall model. On the other hand, the data from HESE southern sky begin to exhibit some critical behavior below 60–70 TeV, because they exceed this limit. Note that in this region, a lot of events were already observed, so it cannot be a problem attributed to the low statistic. To have a more precise idea, let us use the flux that was derived by the IceCube collaboration in Aartsen et al. (2015b), including events above 10 TeV. The energy flux of HESE from the southern sky at 25 TeV is equal to

$$E_\nu^2 \frac{d\phi_{\rm S}}{dE_\nu}(25 \text{ TeV}) = 1.5 \pm 0.4 \times 10^{-7} \frac{\text{GeV}}{\text{cm}^2 \text{ s sr}}. \tag{5}$$

It means that the Waxman and Bahcall limit is violated at 2σ at that energy and the violation is even larger at lower energies: in fact, the flux should extend also at lower energies, if the hypothesis of the single power law holds true. Once again, these considerations also suggest a reconsideration of some of the assumed hypotheses.

Another connected remark emerges comparing the flux of neutrinos with the IGRB (intergalactic gamma-ray background). The first step in this direction was done by Murase et al. (2013) and Murase (2015), using the two-year data set of IceCube events. In this work the pp scenario is considered and the neutrino flux is compared to the diffuse γ-ray background flux measured by *Fermi*. A strong upper limit on the spectral index of the source of neutrinos is obtained, i.e., $\alpha \leqslant 2.2$. A softer spectrum would give an excess of neutrinos with respect to the γ-rays in the 100 GeV region and this is in tension with the theory. Also in Bechtol et al. (2015) this kind of analysis is performed using more recent data. The extrapolated flux of neutrinos exceeds the flux of gamma around 0.1 TeV, even in the most conservative hypothesis in which the extrapolation follows an E^{-2} spectrum. This excess represents a problem in the scenario of star-forming galaxies, where the pp interaction is the dominant mechanism for the production of neutrinos. Moreover, in that energy region the gamma-rays are not absorbed, even if they come from a distance of some Gigaparsec.

The above remarks are our last hint that the null hypothesis requires some critical consideration. The previous issues, concerning the comparison of power-law-distributed neutrinos with protons and γ-rays, can indicate some of the following possibilities:

(i) an exotic source of neutrinos (e.g., exotic dark matter);

[3] The contribution of the tracks that come from ν_τ is of the order of 5% because we have to take into account not only the branching ratio but also the energy of the secondary muon with respect to the energy of the primary neutrino.

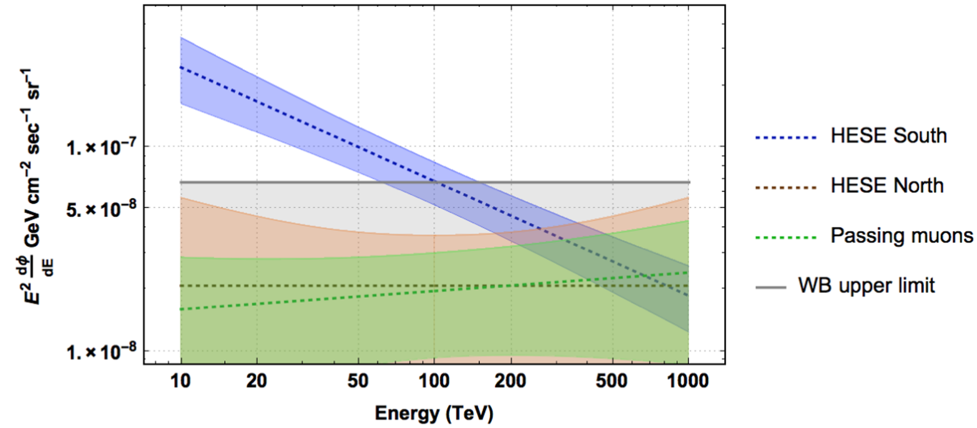

Figure 1. Comparison between the limit given by a source transparent to ultra-high-energy cosmic rays and the power-law fluxes that describe the present IceCube data.

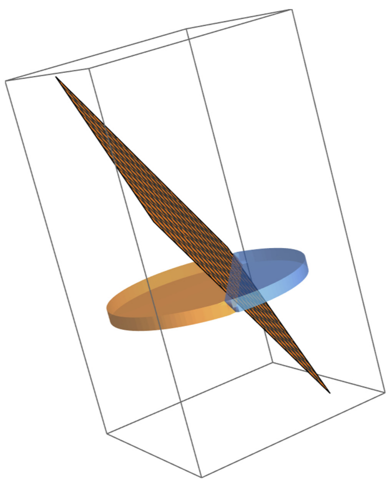

Figure 2. Simplified cylindrical model of the Galaxy. The relative orientation of the Earth's equatorial plane (orthogonal to rotation axis) is also shown.

(ii) absorption of protons and/or gammas into the source[4]; and

(iii) a galactic component of neutrinos on top of the extragalactic component.

In this paper we will explore the last option. It should be stressed that the limit on the neutrino flux, given by the comparison with cosmic rays and IGRBs, only applies to the extragalactic neutrinos.

3. THE HYPOTHESIS OF THE TWO-COMPONENT SPECTRUM

Considering the hints of the previous sections, we explore the hypothesis that the northern sky is seeing an almost purely extragalactic flux of neutrinos and the southern sky is seeing both extragalactic and galactic neutrinos. This hypothesis can justify the spectral differences in the IceCube data set and the angular anisotropy of the observed events. Moreover, this hypothesis "saves" the upper limit of Waxman and Bahcall, which is in strong disagreement with the IceCube global fit already at $\simeq$100 TeV. At the same time, it solves the problems arising from the comparison with the IGRB.

3.1. *General Motivations*

Our hypothesis is motivated by the position of the Earth in the Galaxy and by the location of the IceCube detector. Let us begin with a standard model for the Galaxy, given by a cylinder of radius 15 kpc and a variable height of some kiloparsecs. The axis of the rotation of the Earth forms an angle of 30° with the Galactic plane, so the equator of the Earth makes an angle of 60° with it. The equator is represented by a plane, perpendicular to the normalized vector $(u_x, u_y, u_z) = (0.484, 0.747, 0.456)$. Imposing the condition that the plane includes the Earth, which is at 8.5 kpc from the center of the Milky Way, the equation of the plane is obtained (see Figure 2). At this point it is easy to evaluate the volume of the Galaxy seen by the northern sky and the one seen by the southern sky; the result is about 70% from the south and 30% from the north.

In addition to this, one can take into account that the matter is not distributed uniformly, as can be seen, e.g., from Figure 2 of Vissani et al. (2011). This is another good reason to believe that the southern sky is seeing also a galactic component of neutrinos, which is on the contrary negligible from the northern sky.

However, the above modeling is good only for a cursory exploration and to obtain a first idea. The region where the Galactic neutrino emission is more intense is not known a priori and should be eventually measured by neutrino observatories. For example, it is natural to modify and extend the above model of the region where neutrino emission is more intense, by adding two parameters: the radius of the cylinder and its galactic height.[5] Of course if the radius is well below the distance from the center, $R \sim 8.5$ kpc, all of the emitting region will lie in the Southern Hemisphere. The values of these parameters have to be obtained from the analysis of high-energy neutrino data.

Note, incidentally, that the *Fermi* satellite sees an intense, diffuse galactic emission until 100 GeV. At higher energies, closer to those explored by neutrino telescopes, HESS has proved that the galactic neutrino emission due to point sources

[4] It could happen especially in the $p\gamma$ mechanism of production because the γ-rays produced by the decay of π^0 can interact themselves with the target photons contained in the source and they can be trapped in the source. In this kind of opaque source the connection with neutrinos can be difficult, as discussed in Murase et al. (2015).

[5] An alternative parameterization uses two exponential distributions $n_\nu \propto \exp(-r/\delta r)\exp(-|h|/\delta h)$ where r is the distance from the Galactic center and h is the Galactic height; δr and δh are model parameters.

is quite intense; a diffuse galactic component at high energy, unfortunately, is not yet probed and is hard to probe. In short, the existing gamma-ray data indicate the existence of a significant component of high-energy radiation that can be attributed to the Galaxy, and this consideration does not contradict (but rather supports) the hypothesis that something similar happens for high-energy neutrinos. We will come back to these considerations later.

3.2. Galactic Component

3.2.1. Spectrum and Intensity of the Galactic Component

Elaborating on the hypothesis that IceCube is observing two populations of neutrinos, it seems natural to describe the neutrinos that come from the northern sky, which are assumed to be extragalactic, with an unbroken power law, distributed isotropically over the sky. Moreover, it would be natural to describe the observations of IceCube by adding a new a power-law flux in the southern sky, for which the galactic population of neutrinos provides an important contribution. For the extragalactic component, the spectral index $\alpha = 2$ is adopted, since it coincides with the value measured for the northern sky flux at the best fit. For the galactic component, the spectral index $\alpha = 2.7$ is adopted, i.e., the value of the galactic cosmic rays. It follows that

$$\frac{d\phi_{\rm N}}{dE_\nu} = \frac{d\phi_{\rm EG}}{dE_\nu}$$
$$\frac{d\phi_{\rm S}}{dE_\nu} = \frac{d\phi_{\rm G}}{dE_\nu} + \frac{d\phi_{\rm EG}}{dE_\nu}$$

or more explicitly

$$\frac{d\phi_{\rm N}}{dE_\nu} = F_{\rm EG}\left(\frac{E_\nu}{100\ {\rm TeV}}\right)^{-2} \cdot 10^{-18}\frac{1}{{\rm GeV\ cm^2\ s\ sr}} \tag{6}$$

$$\frac{d\phi_{\rm S}}{dE_\nu} = \left[F_{\rm G}\left(\frac{E_\nu}{100\ {\rm TeV}}\right)^{-2.7} + F_{\rm EG}\left(\frac{E_\nu}{100\ {\rm TeV}}\right)^{-2}\right] \times 10^{-18}\frac{1}{{\rm GeV\ cm^2\ s\ sr}}. \tag{7}$$

Considering the events with deposited energy above 60 TeV,[6] IceCube has observed 32 events in four years, 9 from the northern sky, 22 from the southern sky and one at latitude zero. Using the effective areas for each hemisphere, which recently became available on the IceCube website,[7] the likelihoods for the extragalactic and the galactic normalizations ($F_{\rm EG}$, $F_{\rm G}$) can be obtained. The likelihood of the normalization at 100 TeV for the flux from the northern sky is defined as

$$\mathcal{L}_{\rm EG}(s) \propto (b + 3.6s)^{9.5} \cdot \exp[-(b + 3.6s)] \tag{8}$$

where $b = 2.1$ is the background due to conventional atmospheric neutrinos, considering the best-fit value.[8] The coefficient 3.6, instead, is the number of events expected in the case of normalization equal to one at 100 TeV. The result is $F_{\rm EG} = 2.1^{+1.1}_{-0.6}$ at the 68% confidence level. This extragalactic component is perfectly compatible with the upper limit given by the IGRB observed by *Fermi* (see Bechtol et al. 2015). Let us remark that we are using the HESE both for the Northern and Southern Hemisphere because for this data set the effective areas are available and we can perform our calculation using $\alpha = 2$ for the extragalactic component. The inclusion of the passing muons could certainly reduce the uncertainties on the galactic component that we will calculate later, but it is not possible at the moment; for this reason, our results are conservative.

[6] Following Palladino et al. (2015b), we limit the analysis to this subset of data, for which the atmospheric background gives a small contribution.
[7] https://icecube.wisc.edu/science/data/access
[8] We scaled in time the expectation for the backgrounds contained in Aartsen et al. (2014). We are not considering the charm component, which is zero at the best fit.

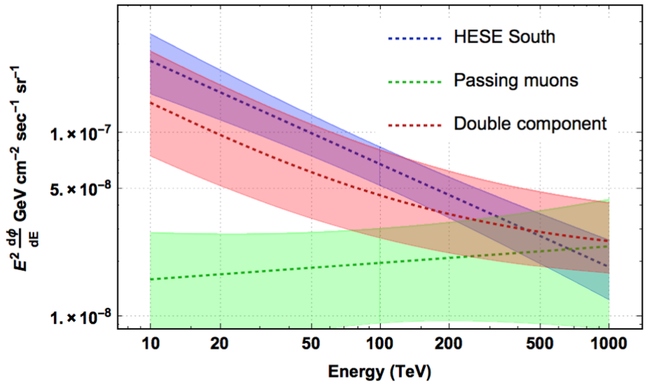

Figure 3. Comparison between the fluxes measured by IceCube from the northern sky and southern sky with the two-component flux.

Taking into account the uncertainties given by the supposed extragalactic component, the likelihood for the normalization at 100 TeV of the galactic component, seen from the southern sky, is given by

$$\mathcal{L}_{\rm G}(s) \propto \int_0^\infty (b + 6.0s_{\rm eg} + 3.1s)^{22.5} \times \exp[-(b + 6.0s_{\rm eg} + 3.1s)] \cdot \mathcal{L}_{\rm EG}(s_{\rm eg})ds_{\rm eg} \tag{9}$$

where the background is equal to the previous case and the coefficients 6.0 and 3.1 are the number of events expected from the southern sky in the case of normalization equal to one at 100 TeV for spectral index $\alpha = 2$ and $\alpha = 2.7$, respectively. The result is $F_{\rm G} = 2.5^{+2.4}_{-1.3}$ at the 68% confidence level. Summarizing, with present information, the best model two-component flux is

$$\frac{d\phi_{\rm S}}{dE_\nu} = \left[2.5\left(\frac{E_\nu}{100\ {\rm TeV}}\right)^{-2.7} + 2.1\left(\frac{E_\nu}{100\ {\rm TeV}}\right)^{-2}\right] \times 10^{-18}\frac{1}{{\rm GeV\ cm^2\ s\ sr}} \tag{10}$$

compared with the fluxes from the northern and southern sky measured by IceCube, which are shown in Figure 3. We show in Figure 3 only the flux that describes the passing muons, which, as discussed above, has a much smaller uncertainty than the flux that describes the HESE from northern sky, and it is perfectly compatible with the latter. The red band is simply obtained by propagating the uncertainties on the normalization, since the shape is fixed. From this figure, the following two features are evident: At low energies, there is a good agreement between the two-component flux and the flux already measured from the southern sky. At high energy, instead, there is a good

Table 1
Comparison of the Two-Component Model with the Events from the Southern Sky with Deposited Energy above 60 TeV of the Four-year Dataset of IceCube

	Gal. $E^{-2.7}$	Extragal. E^{-2}	Backg.	Prompt	Expected	Observed
Tracks	1.2	2.5	1.5	<0.8	5.2	4.5
Showers	6.5	10.1	0.5	<3.1	17.2	18
Total	7.7	12.6	2.0	<3.9	22.3	22.5
Fraction	34.5%	56.5%	9%			

Note. For the galactic and the extragalactic components the best-fit value are used; see Equation (10). For the prompt neutrinos the number is at the 90% confidence level and the best-fit value is 0.

agreement between the two-component flux and the flux already measured from the northern sky.

Using the best-fit values, the number of events from the southern sky due to the extragalactic component is 12.6 (56.5%), whereas the number of events due to the galactic component is 7.7 (34.5%). The expected number of events are calculated using the IceCube effective areas $A^{\ell}_{\rm eff}$ (where ℓ denotes the flavor) of the southern sky (see footnote 7), and the fluxes found in this section, separating the contribution of the extragalactic and galactic components as follows.

$$N_{\rm EG,G} = 2\pi\ T \int_0^\infty \frac{1}{3}\frac{d\phi_{\rm EG,G}}{dE_\nu}(A^e_{\rm eff} + A^\mu_{\rm eff} + A^\tau_{\rm eff})dE_\nu \quad (11)$$

where $T = 4$ yr is the exposure time. We suppose that the flux for each flavor is one-third of the total flux, as expected in good approximation from a production mechanism due to pion decay, after the occurrence of neutrino oscillations.

Due to the great uncertainties on the extragalactic component, i.e., the flux from the northern sky, the normalization of the galactic component (at 100 TeV) can fluctuate inside the interval between 1.2 and 4.9, giving a contribution between 17% and 67%, to the number of events observed from the southern sky. In Table 1 the number of events from the southern sky observed by IceCube over four years is compared with theoretical prediction of the two-component neutrino flux.

3.2.2. Test with IceCube

In our hypothesis, a large fraction of the low-energy events seen by IceCube have to come from a region close to the Galactic disk. The primary test is just the study of the angular distribution of the HESE events detected by IceCube in the southern sky, which is the region where galactic events are supposed to lie.

We begin by describing the model for the angular distribution. We use the galactic coordinates b (latitude) and ℓ (longitude) to identify the events, and consider the distribution in the variable

$$x = \sin(b). \quad (12)$$

First of all, we are interested in considering one population of galactic neutrino events characterized by a low galactic latitude and localized mostly in the southern sky. We assume the (normalized) differential distribution,

$$\lambda(x) = \frac{e^{-\frac{x^2}{2\delta x^2}}}{\sqrt{2\pi}\ \delta x} \text{with } \delta x = \sin 10° \quad (13)$$

where we have fixed the value of angular extent of this population, δx, in view of IceCube's angular resolution for shower events. For isotropic events, instead, we expect a flat distribution in x but localized in the southern sky. So we have $dN/d\Omega = \Theta(\boldsymbol{su})/(2\pi)$, where $d\Omega = d\ell\ d\sin(b)$, Θ is the Heaviside function, $\boldsymbol{u} = (\cos b\cos\ell, \cos b\sin\ell, \sin b)$ is the direction of the event in galactic coordinates, and $\boldsymbol{s}$is the direction of the celestial south pole.[9] Integrating $dN/d\Omega$ over ℓ we get

$$\mu(x) = \frac{1}{\pi}{\rm Re}\left[\arccos\left(-\frac{x}{\sqrt{1-x^2}} \times \tan(b_{\rm CS})\right)\right]$$
$$\text{with } \tan(b_{\rm CS}) = -0.512. \quad (14)$$

We can use the normalized angular distribution

$$\rho(x, f) = f\ \lambda(x) + (1-f)\ \mu(x) \text{ with } 0 \leqslant f \leqslant 1 \quad (15)$$

to model the cases where one or both components are present, depending on the value of the fraction of events due to the galactic component, f.

The IceCube data (of which we present some graphical summaries in Figure 4) can be compared with this model. The results are as follows.

(a) When we fit to the data using the χ^2 described in the Appendix, we find $f = 0.26 \pm 0.15$. This is consistent with the previous independent determination obtained with the spectral information, which gave $f = 0.35^{+0.32}_{-0.18}$. Note that with the angular analysis the uncertainty decreases.

(b) Moreover, we have tested the goodness of fit for the three hypothesized angular distributions: (1) the extragalactic (isotropic) distribution, (2) the purely galactic (low latitude distribution), and (3) the mixed model (with both components). It should be noted that the model with mixed composition is not the result of a fit to the angular data, but it is determined independently from the fit of the energy spectrum. We find that all statistical tests in Figure 5 give very similar results. A purely galactic component can be excluded. On the other side, both the isotropic and the mixed model are compatible with the data, although the model with two components fares better. The results of the hypothesis tests are reported in Figure 5, where also a plot is shown with the different cumulative distributions compared with the experimental one.

(c) Finally, we note that our hypothesis implies a correlated angle-energy distribution in IceCube, namely a large fraction of the low-energy events seen by IceCube in the southern sky have to come from a region close to the Galactic disk. The

[9] Its galactic latitude $b_{\rm CS}$ equals the declination of galactic south (in equatorial coordinates) $b_{\rm CS} = \delta_{\rm GS} = -27°.13$.

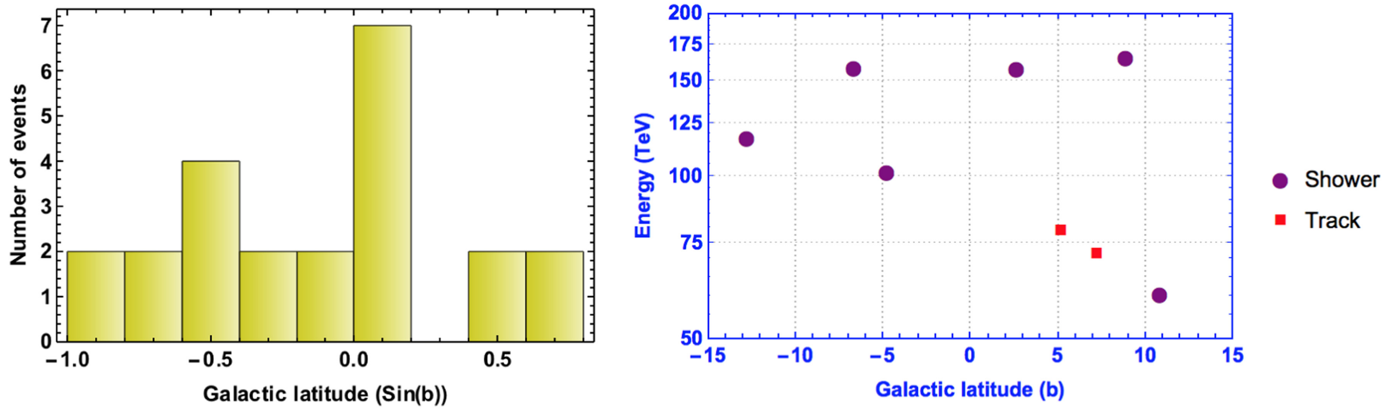

Figure 4. Two presentations of events seen by IceCube in the southern sky and with deposited energy above 60 TeV. In the left panel we show the galactic latitude of all events. In the right panel we show those with energy below 200 TeV and in the region close to the Galactic plane.

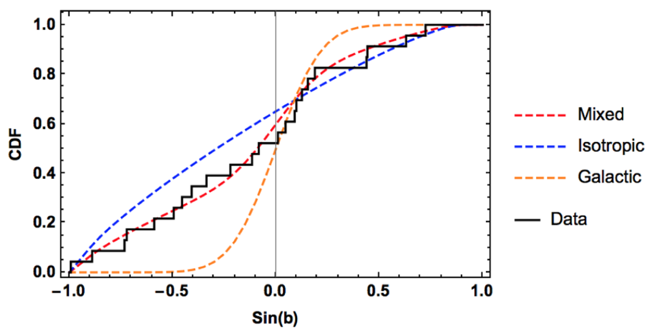

Figure 5. Results of different hypothesis tests for isotropic, galactic, and mixed models. In the bottom of the panel, the comparison is made between the cumulative density function of the three models and southern sky data above 60 TeV.

distribution of the data, given in Figure 4, shows that this is indeed the case. In fact, there are 8 events below 200 TeV and with $|b| < 15°$. This is not so far to the expectation of 7.7 events, that one reads from Table 1.

Summarizing, we have performed various independent tests and analyses of the angular distribution of the events observed by IceCube. Our results are slightly more conservative but consistent with those obtained by Troitsky (2015) and do not contradict the ones of Neronov & Semikoz (2016; see Section 2.1). The main findings are three: (1) The angular distribution of the southern sky (HESE) events is in excellent agreement with the hypothesis of a two-component distribution, formulated previously. (2) A purely extragalactic (isotropic) distribution cannot be excluded by the existing observations of the angular distribution. (3) The fraction of galactic neutrino events can be independently determined using only the spectra or using only the angular distribution; the two determinations are in agreement, and the second method gives a better estimation of the galactic contribution because the uncertainty is smaller. The interest in significantly increasing the present statistics is quite evident.

3.2.3. Test with ANTARES

The data of ANTARES are important to probe the hypothesis of a Galactic component, as remarked in Spurio (2014). A search for an excess of through-going muons was performed using 1288 days of data, in the region $|b| < 4\overset{\circ}{.}5$ and $|l| < 39°$ around the plane of the Milky Way (Visser 2015; Visser & ANTARES Collaboration 2015). A non-significant excess was seen and a bound was thus derived. Since this bound is 1.9 times tighter than the signal expected assuming that all IceCube events come from this region (see Equation (5.25) in Visser 1989) this extreme hypothesis is disfavored by the ANTARES results.

In our hypothesis only a fraction of the high-energy neutrino signal derives from the region in the vicinity of the Galactic plane. An approximate Gaussian description of Visser's (1989) results is given by $\chi^2(f) \approx 30(f - 1/6)^2$, where f is the fraction of the events in the search region used by ANTARES. There is no contradiction with the hypothesis that a large part of the Galactic neutrino signal is contained in the region investigated by ANTARES. Moreover, the total fraction of Galactic events could be larger if the emitting region is larger. It is extremely interesting to continue the search for a diffuse neutrino signal in the surroundings of the Galactic plane.

3.3. Extragalactic Component

In this section, we show that the hypothesis of an extragalactic flux of neutrino E^{-2} is in good agreement with the theory and with the observations.

3.3.1. Comparison with the Theory

We are not interested in considering a specific model here, but it is useful to reference a recent work in this field, where the plausibility of the the above hypothesis is clearly assessed. In the Kalashev et al. (2015), it is shown that a power-law spectrum E^{-2}, in an energy range between $E_{\min} \simeq 100$ TeV and some PeV, can be obtained in the context of AGNs. At the hearth of an AGN resides a super-massive black hole, surrounded by a hot accretion disk that emits thermal radiation. The AGN can accelerate protons up to highest energies and photo-nuclear reactions with subsequent neutrino emission can occur. Considering the interaction between accelerated protons and radiation with energy of 10^2–10^3 eV, it is possible to obtain an E^{-2} flux of neutrinos, as illustrated in Figure 4 of Kalashev et al. (2015), where these kinds of models are exhaustively discussed.

3.3.2. Comparison with the Observations at the Highest Energies

In order to check the validity of our assumption, the theoretical prediction has to be compared with the observed data at high energy, where the extragalactic component

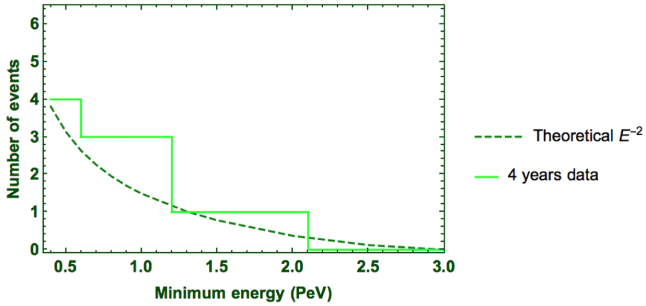

Figure 6. Comparison between theoretical prediction and data observed between a minimum energy and 3 PeV. The line represents the energy in which the number of events between $E_{\min}$ and 3 PeV decreases by 1.

dominates. Let us analyze the number of showers, observed by IceCube over four years, between a minimum energy $E_{\min}$ and a certain maximum energy. The latter is chosen to be $E_{\min} = 3$ PeV, in order to include all the events observed by IceCube and to avoid the Glashow resonance, that will be discussed in a separate section. In order to analyze events, it is convenient to examine the cumulative function, which is more appropriate for analyzing sparse events. The comparison with the predictions can be done using the procedure described in Palladino et al. (2015a). Let us recall that the showers are mostly given by CC interactions of ν_e and ν_τ. The theoretical prediction is given by the formula

$$N_S(E_{\min}) = 4\pi\ T \int_0^{3\text{ PeV}} \frac{1}{3}\frac{d\phi_{\text{EG}}}{dE_\nu} A_{\text{eff}}^{\tau}\ [r(E_{\min}, E_\nu) + r(E_{\min} \times 0.8, E_\nu)]\ dE_\nu \quad (16)$$

where $r(E_{\min}, E_\nu)$ is an energy resolution function defined in Palladino et al. (2015a) and each flavor carries a fraction one-third of the total flux $d\phi_{\text{EG}}/dE_\nu$ defined previously. In this case the effective areas are the ones over 4π and not the effective areas of the southern sky. The results are illustrated in Figure 6 where the minimum energy is 400 TeV, namely already in the region where the extragalactic component dominates.

The spectrum E^{-2} does not contradict the data at very high energy and fits the experimental data well until 3 PeV. A softer spectrum works well at low energies but it has trouble explaining the three events above 1 PeV. Let us remark that with the cumulative analysis there is no lack of events between 0.5 and 1 PeV, a feature that has been emphasized in many theoretical works. The lack is simply related to the choice of the bins and the low statistic of the events and for this reason it is not necessary to introduce an artificial component to explain the deficit of events.

4. PREDICTIONS

Here we will summarize our hypothesis. The high-energy neutrinos are distributed according to a double component, with a softer spectrum below some 100 TeV, where the galactic component dominates, and a harder spectrum above $\simeq$0.5 PeV, where the extragalactic component gives the main contribution. The latter is distributed as E^{-2} while the second is distributed as $E^{-2.7}$. For what concerns the angular distribution, the extragalactic component is distributed isotropically whereas the galactic component is distributed around the Milky Way center and plane; the precise distribution (or, in first approximation, the extension of this component galactic latitude and longitude) will be a matter for observational investigations.

There are many interesting consequences of this hypothesis, for instance:

(1) In the next years an event at Glashow resonance due to the extragalactic component must be observed.

(2) The observation of double pulse events will provide the definitive proof of the cosmic origin of high-energy neutrinos. Evidently, this will be very important, even if it is not a specific test of our hypothesis.

(3) The difference of the track-to-shower ratio of the two components can be measured with IceCube.

(4) There must also be high-energy gammas associated with the galactic neutrinos.

(5) In order to measure precisely the galactic component at low energy, an important contribution could come from telescopes located in the Northern Hemisphere, which has the possibility of measuring the neutrinos from the southern sky by means of passing muons, namely in a more clean way than the HESE, which is instead the observational tool exploited by IceCube.

We will discuss these tests of validation in this section.

4.1. Showers above 3 PeV and Glashow Resonance

Using our estimation of the extragalactic flux, over four years 0.9 events due to deep inelastic scattering and 0.7 due to Glashow resonance ($p\gamma$ hypothesis) are expected. That means a total of 1.6 events not yet observed by IceCube. At the present, this theoretical excess is not important and its significance is of 1.3σ. Following the approach of Palladino et al. (2015a) it is possible to evaluate the number of years required to observe a shower above 3 PeV with a probability greater than 95%, which corresponds to 2σ in a Gaussian approach. In the case of the $p\gamma$ mechanism of production, the number of events per year is about $\mu = 0.4$. Using the Poissonian statistic, the probability of observing at least one event is given by $P(n > 0) = 1 - \exp(-\mu)$, which means about eight years for a probability greater than 95%. So four more years are necessary to confirm our hypothesis or to disfavor it at 2σ of significance. It is important to notice that in the case of $\alpha = 2.5$ more than 40 yr are required to observe at least one shower above 3 PeV, with a confidence level $\geqslant$ 95%; so there is an important difference between the single-power-law predictions and the two-component spectrum prediction. The contribution of the galactic component to this class of events is negligible, due to the harder spectrum.

4.2. Double Pulse

The double pulse is generated by a ν_τ that interacts in CC, as clarified in Aartsen et al. (2015a) and further quantified in Palladino et al. (2015a). In the hypothesis of $p\gamma$ mechanism of production, neutrino oscillation gives an about equal amount of ν_e, ν_μ, and ν_τ at the Earth. Our assumption on the extragalactic component of neutrinos implies that the expected number of events per years is $\simeq$0.12. Using the same considerations of the previous section, about 20 yr are required to observe a double pulse with a confidence level $\geqslant$ 95% and with today's technology. Recall that the double pulse is less sensitive to the shape of the neutrino spectrum with respect to the Glashow resonance events and cascades above 3 PeV because about half of the signal is given by the already observed neutrinos, with

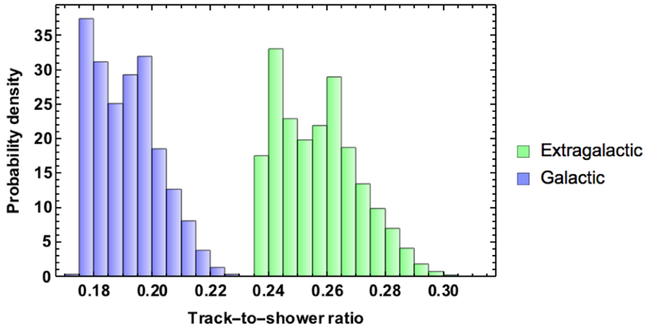

Figure 7. Comparison between the track-to-shower ratio of galactic and extragalactic components, assuming the same mechanism of production, i.e., pion decay.

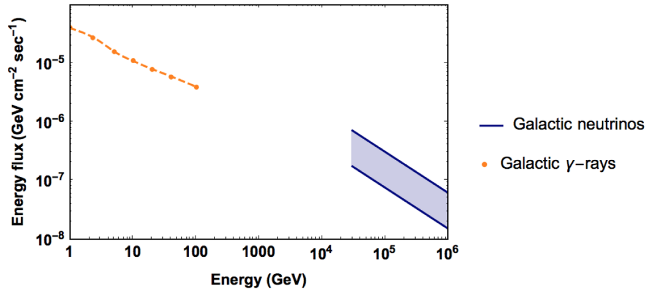

Figure 8. In orange, the spectrum of diffuse gamma-ray emission from the inner Galaxy ($-80° \leqslant l \leqslant 80°$, $-8° \leqslant b \leqslant 8°$), measured by *Fermi*-LAT. In blue, the spectrum of galactic neutrinos with their uncertainties.

energy between 100 TeV and 1 PeV (Palladino et al. 2015a). Note that the galactic component gives a small contribution to the double pulse events, i.e., $\simeq$0.03 expected events per year.

4.3. The Track-to-shower Ratio

It is important to notice that the track-to-shower ratio for the galactic and the extragalactic components is different, even if the mechanism of production is the same—we refer to the standard case of pion decay in our paper. This remains true despite the uncertainties on the oscillation parameters, as can be seen in Figure 7. This difference is due to the different spectral shape, since the larger spectral index (namely the case of galactic ν) penalizes track events.

4.4. Implication for γ Astronomy

The galactic component of neutrinos implies the existence of a galactic component of very high-energy gamma-rays. These gamma-rays can be observed because their path in the Galaxy is less than their mean free path. The same consideration is not true for the extragalactic gamma-rays associated with neutrinos because when the energy is above 10 TeV they are absorbed after about 100 Mpc, due to the interaction with the extragalactic background light. It is possible to give a lower limit on the flux of gammas, which corresponds to the case of *pp* interaction at the source. The ratio between the flux of gamma-rays and the flux of neutrinos is given by the following equation based on Villante & Vissani (2008),

$$\frac{\phi_\gamma}{\phi_\nu} = 1.009 + 0.622(\alpha - 2.3) + 0.140(\alpha - 2.3)^2 + 0.009(\alpha - 2.3)^3 \quad (17)$$

which takes the value $\phi_\gamma/\phi_\nu = 1.28$ if $\alpha = 2.7$. Therefore, the flux of galactic gamma must be of the same order of the neutrino flux, more precisely,

$$\phi_\gamma(E) \geqslant 1.28\ \phi_{\rm G}(E). \quad (18)$$

Let us notice that the cutoff on the ϕ_γ is at higher energy with respect to the cutoff of the galactic neutrino spectrum. In fact, the energy of γ-rays is typically $E_\pi/2$, whereas the energy of neutrinos is $\simeq E_\pi/4$ (see Kelner et al. 2006 for a complete description of the secondary neutrinos spectrum). The relationship between galactic HE neutrinos and HE γ-rays is also discussed in Sahakyan (2015), focusing on SNRs, PWN, and binary systems. Recall also that in the case of a binary system, if the γ-rays are produced from protons, the flux of HE neutrinos can significantly exceed that of γ-rays (by a factor of e^τ where τ is the optical depth) considering that they escape from the region without absorption, unlike the γ-rays.

In the other case, the flux of gamma must be greater than the flux of neutrinos to have a consistent theory. At present there are no measurements of the gamma-rays in the same energy range of neutrinos (above 30 TeV), so a direct comparison is not possible. Figure 8 shows the diffuse flux of galactic gamma-rays (see Abdo et al. 2009; Huntemeyer & Milagro Collaboration 2010) and the estimated flux of galactic neutrinos in the energy range in which they are already measured. No firm conclusion can be derived at present from these results.

4.5. Future km^3-class Telescopes in the Northern Hemisphere

Our hypothesis requires that, in the vicinity of the galactic disk, there is a population of high-energy neutrinos with energies below a few 100 of TeV. The present data are compatible with a large fraction of events from this origin. In our reference hypothesis, the fraction is about 30% but with great uncertainties, since the 1σ interval includes values that differ by a factor of two (above or below). This means that the existence of a galactic component is reasonable even if today it is hard to quantify it precisely; future experiments in the Northern Hemisphere will be necessary to prove its existence and to measure this component precisely.

Let us remark that the $\simeq$30% refers to the fraction of supposed galactic events with respect to the total number of events observed from the southern sky. If all the events (both from south and north) are considered, the galactic contribution becomes 24% at best fit. However, let us repeat that the precise extension of this signal (in angle and in energy) is not known and this requires further experimental investigations. In order to validate this assumption, neutrino telescopes of sufficient volume and located in the Northern Hemisphere are needed.

The KM3NeT (Adrián-Martínez et al. 2016) and the BAIKAL-GVD (Avrorin et al.) are such kinds of experiments; the first will be in the Mediterranean Sea, the second one in Lake Baikal. They will be in the right geographical position to validate the presence of a diffuse galactic neutrino component, thanks to the possibility of using passing muons of relatively low energies. Note that in water the angular resolution is much better than in ice and it attains sub-degrees (Adrián-Martínez et al. 2016). In this manner, the precise angular extension will be investigated. Moreover, also the HESE below a few 100 TeV will be useful for the same purpose.

Also, the extragalactic component can be measured with a similar precision to the IceCube experiment using the HESE in the energy range above some 100 TeV, where the background due to atmospheric neutrinos is negligible.

5. DISCUSSION AND SUMMARY

The main goal of IceCube has been to show conclusively that high-energy neutrinos exist. To consolidate this goal, a simple assumption proved to be reasonably appropriate to summarize the data and the present understanding, namely, the assumption of a single new population, distributed isotropically and with a power-law spectrum with slope significantly different from $\alpha = 2$. This is a minimal modification of the hypothesis suggested by the scenario of extragalactic origin, but it leads to a certain number of issues, examined and discussed in Section 2.

However, there is no strong theoretical reason to omit the galactic contribution to the observed high-energy neutrinos; on the contrary, astronomical data, gamma-rays included, suggest including it. We have shown that the inclusion of another significant component of the high-energy neutrino flux, with the slope of the observed galactic cosmic-ray spectrum, and with an extension up to a few 100 TeV, is consistent with the known IceCube data.

The angular distribution of the total galactic contribution (including point sources and diffuse component) is not precisely known but the galactic latitude should be within $|b| < (5\text{–}20)^\circ$ and the galactic longitude extends from $|l| < 40^\circ$ or an even wider region. Although we would like to observe point sources eventually, the high-energy neutrino astronomy is in a similar position to the ordinary one more than 200 yr ago, at the time of T. Wright, I. Kant, and W. Herschel, when the shape of the Milky Way was understood. It should be stressed, however, that neutrinos will give unique information on galactic and extragalactic cosmic rays.

We have shown that the dominant extragalactic component predicts observable events of a new type with IceCube, those from Glashow resonance and those of a double-pulse type. We have emphasized that the galactic component is more important at comparably low energies (below a few 100 TeV) and can be observed by exploiting through-going muons from the Northern Hemisphere. It leads to a peculiar flavor ratio and should be accompanied by a (diffuse) emission of very high-energy gamma-rays, calculable from the neutrinos, and extended up to a few 100 TeV.

In the near future, a combined analysis of the data of IceCube and ANTARES will permit us to obtain more precise inferences on the parameters of the two-component model. It is important to emphasize that within this model, the angular features and those of the energy spectrum are closely entangled and should be analyzed together.

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APPENDIX
STATISTICAL PROCEDURES

In the main text, we considered various questions: (1) We want to test whether or not two distributions, assumed to be power law, but measured by means of two different data sets, are compatible among them. (2) We want to study the angular distribution of the events from the southern sky and to obtain the optimal combination of galactic and extragalactic neutrino fluxes from the angular distribution of the data. Below, we describe the procedures we have followed to answer them.

A.1. Compatibility of Two Measurements

We have two measurements of one quantity x that are summarized by two likelihood functions $\ell_1(x)$ and $\ell_2(x)$. We want to test whether the two measurements are compatible or not. Therefore, we introduce the variable that quantifies the difference of values δx and associate to this variable a new likelihood function by mean of the following overlap integral,

$$\ell(\delta x) = \int_{-\infty}^{+\infty} dx\ \ell_1(x)\ell_2(x - \delta x) \tag{19}$$

where the integrals are taken over the range of variation of the variable x that we assume to be the whole real axis. In the common case when the two likelihoods are approximated by Gaussian functions, $\ell_i(x) = g(x, \mu_i, \sigma_i)$, where

$$g(x, \mu, \sigma) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma} \tag{20}$$

it is straightforward to show that

$$\ell(\delta x) = g(\delta x, \mu_1 - \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2}). \tag{21}$$

This result is very reasonable and admits the following interpretation: the value $\delta x = 0$, which can be considered as the case when there is no difference between the two likelihoods, deviates from the best-fit value by the following "numbers of sigma,"

$$N_\sigma = \frac{|\mu_1 - \mu_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}}. \tag{22}$$

Supposing $\mu_1 > \mu_2$ (and quite similarly if $\mu_1 < \mu_2$) we can use the tail of the likelihood ℓ to estimate the statistical significance of a similar (or stronger) discrepancy,

$$p\text{-value} = 2\int_{-\infty}^{0} dy\ \ell(y), \tag{23}$$

which corresponds to N_σ if the Gaussian case applies and of course should be used only when the outcome is less than 1. This quantifies the significance of the null hypothesis, that the two measurements are compatible. If the function ℓ does not deviate strongly from a Gaussian distribution, we can generalize the procedure and use the above p-value to quantify the difference between two likelihood functions with different medians $\mu_1 > \mu_2$.

A.2. Analysis of a Two-component Model

Suppose that we have a set of measurements of a certain observable quantity x, with values $x = x_1, x_2, x_3...$, and know that they come from two different populations, with known distributions $\lambda(x)$ and $\mu(x)$ normalized to 1. We ask which is the optimal combination of these distributions that reproduces the set of measurements. This can be answered introducing the fraction of events of the first type $0 \leqslant f \leqslant 1$, corresponding to a fraction of $1-f$ events of the second type, and using the following χ^2:

$$\exp\left(-\frac{\chi^2(f)}{2}\right)=\prod_i \ [f\lambda_i+(1-f)\mu_i] \quad \text{where } \lambda_i=\lambda(x_i) \text{ and } \mu_i=\mu(x_i). \qquad (24)$$

This χ^2 can be used to estimate the fraction f. Supposing that there is a minimum for some value $f=\bar{f}$ internal to the physical interval $0<\bar{f}<1$ and such that

$$\chi^2(f)=\text{constant}+\frac{(f-\bar{f})^2}{\delta f^2}+o((f-\bar{f})^2), \qquad (25)$$

it is easy to obtain the following analytical expressions:

$$\sum_i \xi_i(\bar{f})=0 \text{ and } \delta f^2 = \frac{1}{\sum_i \xi_i(\bar{f})^2} \text{ where } \xi_i(f)=\frac{\lambda_i-\mu_i}{\mu_i+f(\lambda_i-\mu_i)}. \qquad (26)$$

The two quantities $\bar{f}$ and δf characterize which is the optimal combination of the two populations that resembles more closely the given set of measurements, and they are usually quoted as $f=\bar{f}\pm\delta f$.

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