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Mathematical Theory of Networks and Systems

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Technical Program for Tuesday July 12, 2016

TuPL1	Willey 175
Prof. Caines Plenary (PL1)	Plenary Session

09:00-10:00, Paper TuPL1.1

Mean Field Games and the Control of Large Scale Systems: An Overview

Caines, Peter E. — McGill Univ

Keywords: Large Scale Systems

Abstract: Mean Field Game (MFG) theory provides tractable strategies for the decentralized control of large scale systems. The power of the formulation arises from the relative tractability of its infinite population McKean-Vlasov (MV) Hamilton-Jacobi-Bellman equations and the associated MV-Fokker-Planck-Kolmogorov equations, where these are linked by the distribution of the state of a generic agent, otherwise known as the system's mean field. The resulting decentralized feedback controls yield approximate Nash equilibria and depend only upon an agent's state and the mean field. Applications of MFG theory are being investigated within engineering, finance, economics and social dynamics, while theoretical developments include existence and uniqueness theory for solutions to the MFG equations, major-minor (MM) agent systems containing asymptotically non-negligible agents, non-linear estimation theory for MM-MFG systems, and the comparison of centralized (optimal) control and MFG control performance.

TuA01	Blegen 110
Constructions of Strict Lyapunov Functions: Stability, Robustness, Delays, and State Constraints	Mini-course session

Chair: Malisoff, Michael — Louisiana State Univ

Co-Chair: Mazenc, Frederic — INRIA-SUPELEC

10:25-12:30, Paper TuA01.1

PDF *Constructions of Strict Lyapunov Functions: Stability, Robustness, Delays, and State Constraints*

Mazenc, Frederic — INRIA-SUPELEC
Malisoff, Michael — Louisiana State Univ

Keywords: Stability, Feedback Control Systems, Delay Systems

Abstract: This mini-course presents several developments involving constructions of strict Lyapunov functions for systems with uncertainties, state constraints, or input delays. It will be understandable and beneficial to those who are familiar with the material in a standard graduate course on nonlinear control systems. The three 50-minute lectures are entitled as follows: Matrosov's Approach, Lyapunov-Krasovskii Methods, and Robust Forward Invariance.

TuA02	Blegen 120
Composite Control of Networks Via Singular Perturbation Theory	Mini-course session

Chair: Boker, Almuatazbellah M — Marshall Univ

10:25-12:30, Paper TuA02.1

PDF *Composite Control of Networks Via Singular Perturbation Theory*

Boker, Almuatazbellah M — Marshall Univ
Chakrabortty, Aranya — NC State Univ
Nudell, Thomas — North Carolina State Univ

Keywords: Networked Control Systems, Large Scale Systems, Systems on Graphs

Abstract: Our goal in this mini-course is to educate control engineers about modeling and control of multi-time-scale

networked systems, to formulate problems on making the control systems distributed and scalable, and to point out how singular perturbation theory can play an instrumental role in solving these problems. Our target audience for this course are faculty members, postdoctoral researchers, graduate students, and industry practitioners who want to learn about the recent advancements in the field of control of clustered networks and the constructive role that singular perturbation theory can play in this field. The session will provide an insightful thrust to network system education for control engineers, especially for new graduate students who are looking for fresh research problems.

TuA03	Blegen 130
Distributed Parameter Systems-I	Invited Session
Chair: Demetriou, Michael A.	Worcester Pol. Inst
Co-Chair: Jacob, Birgit	Bergische Univ. Wuppertal
Organizer: Demetriou, Michael A.	Worcester Pol. Inst
Organizer: Jacob, Birgit	Bergische Univ. Wuppertal
Organizer: Fahroo, Fariba	Naval Postgraduate School

10:25-10:50, Paper TuA03.1

Distributed Parameter Systems I

Demetriou, Michael A.	Worcester Pol. Inst
Jacob, Birgit	Bergische Univ. Wuppertal
Fahroo, Fariba	Naval Postgraduate School

10:50-11:15, Paper TuA03.2

Optimal Actuator Design with a Linear-Quadratic Performance Measure (I)

Morris, Kirsten A.	Univ. of Waterloo
Vest, Ambroise	Lycée Henri Poincaré

Keywords: Control of Distributed Parameter Systems, Optimization : Theory and Algorithms

Abstract: In control of distributed parameter systems (DPS's), the location, size, distribution and number of actuating and sensing devices constitute a challenging design problem. For instance, in control of flexible structures and acoustic noise reduction, both the type of actuators and sensors, as well as their locations, can be chosen. Furthermore, due to advances in materials, the shape of the hardware is sometimes also a design variable. Since the final system is affected by the controller design, it is sensible to design the actuators using the same criterion as used to design the controller.

Previous work on optimal actuator location is here generalized in several different important directions. First, the control operator is not assumed fixed, which allows for the consideration of such problems as optimizing the shape of a smart material used as an actuator. Also, in earlier work, it was assumed that the actuator did not affect the internal dynamics of the system. This assumption is relaxed here, allowing for the consideration of a wider class of problems, for instance, those where the actuator has a non-negligable mass.

11:15-11:40, Paper TuA03.3

Lyapunov Stabilization Via Boundary Control for Scalar Conservation Laws (I)

Blandin, Sebastien	IBM Res
Litrico, Xavier	Cemagref
Delle Monache, Maria Laura	Rutgers Univ
Piccoli, Benedetto	Rutgers Univ
Bayen, Alexandre M.	Univ. of California at Berkeley

Keywords: Control of Distributed Parameter Systems, Stability, Infinite Dimensional Systems Theory

Abstract: We consider the problem of Lyapunov boundary stabilization of the weak entropy solution to a scalar conservation law with strictly convex flux in one dimension of space, around a uniform equilibrium. We show that for a specific class of boundary conditions, the solution to the initial-boundary value problem for an initial condition with bounded variations can be approximated arbitrarily closely in the L^{1} norm by a piecewise smooth solution with finitely many discontinuities. The constructive method we present designs explicit boundary conditions in this class, which guarantee Lyapunov stability of the weak entropy solution to the initial-boundary value problem. The stabilization

result accounts for the proper treatment of boundary conditions in the weak sense. We also design a greedy controller obtained by maximizing the instantaneous decrease rate of the Lyapunov function, and illustrate the limitations of such simple controllers. Finally, we design improved boundary feedback controller which guarantees Lyapunov asymptotic stability while accounting for proper treatment of weak boundary conditions. Controllers performance is illustrated on numerical benchmarks using the Godunov scheme.

11:40-12:05, Paper TuA03.4

PDF *Optimal Control Locations for Time-Varying Systems on a Finite-Time Horizon (I)*

Jacob, Birgit	Bergische Univ. Wuppertal
Wu, Xueran	Forschungszentrum Juelich/Univ. of Wuppertal
Elbern, Hendrik	RIU at the Univ. of Cologne/Forschungszentrum Juelich

Keywords: Infinite Dimensional Systems Theory, Control of Distributed Parameter Systems

Abstract: The choice of the location of controllers is of great importance for designing control systems. We study the linear-quadratic optimal location control problem for deterministic as well as stochasticsystems and develop conditions guaranteeing the existence of optimal locations of linear quadratic control problems. Associated with practical applications, since optimal control problems cannot be solved directly in infinite-dimensional spaces, a sequence of approximations of the original time-varying system have to be considered. Thus, we introduce approximation conditions for evolution operators to ensure that the approximated control problems converge to the optimal control problem of the original infinite-dimensional time- varying system. Further, we show the convergence of minimal costs and optimal locations of the sequence of approximations.

12:05-12:30, Paper TuA03.5

PDF *Optimal Sensor Selection for Optimal Filtering of Spatially Distributed Processes (I)*

Demetriou, Michael A.	Worcester Pol. Inst
Fahroo, Fariba	Naval Postgraduate School

Keywords: Control of Distributed Parameter Systems, Infinite Dimensional Systems Theory, Optimization : Theory and Algorithms

Abstract: We consider the problem of the integrated design of an optimal state estimator and its associated optimal sensor positioning for a class of distributed parameter systems. The optimal estimator is parameterized by the admissible sensor locations and an appropriate performance measure is used to obtain the optimal sensor location. It turns out that this performance measure is given by the trace of a location parameterized solution to an operator Riccati equation. The conditions ensuring that the optimal sensor of the finite dimensional optimization problem converges to the optimal sensor of the infinite dimensional optimization problem are summarized. Studies of the effects of the spatial distribution of the process noise on 1D diffusion PDE are performed in order to shed light on the effects of the optimal sensor location.

TuA04	Blegen 135
Algebraic Methods and Symbolic-Numeric Computation in Systems Theory-I	Invited Session
Co-Chair: Zerz, Eva	RWTH Aachen Univ
Organizer: Quadrat, Alban	INRIA Lille
Organizer: Zerz, Eva	RWTH Aachen Univ

10:25-10:50, Paper TuA04.1

PDF *Musical Wind Instruments, Graphs and Orthogonal Polynomials (I)*

Le Vey, Georges	Ec. Des Mines De Nantes/IRCCyN CNRS

Keywords: Physical Systems Theory, Systems on Graphs

Abstract: Orthogonal polynomials are ubiquitous in mathematical physics since the XIX^{rm th} century and are still an active research field. In this work relationship between discrete acoustical resonators, harmonicity of resonance frequencies and ultraspherical polynomials is established. It relies upon recent work done by the author on modelling musical wind instruments within graph theory. The approach originates from the mathematical study of evolution equations on networks. It furnishes closed-form expressions, even for complex instruments, useful for musical acoustics purposes. When searching for resonators with harmonicity property, it was found that a huge class of resonators meet the requirement. Thus, the additional property of orthogonality is imposed for restricting this class

and related consequences are investigated.

10:50-11:15, Paper TuA04.2

A Symbolic-Numeric Method for the Parametric H_∞ Loop-Shaping Design Problem (I)

Rance, Guillaume — Sagem Defense Securite
Bouzidi, Yacine — INRIA
Quadrat, Alban — INRIA Lille
Quadrat, Arnaud — Sagem Ds

Keywords: Robust and H-Infinity Control, Numerical and Symbolic, Optimal Control

Abstract: In this paper, we present a symbolic-numeric method for solving the H_∞ loop-shaping design problem for low order single-input single-output systems with parameters. Due to the system parameters, no purely numerical algorithm can indeed solve the problem. Using Gröbner basis techniques and the Rational Univariate Representation of zero-dimensional algebraic varieties, we first give a parametrization of all the solutions of the two Algebraic Riccati Equations associated with the H_∞ control problem. Then, a certified symbolic-numeric algorithm is obtained for the computation of the positive definite solutions of these two Algebraic Riccati Equations. Finally, we present a certified symbolic-numeric algorithm which solves the Hinf loop-shaping design problem for the above class of systems. This algorithm is illustrated with a standard example.

11:15-11:40, Paper TuA04.3

Modelling and Structural Properties of Distributed Parameter Wind Power Systems (I)

Mounier, Hugues — Univ. Paris Sud 11
Greco, Luca — Univ. Paris Sud

Keywords: Algebraic Systems Theory, Control of Distributed Parameter Systems, Delay Systems

Abstract: We examine two distributed parameter models for strings of generators connected to a wind farm. We show that these models boil down to delay systems either with or without continuous dynamics, depending on the type of the chosen boundary conditions. We then investigate the differential flatness of the systems.

11:40-12:05, Paper TuA04.4

Certified Algorithms for Proving the Structural Stability of Two Dimensional Systems Possibly with Parameters (I)

Bouzidi, Yacine — INRIA
Rouillier, Fabrice — INRIA

Keywords: Algebraic Systems Theory, Multidimensional Systems, Numerical and Symbolic

Abstract: In a recent work by Bouzidi et al., a new method for testing the structural stability of multidimensional systems has been presented. The key idea of this method is to reduce the problem of testing the structural stability to that of deciding if an algebraic set has real points. Following the same idea, we consider in this work the specific case of two-dimensional systems and focus on the practical efficiency aspect. For such systems, the problem of testing the stability is reduced to that of deciding if a bivariate algebraic system with finitely many solutions has real ones. Our first contribution is an algorithm that answers this question while achieving practical efficiency. Our second contribution concerns the stability of two dimensional systems with parameters. More precisely, given a two-dimensional system involving a set of parameters, we present a new algorithm that computes regions of the parameters space in which the considered system is structurally stable.

12:05-12:30, Paper TuA04.5

Algebraic Analysis for the Ore Extension Ring of Differential Time-Varying Delay Operators (I)

Quadrat, Alban — INRIA Lille
Ushirobira, Rosane — Inria

Keywords: Algebraic Systems Theory, Delay Systems, Linear Systems

Abstract: To the best of our knowledge, no algebraic (polynomial) approach exists for the study of linear differential time-delay systems in the case of a (sufficiently regular) time-varying delay. Based on the concept of skew polynomial rings developed by Ore in the 30s, the purpose of this paper is to construct the ring of differential time-delay operators as an Ore extension and to analyze its properties. A characterization of classical algebraic properties

of this ring, such as noetherianity, its homological and Krull dimensions and the existence of Gröbner bases, are given in terms of the time-varying delay function. In conclusion, the algebraic analysis approach to linear systems theory allows us to study linear differential time-varying delay systems (e.g. existence of autonomous elements, controllability, parametrizability, flatness, behavioral approach) through methods coming from module theory, homological algebra and constructive algebra.

TuA05	Blegen 145
Optimization: Theory and Algorithms-I	Regular Session
Chair: Karlsson, Johan	Royal Inst. of Tech. (KTH)

10:25-10:50, Paper TuA05.1

PDF *Finite Model Approximations and Asymptotic Optimality of Quantized Policies in Decentralized Stochastic Control*

Saldi, Naci — Univ. of Illinois at Urbana-Champaign
Yuksel, Serdar — Queen's Univ
Linder, Tamas — Queen's Univ

Keywords: Networked Control Systems, Stochastic Control and Estimation, Optimization : Theory and Algorithms

Abstract: In decentralized stochastic control problems, there is a lack of a universal systematic method to obtain optimal solutions unlike in centralized stochastic control problems and even establishing the existence of optimal policies is a challenging problem with few results available in the literature. The intricacies in decentralized stochastic control have been convincingly exhibited in Witsenhausen's celebrated counterexample which has puzzled the control community for more than 40 years with its philosophical impact demonstrating the challenges that arise due to its non-classical information structure, and the formidable difficulty of obtaining an optimal solution or its approximations. In fact, optimal policies and their value are still unknown, even though the existence of an optimal policy has been established using various methods.

In this paper, we consider finite model approximations of a large class of static and dynamic team problems where these models are constructed through uniform quantization of the observation and action spaces of the agents. The strategies obtained from these finite models are shown to approximate the optimal cost with arbitrary precision under mild technical assumptions. In particular, quantized team policies are asymptotically optimal where the quantization operation involves the uniform quantizers, and thus a finite model is obtained through a constructive algorithm.

This result is then applied to Witsenhausen's celebrated counterexample and the Gaussian relay channel problems. For Witsenhausen's counterexample, our approximation approach provides, to our knowledge, the first rigorously established result stating that one can construct an varepsilon-optimal strategy for any varepsilon > 0 through a solution of a simpler problem.

10:50-11:15, Paper TuA05.2

PDF *A Riemannian Optimization Approach for Role Model Extraction*

Marchand, Melissa — Florida State Univ
Huang, Wen — Florida State Univ
Browet, Arnaud — Univ. Catholique De Louvain
Van Dooren, Paul — Univ. Catholique De Louvain
Gallivan, Kyle — Florida State Univ

Keywords: Numerical and Symbolic, Optimization : Theory and Algorithms

Abstract: The ability to compute meaningful clusters of nodes is important in the analysis of large networks. A particular approach to this problem is the use of role models of a graph. For large networks, the algorithms must be specifically designed to extract role models while maintaining efficiency in storage and computations. Browet et al. have investigated the computation of role models for both moderately sized and large networks. They proposed an efficient iteration on low-rank matrices to compute an approximation to the required pairwise node similarity measure at MTNS 2014. In this paper, we summarize a new approach to compute an approximation to the pairwise node similarity measure for large networks based on Riemannian optimization. A comparison of our optimization approach with that of Browet et al. shows that our approach computes the same approximate solution in significantly less time.

11:15-11:40, Paper TuA05.3

PDF *Seeking Saddle-Points for Non-Convex QCQPs Via the Distributed Optimization Dynamics Approach*

Ma, Xu | Iowa State Univ

Elia, Nicola | Iowa State Univ

Keywords: Optimization : Theory and Algorithms, Stability

Abstract: This paper focuses on solving general, non-convex Quadratically Constrained Quadratic Programmings (QCQPs) via the continuous-time optimization dynamics. In general, any equilibrium to the optimization dynamics is a KKT point of the optimization problem. However, because of the non-convexity of QCQPs, KKT conditions are only necessary for locally optimal solutions. Once the QCQP's KKT point is found, we also have to check whether or not it is globally optimal.

In this paper, we first develop an easily checkable necessary and sufficient condition characterizing when the QCQP's KKT point is a saddle-point. Under this condition, a global optimum can be obtained from the saddle-point. Next we analyze locally asymptotic stability of the saddle-point equilibrium with respect to the optimization dynamics. For certain networked QCQPs, we point out that our approach exhibits an intrinsic distributed computational structure. A MAXCUT example is also given to illustrate the distributedness and effectiveness of our approach.

11:40-12:05, Paper TuA05.4

PDF *Super-Resolution Methods and Metric Uncertainty Via Optimal Transport*

Karlsson, Johan | Royal Inst. of Tech. (KTH)

Ning, Lipeng | Harvard Medical School

Keywords: Optimization : Theory and Algorithms, System Identification, Signal Processing

Abstract: The use of regularization in sparse estimation methods has recently received huge attention impacted virtually all fields of applied mathematics. This interest was sparked by the recovery results of Candès, Donoho, Tao, Tropp, et al. and has resulted in a framework for solving a set of combinatorial problems in polynomial time by using convex relaxation techniques.

In this work we study the use of total-variation regularization methods for inverse problems in high-resolution imaging, widely used in remote sensing. For these problems, the dictionary is typically coherent and existing theory for robust/exact recovery does not apply. In fact, the robustness cannot be guaranteed in the usual strong sense. Instead, we consider metrics inspired by the Monge-Kantorovich transportation problem and show that the solution can be robustly recovered if the original signal is sufficiently sparse and separated.

12:05-12:30, Paper TuA05.5

PDF *Distributed Optimization with Multi-Agent Gradient Information*

Sajjanshetty, Kiran | Univ. of Southern California

Tatikonda, Sekhar | Yale Univ

Keywords: Optimization : Theory and Algorithms, Systems on Graphs, Large Scale Systems

Abstract: We propose a distributed optimization model wherein agents exchange gradient information along with their states to minimize a sum of convex functions. It is shown that the local state update equations converge to the classical gradient descent algorithm if an agent’s degree information is also transmitted. Theoretical results showing the convergence rate estimates for the proposed update rule are provided. Simulation examples with various fixed graph structures are shown which indicate significant improvement in the speed of convergence when compared to the model which uses just the state information of its neighbors.

TuA06 | Blegen 150

Model Reduction-I | Invited Session

Chair: Kawano, Yu | Kyoto Univ

Organizer: Kawano, Yu | Kyoto Univ

Organizer: Scherpen, Jacquelien M.A. | Univ. of Groningen

Organizer: van der Schaft, Arjan J. | Univ. of Groningen

10:25-10:50, Paper TuA06.1

PDF *Algebraic Gramians for Quadratic-Bilinear Systems and Their Application in Model Order Reduction (I)*

Benner, Peter — Max Planck Inst. for Dynamics of Complex Tech. Systems
Goyal, Pawan — Max Planck Inst. for Dynamics of Complex Tech. Systems

Keywords: Computations in Systems Theory, Nonlinear Systems and Control, Large Scale Systems

Abstract: We discuss algebraic Gramians for continuous time quadratic-bilinear (QB) systems in the context of model order reduction (MOR) based on system balancing. MOR for linear systems via balanced truncation requires the computation of the Gramians of the system, namely controllability and observability Gramians, which are extended to the general nonlinear setting in Scherpen, 1993. Therein, it is shown that Gramians for nonlinear systems are solutions to state-dependent Hamilton-Jacobi equations. However, determining solutions to these equations is very challenging in large-scale settings, and also they are not easy to apply in the MOR framework. In this work, we derive alternative controllability and observability Gramians based on the kernels of the Volterra series of QB systems and their Hilbert adjoint QB systems, respectively. We also show connections between Gramians and solutions of Lyapunov equations. Moreover, we relate input/output energy functionals with these algebraic Gramians. Furthermore, we employ these Gramians for balancing of QB systems and for determining reduced-order systems. We illustrate the efficiency of the proposed balancing method by applying it to various systems with dynamics modeled by nonlinear partial differential equations.

10:50-11:15, Paper TuA06.2

Moments at ``discontinuous Signals'' with Applications: Model Reduction for Hybrid Systems and Phasor Transform for Switching Circuits (I)

Scarciotti, Giordano — Imperial Coll. London
Astolfi, Alessandro — Imperial Coll. London

Keywords: Hybrid Systems, Mathematical Theory of Networks and Circuits

Abstract: We provide an overview of the theory and applications of the notion of moment at "discontinuous interpolation signals", i.e. the moments of a system for input signals that do not satisfy a differential equation. After introducing the theoretical framework, which makes use of an integral matrix equation in place of a Sylvester equation, we discuss some applications: the model reduction problem for linear systems at discontinuous signals, the model reduction problem for hybrid systems and the discontinuous phasor transform for the analysis of circuits powered by discontinuous sources.

11:15-11:40, Paper TuA06.3

Kron-Based Model Reduction of Physical Network Systems (I)

van der Schaft, Arjan J. — Univ. of Groningen

Keywords: Physical Systems Theory, Mathematical Theory of Networks and Circuits

Abstract: This work focusses on structure-preserving model reduction of physical network systems using Kron reduction. It is well-known that any Schur complement of a symmetric weighted Laplacian matrix results in another symmetric weighted Laplacian matrix. The same turns out to hold for an asymmetric Laplacian matrix, and for a balanced asymmetric Laplacian matrix. Kron reduction is also shown to have close relations with recently studied notions of effective resistance. Kron reduction yields a method for model reduction of physical network systems, where the Hamiltonian (stored energy) may be non-quadratic, corresponding to nonlinear dynamics. In the present paper close attention is paid to the relation of Kron reduction with singular perturbation balanced model reduction of physical network dynamical systems; in particular for network systems described as gradient or reciprocal port-Hamiltonian systems.

11:40-12:05, Paper TuA06.4

Clustering-Based Model Reduction of Network Systems with Error Bounds (I)

Cheng, Xiaodong — Univ. of Groningen
Kawano, Yu — Kyoto Univ
Scherpen, Jacquelien M.A. — Univ. of Groningen

Keywords: Systems on Graphs, Mathematical Theory of Networks and Circuits, Linear Systems

Abstract: In this paper, we present a structure-preserving model reduction procedure for network systems by means of graph clustering. In our approach, vertices in the graph are iteratively aggregated if they have similar frequency responses to the external inputs. The resulting reduced-order system can still be interpreted as a network system but defined on a graph with fewer vertices. Furthermore, we establish an approximation error bound of one-step clustering.

TuA07	Blegen 155
Control of Large-Scale Systems-I	Invited Session

Chair: Seiler, Peter — Univ. of Minnesota
Co-Chair: Khong, Sei Zhen — Lund Univ
Organizer: Khong, Sei Zhen — Univ. of Minnesota
Organizer: Seiler, Peter — Univ. of Minnesota

10:25-10:50, Paper TuA07.1

Solidifying and Expandable Network System Constructed by Interconnection of Gamma-Passive Systems (I)

Urata, Kengo — Keio Univ
Inoue, Masaki — Keio Univ

Keywords: Stability, Large Scale Systems, Feedback Control Systems

Abstract: We study a network system whose dissipation performance is strictly improved by network expansion. Assuming that each subsystem is gamma-passive: a special class of dissipative systems, we analyze dissipation performance of the entire network system. We show that the performance is strictly improved and the entire network system is solidifying with the increase of the number of subsystems. This analysis can be a basis for decentralized design of a stable and expandable network system with guaranteeing high control performance.

10:50-11:15, Paper TuA07.2

Cluster Consensus Over Strongly Connected Voltage Graphs (I)

Chen, Xudong — Univ. of Illinois at Urbana-Champaign
Belabbas, Mohamed Ali — Univ. of Illinois at Urbana-Champaign
Basar, Tamer — Univ. of Illinois at Urbana-Champaign

Keywords: Networked Control Systems, Systems on Graphs, Linear Systems

Abstract: A cluster consensus system is a multi-agent system in which the autonomous agents communicate to form multiple clusters, with each cluster of agents asymptotically converging to the same clustering point. We introduce in this paper a special class of cluster consensus dynamics, termed the G-clustering dynamics for G a point group, whereby the autonomous agents can form as many as |G| clusters, and moreover, the associated |G| clustering points exhibit a geometric symmetry induced by the point group. The definition of a G-clustering dynamics relies on the use of the so-called voltage graph. We recall that a G-voltage graph is comprised of two elements---one is a directed graph (digraph), and the other is a map assigning elements of a group~G to the edges of the digraph. For example, in the case when G = {1, -1}, i.e., a cyclic group of order 2, a voltage graph is nothing but a signed graph. A G-clustering dynamics can then be viewed as a generalization of the so-called Altafini's model, which was originally defined over a signed graph, by defining the dynamics over a voltage graph. One of the main contributions of this paper is to identify a necessary and sufficient condition for the exponential convergence of a G-clustering dynamics. Various properties of voltage graphs that are necessary for establishing the convergence result are also investigated, some of which might be of independent interest in topological graph theory.

11:15-11:40, Paper TuA07.3

On the Positive Systems and Integral Linear Constraints (I)

Kao, Chung-Yao — National Sun Yat-Sen Univ
Khong, Sei Zhen — Lund Univ

Keywords: Robust and H-Infinity Control, Networked Control Systems, Large Scale Systems

Abstract: The theory of integral linear constraints (ILCs) for robustness analysis of positive feedback systems is generalized to accommodate multipliers taking the forms of linear time-varying operators. This serves as a counterpart to the well-studied theory of integral quadratic constraints. A list of ILCs characterizing commonly encountered uncertainties and sufficient conditions for verifying ILCs are provided. This note also demonstrates how ILCs can be used for scalable analysis of network systems manifesting the positivity property.

11:40-12:05, Paper TuA07.4

Robust Consensus in Multi-Agent Networked Systems with Nonlinear or Time-Varying Uncertainties (I)

Hu, Bin Univ. of Minnesota, Twin Cities
Khong, Sei Zhen Lund Univ

Keywords: Networked Control Systems, Stability, Operator Theoretic Methods in Systems Theory

Abstract: This paper presents an input-output approach for convergence analysis of robust consensus in multi-agent networks with nonlinear or time-varying uncertainties. The robustness analysis of the consensus algorithm is reformulated as an input-output stability verification problem involving two related interconnections. The resultant input-output stability tests are derived using integral quadratic constraints. The key feature of the proposed input-output approach is that only time domain arguments are used, whereby time-varying/nonlinear uncertainties can be directly incorporated.

TuSL1 Willey 175

Prof. Camlibel Semi-Plenary Semi-plenary session

14:00-15:00, Paper TuSL1.1

Controllability of Systems Defined on Graphs

Camlibel, Kanat Univ. of Groningen

Keywords: Networked Control Systems

Abstract: The study of networks of dynamical systems became one of the most popular themes within systems and control theory in the last two decades. This talk focuses on controllability of networks consisting of identical dynamical systems. For such networks, the overall dynamics determined by the (identical) dynamics of the individual systems as well as the graph capturing the network structure. First, we will focus on a particular system defined over graphs, namely diffusively coupled leader/follower systems. These systems admit models in which graph Laplacians play an important role. By studying certain partitions of the underlying graph, we will provide purely graph theoretical necessary as well as sufficient conditions for the controllability of diffusively coupled leader/follower multi-agent systems. After that, we shift our attention to more general graph related matrices than Laplacians and look at the problem of minimal leader selection, that is rendering the overall network controllable by choosing as few as possible number of leaders.

TuSL2 Willey 125

Prof. Salapaka Semi-Plenary Semi-plenary session

14:00-15:00, Paper TuSL2.1

Reconstruction of Interconnectedness in Networks of Dynamical Systems Based on Passive Observations

Salapaka, Murti V. Univ. of Minnessota

Keywords: Linear Systems, System Identification

Abstract: Determining interrelatedness structure of various entities from multiple time series data is of significant interest to many areas. Knowledge of such a structure can aid in identifying cause and effect relationships, clustering of similar entities, identification of representative elements and model reduction. In this talk, a methodology for identifying the interrelatedness structure of dynamically related time series data based on passive observations will be presented. The framework will allow for the presence of loops in the connectivity structure of the network. The quality of the reconstruction will be quantified. Results on the how the sparsity of multivariate Wiener filter, the Granger filter and the causal Wiener filter depend on the network structure will be presented. Connections to graphical models with notions of independence posed by d-separation will be highlighted.

TuB01 Blegen 110

Stochastic Modeling and Control Regular Session

Chair: Picci, Giorgio Univ. Di Padova

Co-Chair: Camlibel, Kanat Univ. of Groningen

15:30-15:55, Paper TuB01.1

PDF *An Alternating Minimization Algorithm for Structured Covariance Completion Problems*

Zare, Armin Univ. of Minnesota

Chen, Yongxin Univ. of Minnesota
Jovanovic, Mihailo Univ. of Minnesota
Georgiou, Tryphon T. Univ. of Minnesota

Keywords: Optimization : Theory and Algorithms, Linear Systems, Stochastic Modeling and Stochastic Systems Theory

Abstract: State statistics of linear systems satisfy certain structural constraints that arise from the underlying dynamics and the directionality of input disturbances. These statistics are relevant in understanding the fundamental physics and can be used to develop control-oriented models for large-scale dynamical systems, e.g., stochastically forced linearized Navier-Stokes equations. The problem of completing partially known state statistics via stochastically driven linear time-invariant systems gives rise to a class of structured covariance completion problems. In this, nuclear norm minimization is used to identify forcing models of low complexity. Herein, we develop a customized alternating minimization algorithm (AMA) to solve this optimization problem for large-scale systems. We interpret AMA as a proximal gradient for the dual problem which allows us to prove convergence for the algorithm with fixed step-size.

15:55-16:20, Paper TuB01.2

PDF *Towards Realization Theory of Interconnected Linear Stochastic Systems*

Jozsa, Monika Univ. of Groningen
Petreczky, Mihaly CNRS
Camlibel, Kanat Univ. of Groningen

Keywords: Linear Systems, Stochastic Modeling and Stochastic Systems Theory

Abstract: In this paper we present necessary and sufficient conditions for an output process to admit a minimal realization by a coordinated linear stochastic system in forward innovation form.

16:20-16:45, Paper TuB01.3

PDF *A New Approach to Circulant Band Extension*

Picci, Giorgio Univ. Di Padova

Keywords: Stochastic Modeling and Stochastic Systems Theory, Algebraic Systems Theory, Operator Theoretic Methods in Systems Theory

Abstract: The circulant band-extension problem has been object of intense study in recent years which have led to a solution in terms of optimization of an Entropy-like functional. It is shown here that the problem can also be solved in terms of a special kind of matrix spectral factorization. The extension can be computed via the matrix Levinson-Whittle algorithm and by solving a two point boundary value problem.

16:45-17:10, Paper TuB01.4

PDF *On the Robustness of Mean Square Exponential Stability of a Discrete-Time Linear System with Markovian Jumping Subject to White Noise Perturbations*

Dragan, Vasile Romanian Acad
Morozan, Toader Inst. of Mathematics "Simion Stoilow", Romanian Acad. Buc
Stoica, Adrian-Mihail Univ. Pol. of Bucharest

Keywords: Stochastic Modeling and Stochastic Systems Theory, Stability, Linear Systems

Abstract: In the present paper the problem of robust stability in mean square of the discrete-time linear dynamic systems with Markov jumps and corrupted with multiplicative (state-dependent) white noise perturbations is considered. The robustness analysis is performed with respect to the intensity of the white noises. The theoretical developments are illustrated by a numerical example.

17:10-17:35, Paper TuB01.5

PDF *A Chance-Constrained Approach to the Quantized Control of a Heat Ventilation and Air Conditioning System with Prioritized Constraints*

Brocchini, Caterina Pol. Di Milano
Falsone, Alessandro Pol. Di Milano

Manganini, Giorgio — Pol. Di Milano
Holub, Ondrej — Honeywell
Prandini, Maria — Pol. Di Milano

Keywords: Stochastic Control and Estimation, Linear Systems

Abstract: This paper addresses quantized control of a heat ventilation and air conditioning system. The objective is to guarantee comfort, defined in terms of desired temperature and humidity, with a higher priority assigned to the temperature control. The system is described by a linear model with a stochastic input to account for model uncertainty. A chance-constrained control design strategy is proposed where constraints on the temperature and humidity ranges are enforced over some look-ahead time horizon with a predefined (high) probability with respect to the uncertain initial state and the stochastic input. Feasibility of the constraints is guaranteed by minimizing the temperature and humidity variability around the desired set-points, with the variability range on the humidity eventually enlarged when needed to squeeze the one on the temperature. The resulting quantized control is applied in a receding horizon fashion, leading to a closed-loop solution that integrates state filtering to reduce on the fly the uncertainty on the state.

TuB02	Blegen 120
Positive Systems - Convex Invertible Cones Point of View	Invited Session

Chair: Lewkowicz, Izchak — Elect. Eng. Dept., Ben-Gurion Univ
Organizer: Alpay, Daniel Aron — Ben-Gurion Univ
Organizer: Lewkowicz, Izchak — Elect. Eng. Dept., Ben-Gurion Univ

15:30-15:55, Paper TuB02.1

Convex Invertible Cones in System Theory - an Overview (I)

Alpay, Daniel Aron — Ben-Gurion Univ.
Lewkowicz, Izchak — Elect. Eng. dept., Ben-Gurion Univ.

15:55-16:20, Paper TuB02.2

Convex Invertible Cones of Matrices (I)

Alpay, Daniel Aron — Ben-Gurion Univ
Lewkowicz, Izchak — Elect. Eng. Dept., Ben-Gurion Univ

Keywords: Nonlinear Systems and Control

Abstract: Convex Invertible Cones of Matrices

16:20-16:45, Paper TuB02.3

Nevanlinna-Pick Interpolation and Lyapunov Orderings: The Noncommutative Setting (I)

Ball, Joseph A. — Virginia Tech

Keywords: Robust and H-Infinity Control, Cellular Automata, Multidimensional Systems

Abstract: The classical Nevanlinna-Pick theorem, now approaching 100 years old, gives a criterion for the existence of a holomorphic function mapping the open unit disk into the closed unit disk and simultaneously satisfying finitely many interpolation conditions in terms of the positive-semidefiniteness of the associated so-called Pick matrix. Fairly recent work of Cohen and Lewkowicz as part of their study of invertible cones of Hermitian matrices considered an operator-valued/operator-argument version of the Nevanlinna-Pick problem and obtained a criterion related to a Lyapunov ordering on Hermitian matrices for solution of the problem in lieu of the Pick-matrix test. More recently there has been a slew of activity on a new kind of function theory called (e.g. free noncommutative function theory} (with motivation from free probability theory, noncommutative functional calculus, the theory of automata and formal languages inter alia) and there have now appeared Nevanlinna-Pick interpolation theorems for this setting. We survey here some of these results, and in particular an analogue of the Cohen-Lewkowicz Lyapunov-ordering criterion for the solvability of a noncommutative version of the Nevanlinna-Pick interpolation problem.

16:45-17:10, Paper TuB02.4

Sector Preserving and Monotonicity (I)

Alpay, Daniel Aron — Ben-Gurion Univ
Lewkowicz, Izchak — Elect. Eng. Dept., Ben-Gurion Univ

Keywords: Nonlinear Systems and Control

Abstract: Sector Preserving and Monotonicity

17:10-17:35, Paper TuB02.5

The Kalman - Yakubovich - Popov Lemma (I)

Alpay, Daniel Aron — Ben-Gurion Univ
Lewkowicz, Izchak — Elect. Eng. Dept., Ben-Gurion Univ

Keywords: Nonlinear Systems and Control

Abstract: The Kalman - Yakubovich - Popov Lemma

17:35-18:00, Paper TuB02.6

Positive Systems - Convex Invertible Cones Point of View

Alpay, Daniel Aron — Ben-Gurion Univ
Lewkowicz, Izchak — Elect. Eng. Dept., Ben-Gurion Univ

TuB03 — Blegen 130

Distributed Parameter Systems-II — Invited Session

Chair: Demetriou, Michael A. — Worcester Pol. Inst
Co-Chair: Jacob, Birgit — Bergische Univ. Wuppertal
Organizer: Demetriou, Michael A. — Worcester Pol. Inst
Organizer: Jacob, Birgit — Bergische Univ. Wuppertal
Organizer: Fahroo, Fariba — Naval Postgraduate School

15:30-15:55, Paper TuB03.1

Robust Regulation for Port-Hamiltonian Systems of Even Order (I)

Humaloja, Jukka-Pekka — Tampere Univ. of Tech
Paunonen, Lassi — Tampere Univ. of Tech
Pohjolainen, Seppo — Tampere Univ. of Tech

Keywords: Control of Distributed Parameter Systems, Robust and H-Infinity Control, Feedback Control Systems

Abstract: We present a controller that achieves robust regulation for a port-Hamiltonian system of even order. The controller is especially designed for impedance energy-preserving systems. By utilizing the stabilization results for port-Hamiltonian systems together with the theory of robust output regulation for exponentially stable systems, we construct a simple controller that solves the Robust Output Regulation Problem for an initially unstable system. The theory is illustrated on an example where we construct a controller for one-dimensional Schrödinger equation with boundary control and observation.

15:55-16:20, Paper TuB03.2

Realization of Infinite-Dimensional Systems, Part I: Input/output Approach Based on Fractional Representations (I)

Yamamoto, Yutaka — Kyoto Univ

Keywords: Infinite Dimensional Systems Theory, Delay Systems, Control of Distributed Parameter Systems

Abstract: This two part presentation gives an overview of an approach to infinite-dimensional systems. This first part gives a brief overview of realization theory based on a class of fractional representations called {em pseudorational}. This class can be expressed as a ratio of distributions of compact support in the convolution algebra of distributions. We start with a description of Hankel operators, and it leads to a natural state space realization as a closed subspace of output functions derived from a denominator of a transfer function. It is seen that this class yields a natural input/output approach to delay-differential systems. For example, the well-known M_2 space realization can

be naturally derived from such a realization procedure. A close connection with complex analysis, such as the Paley-Wiener theorem, is exhibited, and it in turn leads to a natural characterization of the spectrum of the infinitesimal generators of such a realization.

16:20-16:45, Paper TuB03.3

Realization of Infinite-Dimensional Systems, Part II: - Invariant Subspaces, Coprimeness, and Bezoutidentities (I)

Yamamoto, Yutaka — Kyoto Univ

Keywords: Infinite Dimensional Systems Theory, Control of Distributed Parameter Systems, Delay Systems

Abstract: This second part discusses various coprimeness issues in the class of pseudorational transfer functions. Given a pseudorational impulse response p/q, there can be variaed notions of coprimeness. When there are no common zeros between hatp and hatq, the pair is spectrally coprime. If the pair satisfies an additional condition, the pair is approximately coprime in the sense that there exists a sequence phi_{n} and psi_{n} such that

p*phi_{n} + q*psi_{n} rightarrow delta. There is a close connection of these notions with some reachability properties. As an application of a representation of the standard state space X^{q}, we give a new characterization of invariant subspaces of H^{2}. This in turn leads to a new formula for the one-block H^{infty} weighted sensitivity problem. We then give a condition under which the pair (p, q) satisfies a Bezout identity. We conclude the paper with applications to behaviors, particularly characterizations of controllable behaviors defined by pseudorational pairs.

16:45-17:10, Paper TuB03.4

A New Lyapunov-Barbalat Theorem with Application to Adaptive Control of Mildly Nonlinear Infinite-Dimensional Systems on Hilbert Space (I)

Balas, Mark — Embry Riddle Aeronautical Univ
Frost, Susan — NASA Ames Res. Center

Keywords: Control of Distributed Parameter Systems, Adaptive Control, Infinite Dimensional Systems Theory

Abstract: This paper presents an exponential stability result for distributed parameter systems based on a version of Barbalat's lemma for infinite dimensional Hilbert spaces. Given a mildly nonlinear continuous-time infinite-dimensional plant on a Hilbert space and disturbances of known and unknown waveform, we will use this infinite-dimensional Lyapunov-Barbalat Lemma to show that there is an exponentially stabilizing direct adaptive control law with certain disturbance rejection and robustness properties.

17:10-17:35, Paper TuB03.5

Control Preserving Positivity in Diffusion and Population Dynamics Systems (I)

Tucsnak, M. — Univ. of Bordeaux/CNRS

Keywords: Infinite Dimensional Systems Theory

Abstract: The motivation of this study comes from various infinite dimensional systems modelling heat propagation, diffusion processes or population dynamics, in which the state variable is necessarily described by a positive function. Most of the controllability or stabilization results on these systems neglect these constraints so that the controlled trajectories do not necessarily describe physically or biological relevant situations. In this work we give simple examples, involving parabolic or transport partial differential equations, for which we are able to design controls preserving positivity constraints. We show that satisfying such constraints requires, in general, a large enough controllability time.

TuB04	Blegen 135
Algebraic Methods in Symbolic/Numeric Computations in Systems Theory-II	Invited Session

Co-Chair: Zerz, Eva — RWTH Aachen Univ
Organizer: Quadrat, Alban — INRIA Lille
Organizer: Zerz, Eva — RWTH Aachen Univ

15:30-15:55, Paper TuB04.1

Gröbner Bases Over Finitely Generated Affine Monoids and Applications. the Direct Sum Case (I)

Scheicher, Martin — Univ. Innsbruck

Keywords: Computations in Systems Theory, Multidimensional Systems, Algebraic Systems Theory

Abstract: For multidimensional linear systems with constant coefficients, Gröbner bases are the universal tool to solve algorithmically a multitude of problems which arise in control theory. Gröbner bases are defined over the polynomial ring which means that the domain of definition of discrete systems is the positive orthant. However, often the individual variables are interpreted diversely, e.g., some as time, which should be nonnegative, and some as space, which extends in positive as well as in negative directions. In these situations, the usefulness of conventional Gröbner bases is limited.

In response to this, we consider monoid algebras of finitely generated submonoids of the integer lattice, define Gröbner bases for modules over these monoid algebras and present a method for computing them. Finally, we apply these results to three tasks from multidimensional systems theory, namely, to decide when two systems of homogeneous equations are equivalent, to determine when a system is controllable and to the Cauchy problem. We specialise to the case that the submonoid is a direct sum of a pointed monoid and a group, which includes the most relevant examples.

15:55-16:20, Paper TuB04.2

Controlled and Conditioned Invariant Varieties for Polynomial Control Systems with Rational Feedback (I)

Schilli, Christian — RWTH Aachen Univ
Zerz, Eva — RWTH Aachen Univ
Levandovskyy, Viktor — RWTH Aachen Univ

Keywords: Nonlinear Systems and Control, Applications of Algebraic and Differential Geometry in Systems Theory, Feedback Control Systems

Abstract: The present paper deals with polynomially non-linear state-space systems with rational feedback. A given variety V is said to be controlled invariant for such a system if we can find a rational state feedback law that causes the resulting rational closed loop system to have V as an invariant set. If this task can be achieved by a rational output feedback law, V is called controlled and conditioned invariant with rational feedback. Under some additional assumptions on the variety, we will give algorithms, which allow to decide whether it is controlled (and conditioned) invariant and which compute a rational (output) feedback law achieving the task.

16:20-16:45, Paper TuB04.3

Multiplicity and Stable Varieties of Time-Delay Systems: A Missing Link (I)

Boussaada, Islam — IPSA & Lab. Des Signaux Et Systèmes
Unal, Hakki Ulas — Anadolu Univ
Niculescu, Silviu-Iulian — Umr Cnrs 8506, Cnrs-Supelec

Keywords: Delay Systems, Linear Systems, Stability

Abstract: Multiple spectral values in dynamical systems are often at the origin of complex behaviors as well as unstable solutions. However, in some recent studies, an unexpected property is emphasized. More precisely, an example of real scalar delay system is constructed, where the maximal multiplicity of an appropriate delay-dependant real and negative spectral value leads to a negative spectral abscissa and, as a consequence, the asymptotic stability of the corresponding steady state solution. In algebraic terms (with respect to the parameter space), the variety corresponding to such a multiple root defines a stable variety for the steady state. Furthermore, for the reduced examples we consider, we show that, under mild assumptions, such a multiple spectral value is nothing else than the spectral abscissa. To the best of the authors' knowledge, such a property was not deeply investigated. Motivated by the potential implication of such a property in control systems applications, this note is devoted to better explore the connexion between those varieties. Finally, the sunflower dynamical equation illustrates the study.

TuB05 — Blegen 145

Optimization: Theory and Algorithms-II — Regular Session

Co-Chair: Enqvist, Per — KTH

15:30-15:55, Paper TuB05.1

Comparing Regularized Spectral Estimation Methods on Voiced Speech

Enqvist, Per KTH

Kleijn, W. Bastiaan Victoria Univ. of Wellington

Keywords: System Identification, Signal Processing, Optimization : Theory and Algorithms

Abstract: In many speech processing applications it is important to estimate the envelope of the power spectral density of speech signals. Voiced speech has a harmonic structure and this causes difficulties for many estimation methods. Hence an increased robustness of the estimator is needed and several regularized methods have been proposed. One of the contributions of this paper is the introduction of a common Bayesian framework to derive a selection of these robust methods from the same general set-up but with different assumptions on a priori information. The main contribution is the evaluation of the methods performed by studying their performance on artificial speech signals with and without noise. Several of the methods have previously only been tested on a few theoretical examples. The results indicate that regularization improves the performance and that the best method depends on the characteristics of the speech signal and the demands on computation speed.

15:55-16:20, Paper TuB05.2

Global Optimality Bounds for ICA Algorithms

Colombo, Nicolo Univ. of Luxembourg

Thunberg, Johan Univ. of Luxembourg

Goncalves, Jorge M. Univ. of Luxembourg

Keywords: Optimization : Theory and Algorithms, Signal Processing, Applications of Algebraic and Differential Geometry in Systems Theory

Abstract: Independent Component Analysis is a popular statistical method for separating a multivariate signal into additive components. It has been shown that the signal separation problem can be reduced to the joint diagonalization of the matrix slices of some higher-order cumulants of the signal. In this approach, the unknown mixing matrix can be computed directly from the obtained joint diagonalizer. Various iterative algorithms for solving the non-convex joint diagonalization problem exist, but they usually lack global optimality guarantees. In this paper, we introduce a procedure for computing an optimality gap for local optimal solutions. The optimality gap is then used to obtain an empirical error bound for the estimated mixing matrix. Finally, a class of simultaneous matrix decomposition problems that admit such relaxation procedure is identified.

16:20-16:45, Paper TuB05.3

Inverse Problems for Matrix Exponential in System Identification: System Aliasing

Yue, Zuogong Univ. of Luxembourg

Thunberg, Johan Univ. of Luxembourg

Goncalves, Jorge M. Univ. of Luxembourg

Keywords: Optimization : Theory and Algorithms, System Identification

Abstract: This note addresses identification of the A-matrix in continuous time linear dynamical systems on state-space form. If this matrix is partially known or known to have a sparse structure, such knowledge can be used to simplify the identification. We begin by introducing some general conditions for solvability of the inverse problems for matrix exponential. Next, we introduce ``system aliasing'' as an issue in the identification of slow sampled systems. Such aliasing give rise to non-unique matrix logarithms. As we show, by imposing additional conditions on and prior knowledge about the A-matrix, the issue of system aliasing can, at least partially, be overcome. Under conditions on the sparsity and the norm of the A-matrix, it is identifiable up to a finite equivalence class.

16:45-17:10, Paper TuB05.4

Masking Method of Private Information for Distributed Optimization and Its Application to Real-Time Pricing

Wada, Kazuma Tottori Univ

Sakurama, Kazunori Tottori Univ

Keywords: Optimization : Theory and Algorithms, Large Scale Systems, Networked Control Systems

Abstract: In this paper, we propose a masking method to protect private information for a distributed optimization. This method enables us to obtain the solution of the optimization problem even if local information is masked by a

signal. Moreover, we evaluate the privacy protection performance by the correlation coefficient between original and masked data, and show that the private information can be protected by the proposed method. Finally, we apply the proposed method to real-time pricing for the supply and demand balance in a power grid.

17:10-17:35, Paper TuB05.5

Matricial Wasserstein and Unsupervised Tracking

Ning, Lipeng	Harvard Medical School
Sandhu, Romeil	Departments of Computer Science and Applied Mathematics/Statisti
Georgiou, Tryphon T.	Univ. of Minnesota
Tannenbaum, Allen	Stony Brook Univ

Keywords: Stability, System Identification

Abstract: The context of this work is spectral analysis of multivariable times-series as this may arise in processing signals originating in antenna and sensor arrays. The salient feature of these time signals is that they contain information about moving scatterers/targets which may not be known a priori. That is, neither the number nor the physical properties of scatterers may be known in advance, a fact which necessitates that analysis needs to be model free. Thus, what is important is to attain reliable and high resolution spectral estimates based on short-time observations due to the expected motion of objects within the scattering field.

Traditional spectral analysis methods such as spectrograms and maximum entropy, Capon, etc. techniques, are often severely constrained by the non-stationary nature of time-series, which necessitates very short observation records. Thus, our goal has been to develop natural regularization techniques that allow smooth interpolation of spectrograms in time, thereby improving resolution and robustness. Since power spectra are matrix-valued measure, we sought to develop geometric tools that are based on weak* continuous metrics, such as Wasserstein metrics, only for matrix-valued functions. The present work is largely based on cite{ning2015matrix} where such a theory was laid out.

TuB06	Blegen 150
Model Reduction-II	Invited Session
Chair: van der Schaft, Arjan J.	Univ. of Groningen
Co-Chair: Kawano, Yu	Kyoto Univ
Organizer: Kawano, Yu	Kyoto Univ
Organizer: Scherpen, Jacquelien M.A.	Univ. of Groningen
Organizer: van der Schaft, Arjan J.	Univ. of Groningen

15:30-15:55, Paper TuB06.1

Structure Preserving Truncation for Linear Port Hamiltonian Systems (I)

Kawano, Yu	Kyoto Univ
Scherpen, Jacquelien M.A.	Univ. of Groningen

Keywords: Large Scale Systems, Linear Systems

Abstract: In this paper we present a novel balancing method for linear port Hamiltonian systems based on the Hamiltonian and the controllability Gramian. This balanced truncation method preserves the port Hamiltonian structure in contrast to the traditional balanced truncation method based on the controllability and observability Gramians. In addition, we study a similar method for gradient systems, and show that our method provides an equivalent reduced order model as the model obtained by traditional balancing.

15:55-16:20, Paper TuB06.2

Balancing of Linear Time-Varying Symmetric Systems (I)

Kawano, Yu	Kyoto Univ
Scherpen, Jacquelien M.A.	Univ. of Groningen

Keywords: Large Scale Systems, Linear Systems

Abstract: In this paper, we introduce the notion of symmetry and the cross Gramian for linear time-varying systems. Then, we show that both the controllability and observability Gramians of symmetric systems are obtained from the

cross Gramian and the time-varying version of the metric. This implies that a uniformly balanced realization can be obtained without separately computing the controllability and observability Gramians. Furthermore, the symmetric structure is preserved by uniform balanced truncation if all Hankel singular values are distinct.

16:20-16:45, Paper TuB06.3

Clustering-Based Model Order Reduction for Multi-Agent Systems with General Linear Time-Invariant Agents (I)

Mlinarić, Petar	Max Planck Inst. for Dynamics of Complex Tech. Systems
Grundel, Sara	Max Planck Inst. for Dynamics of Complex Tech. Systems
Benner, Peter	Max Planck Inst. for Dynamics of Complex Tech. Systems

Keywords: Computations in Systems Theory, Systems on Graphs, Networked Control Systems

Abstract: In this paper, we extend our clustering-based model order reduction method for multi-agent systems with single-integrator agents to the case where the agents have identical general linear time-invariant dynamics. The method consists of the Iterative Rational Krylov Algorithm, for finding a good reduced order model, and the QR decomposition-based clustering algorithm, to achieve structure preservation by clustering agents. Compared to the case of single-integrator agents, we modified the QR decomposition with column pivoting inside the clustering algorithm to take into account the block-column structure. We illustrate the method on small and large-scale examples.

TuB07	Blegen 155
Control of Large Scale Systems-II	Invited Session
Chair: Seiler, Peter	Univ. of Minnesota
Co-Chair: Khong, Sei Zhen	Lund Univ
Organizer: Khong, Sei Zhen	Univ. of Minnesota
Organizer: Seiler, Peter	Univ. of Minnesota

15:30-15:55, Paper TuB07.1

Input-Output Stability of Linear Discrete-Time Consensus Processes (I)

Liu, Ji	Univ. of Illinois at Urbana-Champaign
Basar, Tamer	Univ. of Illinois at Urbana-Champaign

Keywords: Mathematical Theory of Networks and Circuits, Linear Systems, Stability

Abstract: In a network of n agents, consensus means that all n agents reach an agreement on a specific value of some quantity via local interactions. A linear discrete-time consensus process can typically be modeled by a discrete-time linear recursion equation, whose equilibria include nonzero states of the form amathbf{1} where a is a constant and mathbf{1} is a column vector in R^n whose entries all equal 1. Using a suitably defined semi-norm, this work extends the standard notion of input-output stability from linear systems to linear recursions of this type. A sufficient condition for input-output consensus stability is provided. A connection between uniform bounded-input, bounded-output consensus stability and uniform exponential consensus stability is established. A certain type of additive perturbation to a consensus process is also considered.

15:55-16:20, Paper TuB07.2

Products of Doubly Stochastic Matrices (I)

Xia, Weiguo	Dalian Univ. of Tech
Liu, Ji	Univ. of Illinois at Urbana-Champaign
Cao, Ming	Univ. of Groningen
Johansson, Karl Henrik	Royal Inst. of Tech
Basar, Tamer	Univ. of Illinois at Urbana-Champaign

Keywords: Mathematical Theory of Networks and Circuits, Linear Systems, Stability

Abstract: Doubly stochastic matrices constitute an important class of stochastic matrices, playing a critical role in the study of discrete-time distributed averaging and distributed optimization algorithms. In this extended abstract, we report on several properties of such matrices. Furthermore, we utilize these properties to establish necessary and

sufficient conditions for deciding whether a set of doubly stochastic matrices is a consensus set or not.

TuME	Room T1
Memorial for Uwe Helmke - Organizers: L. Gruene, J.Trumpf, Y. Yamamoto	Plenary Session

Chair: Gruene, Lars — Univ. of Bayreuth

Co-Chair: Trumpf, Jochen — The Australian National Univ

A chance-constrained approach to the quantized control of a heat ventilation and air conditioning system with prioritized constraints

Caterina Brocchini[1], Alessandro Falsone[1], Giorgio Manganini[1], Ondrej Holub[2], Maria Prandini[1]

***Abstract*— This paper addresses quantized control of a heat ventilation and air conditioning system. The objective is to guarantee comfort, defined in terms of desired temperature and humidity, with a higher priority assigned to the temperature control. The system is described by a linear model with a stochastic input to account for model uncertainty. A chance-constrained control design strategy is proposed where constraints on the temperature and humidity ranges are enforced over some look-ahead time horizon with a predefined (high) probability with respect to the uncertain initial state and the stochastic input. Feasibility of the constraints is guaranteed by minimizing the temperature and humidity variability around the desired set-points, with the variability range on the humidity eventually enlarged when needed to squeeze the one on the temperature. The resulting quantized control is applied in a receding horizon fashion, leading to a closed-loop solution that integrates state filtering to reduce on the fly the uncertainty on the state.**

I. INTRODUCTION

Energy management is an interesting and challenging problem that has recently attracted the attention of many researchers in academy as well as in industry, with focus on microgrid operation, [1]–[5]. Exhaustion of energy resources and heavy environmental impact are two of the main issues that arise from the growing energy consumption. Almost of 40% of the total energy consumption in developed countries is due to buildings [6], a significant part of it being used by Heating, Ventilation and Air Conditioning (HVAC) systems for maintaining comfort conditions for the building occupants. Indeed, indoor air quality seems to have a direct impact on people productivity [7]. Thus a significant part of the research on optimal energy management has been focused on climate control in buildings (see e.g. [8]–[18])

Building energy regulations were established since the 1970s to guarantee a certain energy efficiency level [19]. Efficient management of HVAC units imposes new requirements on their configuration and operation: new components are added, on-off actuators are replaced by multi-staged ones, and more complex control specifications are given. Control specifications for an HVAC system are naturally given in terms of comfort conditions, i.e., desired ranges for temperature and humidity. Due to the limited control authority of HVAC multi-staged actuators, some priority order need to be assigned to the controlled variables and, in particular, temperature regulation has higher priority since temperature is more relevant to comfort than humidity.

HVAC systems have multiple components, and physical modeling based on first principles becomes quite challenging. A possible solution is to determine relevant operating conditions and identify for each of them a linear model that includes a stochastic input to account for actual noises and also model inaccuracy. The resulting model is a stochastic switched linear system, with switches determined by an endogenous signal in that it depends on the values taken by the state variables, [20], [21].

The joint presence of quantization of the control input, prioritization of constraints, stochastic input, and switching dynamics makes the problem hard to tackle with traditional control design methodologies.

In this paper, we focus on a single operating condition and propose a novel chance-constrained approach to solve the quantized control problem with prioritized constraints. Constraints on temperature and humidity are enforced in probability over some look-ahead time horizon, where the probability is induced on the system evolution by the uncertain initial state and the stochastic input. Since constraints on temperature and humidity might cause infeasibility of the chance-constraint optimization program, inspired by [22], [23], feasibility is enforced by optimizing the temperature and humidity ranges. This also allows to handle the prioritization of the temperature constraint over that on the humidity, by eventually enlarging the humidity range with respect to the desired range if this is needed to squeeze the one on the temperature. The resulting quantized control is applied in a receding horizon fashion, thus leading to a closed-loop solution which allows to incorporate state filtering and progressively reduce the uncertainty on the state.

A numerical instance of the problem shows the superiority of the proposed control design methodology against a Model Predictive Control (MPC) approach (see e.g. [24]) where prioritization is accounted for indirectly, via different weights on temperature and humidity in a quadratic average cost.

The rest of the paper is organized as follows. In Section II, we described the model of the HVAC system. In Section III we describe the control problem and the proposed solution, including algorithmic aspects related to the chance constrained optimization program solution via randomized techniques and its receding horizon application integrating state filtering. In Section IV, a numerical example is presented to show the effectiveness of the proposed approach.

This work is partially supported by the European Commission under the project UnCoVerCPS with grant number 643921.

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Some concluding remarks are drawn in Section V.

II. HVAC MODELING

HVAC systems are used to provide thermal comfort and suitable air quality inside a building. An HVAC is composed of one or more Roof Top Units (RTUs), which are devices used to regulate and circulate air in different zones of the building. Direct-expansion RTUs use a refrigerant vapor expansion-compression cycle to directly cool the supplied air [25], typically via two- to four-compressor units. A supply fan blows the air across the evaporator, which serves as a cooling coil. In the simplest case, the supply air is directly transported to the conditioned zone. Economiser dampers can be used to mix fresh outside air with air returned from the zone to alter the air properties at the intake of the cooling/heating coils. The considered RTU is equipped with a two-stage compressor, a multi-speed supply fan, and a modulating economiser. It comprises a controller that regulates temperature and humidity of the zone. Measurements of both zone temperature and humidity are made available to the controller and our aim here is to provide a new control algorithm for an RTU operating in a zone of a building.

A description of the controlled system based on first principles [26], [27] is given by

$$\begin{cases} C_{\mathrm{ZA}}\dot{T}_{\mathrm{ZA}} = P_{\mathrm{ZA}} - P_{\mathrm{RTU}} \\ \dot{w}_{\mathrm{ZA}} = h_{\mathrm{ZA}} - h_{\mathrm{RTU}} \end{cases} \tag{1}$$

where P, h, T, and w represent the heat gains, moisture gains, temperature, and absolute humidity, respectively; and the subscripts $(.)_{\mathrm{ZA}}$ and $(.)_{\mathrm{RTU}}$ refer to zone and RTU. In practice, these equations are not useful for control design purposes since they are quite involved when making the dependence on the control inputs (compressor power, speed of the fan, dampers positions in the economiser) and disturbance inputs (e.g. occupancy and weather conditions) explicit, and physical parameters entering the description are difficult to determine. For instance, the amount of heat P_{RTU} extracted from the supply air by the cooling coil depends in general on the compressor power, mixed air temperature and humidity, and supply airflow. In turn, properties of the mixed air at the intake of the cooling coil depend on the outside air and return air properties and the mixing ratio for the two air streams, as determined by the economiser position; and the airflow is a function of the fan speed and system resistance, which depends on the economiser positions and the pressure changes in the conditioned space due to windows opening/closing.

To the purpose of control design, first-principle equations are then replaced by a simpler approximate description constituted by a set of linear models to describe the behavior of the controlled system around specific operating points, which can be derived via black box identification. If this is done in a laboratory setup, large sources of uncertainty that affect the real operating system like, e.g., the occupancy and location (weather and shading) of the building, and interaction between conditioned zones, are neglected and an additive stochastic input is introduced to account for them. The controlled system (RTU operating in a zone) is hence reduced to a switching stochastic linear system that changes dynamics when commuting between different operating conditions and is subject to a quantized control input given by the multi-stage controls of the RTU.

In formulas, for each operating condition, the controlled system model is given by:

$$\begin{cases} \dot{\xi} = F\xi + G_{\upsilon}\upsilon + G_{\omega}\omega \\ \zeta = H\xi \end{cases} \tag{2}$$

where ξ is the state vector comprising zone temperature and humidity which are made available as output in ζ, the input υ comprises the discrete multi-stage controls of the RTU, and ω is a stochastic input used to capture inaccuracy of the model (2) with respect to (1) and short-term fluctuations in the operating conditions (primarily the loads P_{ZA} and h_{ZA}). Matrices F, G_{υ}, and G_{ω} depend on the operating condition.

In this paper, we focus on control design for a given operating condition.

III. QUANTIZED CONTROL WITH PRIORITIZED CONSTRAINTS

The addressed control problem consists in operating the HVAC system so as to maintain appropriate comfort conditions in the zone, i.e., to keep the zone temperature and humidity within prescribed ranges around given set points. Since the discrete nature of the available control inputs makes it difficult to keep both humidity and temperature within their prescribed ranges, we assign a different priority to the two controlled variables by allowing the humidity specification to be violated if this is needed to satisfy that on the temperature (prioritized constraints).

The control problem is defined over a finite horizon $[0, t_f]$ which is discretized in M time slot of length Δ_t. A discrete-time version of system (2) is then introduced where state and output variables are sampled every Δ_t time units.
If the control input υ is kept constant in each interval $[k\Delta_t, (k+1)\Delta_t)$ for $k = 0, \dots, M-1$, then a discrete-time system equivalent to (2) is given by

$$\begin{cases} x_{k+1} = Ax_k + B_u u_k + B_d d_k \\ y_k = Cx_k \end{cases} \tag{3}$$

where matrices A, B_u, and B_d are given by

$$A = e^{F\Delta_t},\ B_u = \int_0^{\Delta_t} e^{F\tau}G_{\upsilon}d\tau,\ B_d = \int_0^{\Delta_t} e^{F\tau}G_{\omega}d\tau, \tag{4}$$

and we set $x_k = \xi(k\Delta_t)$, $u_k = \upsilon(k\Delta_t)$, $y_k = \zeta(k\Delta_t)$, $d_k = \omega(k\Delta_t)$, assuming that ω is constant over each sample interval, $k = 0, \dots, M-1$. The control input u_k takes value in some discrete set U. The initial condition x_0 may be uncertain and characterized as a random variable with a certain probability distribution.

The output y_k is composed of the temperature and humidity variables, which are denoted in the following as y_k^T and y_k^H, respectively: $y_k = [y_k^T \; y_k^H]^\top$. Since the system is linear, we can assume without loss of generality that the desired ranges for y_k^T and y_k^H are both symmetric and centered around zero, namely $[-\bar{T}, \bar{T}]$ and $[-\bar{H}, \bar{H}]$, respectively. Our ideal goal is then to design the control input u_k, $k = 0, \dots, M-1$, so as to enforce the following constraints:

$$|y_k^T| \leq \bar{T},\ |y_k^H| \leq \bar{H},\ k = 0, 1, \dots, M.$$

Note, however, that these are constraints posed on variables that depend on the uncertain initial state and stochastic input d_k realizations. To make this dependence explicit, let us introduce some compact notations.

Set

$$\boldsymbol{u} = \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{M-1} \end{bmatrix}, \quad \boldsymbol{d} = \begin{bmatrix} d_0 \\ d_1 \\ \vdots \\ d_{M-1} \end{bmatrix}, \quad \boldsymbol{y} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_M \end{bmatrix}.$$

If we then unroll the dynamics of (3) along the discrete time horizon starting from the initial state x_0, we easily get

$$\boldsymbol{y} = \mathcal{A}x_0 + \mathcal{B}_u\boldsymbol{u} + \mathcal{B}_d\boldsymbol{d},$$

where matrices $\mathcal{A}$, $\mathcal{B}_u$ and $\mathcal{B}_d$ are defined as follows

$$\mathcal{A} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^M \end{bmatrix},$$

$$\mathcal{B}_u = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ CB_u & 0 & \cdots & 0 & 0 \\ CAB_u & CB_u & \cdots & 0 & 0 \\ \vdots & \dots & \dots & \dots & \vdots \\ CA^{M-2}B_u & CA^{M-3}B_u & \cdots & CB_u & 0 \\ CA^{M-1}B_u & CA^{M-2}B_u & \cdots & CAB_u & CB_u \end{bmatrix},$$

$$\mathcal{B}_d = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ CB_d & 0 & \cdots & 0 & 0 \\ CAB_d & CB_d & \cdots & 0 & 0 \\ \vdots & \dots & \dots & \dots & \vdots \\ CA^{M-2}B_d & CA^{M-3}B_d & \cdots & CB_d & 0 \\ CA^{M-1}B_d & CA^{M-2}B_d & \cdots & CAB_d & CB_d \end{bmatrix}.$$

To account for the uncertainty affecting the system evolution, one can opt either for hard constraints or for soft constraints: in the case of hard constraints, they must hold for every and each uncertainty instance, even for very unlikely realizations, while in the case of soft constraints, they are expressed in probability and must hold on a set of uncertainty instances of predefined probability at least $1 - \varepsilon$, with $\varepsilon \in (0, 1)$ set by the user. Since the hard constraint solution may be conservative and hard constraints are indeed not feasible when the stochastic input d_k has unbounded support (d_k enters additively the output and its contribution cannot be canceled exactly), we head for a soft constraint formulation of the form

$$\mathbb{P}_{(x_0,\boldsymbol{d})}\{|\boldsymbol{y}^T| \leq \bar{\boldsymbol{T}},\ |\boldsymbol{y}^H| \leq \bar{\boldsymbol{H}}\} \geq 1 - \varepsilon, \tag{5}$$

with $\mathbb{P}_{(x_0,\boldsymbol{d})}$ denoting the joint probability distribution of the uncertain initial state and the stochastic input. Here

$$\boldsymbol{y}^T = \begin{bmatrix} y_0^T \\ y_1^T \\ \vdots \\ y_M^T \end{bmatrix}, \qquad \boldsymbol{y}^H = \begin{bmatrix} y_0^H \\ y_1^H \\ \vdots \\ y_M^H \end{bmatrix},$$

$\bar{\boldsymbol{T}}$ and $\bar{\boldsymbol{H}}$ are column vectors with $M+1$ elements all equal to $\bar{T}$ and $\bar{H}$, respectively, and absolute value and inequalities should be interpreted componentwise.

Still, it might be the case that the probabilistic constraint (5) is unfeasible since it is violated at time $k = 0$ (the air in the zone starts to be controlled at time $k = 0$) or along the control horizon because of the limited actuation capabilities of the control system and the unboundedness of the stochastic input. Inspired by [22], [23], we address this feasibility issue by relaxing the constraint (5) and replacing the threshold values $\bar{T}$ and $\bar{H}$ with optimization variables, say h_k^T and h_k^H, $k = 0, 1, \dots, M$, representing the bounds on the temperature and the humidity, that are minimized via the introduction of an appropriate cost function.

Interestingly, we can exploit constraint relaxation to account for prioritization of the control specifications. More specifically, we can give more weight to the minimization of the bounds on the temperature with respect to those on the humidity in the cost function, and impose that the bounds on the humidity are not smaller than the desired $\bar{H}$ value. This way, the variability range of the humidity is possibly enlarged with respect to the desired range so as to squeeze that on the temperature.

This finally leads to the following formulation of the control problem:

$$\begin{aligned} &\min_{\boldsymbol{u} \in U^M, \boldsymbol{h} \in \mathbb{R}^{2(M+1)}} \ \boldsymbol{h}^\top \mathcal{W} \boldsymbol{h} \\ &\text{subject to:} \quad \mathbb{P}_{(x_0,\boldsymbol{d})}\{|\mathcal{A}x_0 + \mathcal{B}_u\boldsymbol{u} + \mathcal{B}_d\boldsymbol{d}| \leq \boldsymbol{h}\} \geq 1 - \varepsilon \\ &\qquad\qquad\quad \boldsymbol{h}^H \geq \bar{\boldsymbol{H}} \end{aligned} \tag{6}$$

where the optimization variables are given by the control input $\boldsymbol{u}$ taking values in the discrete set U^M, and by the bounds on temperature and humidity that are collected in $\boldsymbol{h} = \begin{bmatrix} h_0^\top & h_1^\top & \dots & h_M^\top \end{bmatrix}^\top$ with $h_k = [h_k^T\ h_k^H]^\top \in \mathbb{R}^2$. Vector $\boldsymbol{h}^H$ appearing in the optimization problem (6) comprises only the bounds on the humidity, i.e., $\boldsymbol{h}^H = \begin{bmatrix} h_0^H & h_1^H & \dots & h_M^H \end{bmatrix}^\top$. $\mathcal{W}$ is a block diagonal matrix with on the diagonal 2×2 positive definite matrices W_k, each one weighting h_k, $k = 0, 1, \dots, M$. Matrices W_k can be chosen to be all equal to the diagonal matrix $W = \text{diag}(w_T, w_H)$ with $w_T \gg w_H > 0$ so as to weight more the temperature bounds than the humidity bounds.

Note that (6) is a chance-constrained optimization program in that it involves a bound in probability. Also, it is a mixed integer optimization problem since some of the decision variables are discrete. These two aspects pose some challenges to the solution of (6) that will be addressed in the next subsection.

A. Scenario solution to the chance-constrained optimization

Chance-constrained optimization problems are known to be hard to solve except for few particular cases, [28], [29]. In this work we resort on a randomized technique, known as the scenario approach, [30]–[33], to approximately solve (6). Notably, precise guarantees can be given on the feasibility of the scenario solution for the original chance-constrained problem (6).

We next briefly recall the results on the scenario theory given in the literature that are relevant to our problem. Consider a chance-constrained optimization problem of the form

$$\begin{aligned}\min_{\vartheta\in\mathbb{R}^d} \quad & f(\vartheta) \\ \text{subject to:} \quad & \mathbb{P}_\delta\{\vartheta\in\Theta_\delta\}\geq 1-\varepsilon\end{aligned} \tag{7}$$

where $f(\cdot)$ is a convex function, Θ_δ is a convex set depending on an uncertain parameter δ, which takes values in a set Δ according to a (possibly unknown) probability distribution $\mathbb{P}_\delta$.

The idea of the scenario approach is as simple as follows. Suppose that N samples $\delta^{(1)},\dots,\delta^{(N)}$ of the uncertain parameter drawn independently according to $\mathbb{P}_\delta$ are available. Then, a randomized solution to (7) can be found by solving the following convex optimization program

$$\begin{aligned}\min_{\vartheta\in\mathbb{R}^d} \quad & f(\vartheta) \\ \text{subject to:} \quad & \vartheta\in\Theta_{\delta^{(i)}} \quad i=1,\dots,N\end{aligned} \tag{8}$$

and the following Theorem 1 establishes a link between the solution to (8) and its feasibility for (7).

Theorem 1 (Scenario Guarantees): Let problem (8) be feasible for every multi-sample extraction $\delta^{(1)},\dots,\delta^{(N)}$. Choose a *confidence parameter* $\beta\in(0,1)$. If N is selected so as to satisfy

$$\sum_{i=0}^{d}\binom{N}{i}\varepsilon^i(1-\varepsilon)^{N-i}\leq\beta, \tag{9}$$

where d is the number of decision variables, then, with probability at least $1-\beta$, the solution $\vartheta_N^\star$ to (8) is feasible for (7). ■

Note that the explicit bound

$$N\geq\frac{d+1+\ln(1/\beta)+\sqrt{2(d+1)\ln(1/\beta)}}{\varepsilon} \tag{10}$$

derived in [34] from (9) shows that the dependence on the confidence parameter β is logarithmic so that β can be set very small, say $\beta=10^{-6}$, to get a result that holds deterministically (with probability $\simeq 1$) without having a too large sample size N.

The scenario solution $\boldsymbol{h}_N^\star$, $\boldsymbol{u}_N^\star$ to problem (6) is then obtained via the optimization program

$$\begin{aligned}\min_{\boldsymbol{u}\in U^M,\ \boldsymbol{h}\in\mathbb{R}^{2(M+1)}} \quad & \sum_{k=0}^{M} h_k W h_k^\top \\ \text{subject to:} \quad & |\mathcal{A}x_0^{(i)}+\mathcal{B}_u\boldsymbol{u}+\mathcal{B}_d\boldsymbol{d}^{(i)}|\leq\boldsymbol{h}, i=1,\dots,N \\ & \boldsymbol{h}^H\geq\bar{\boldsymbol{H}}\end{aligned} \tag{11}$$

where $(x_0^{(i)},\boldsymbol{d}^{(i)})$, $i=1,\dots,N$ are independently extracted from $\mathbb{P}_{(x_0,\boldsymbol{d})}$.

Unfortunately, the optimization variables $\boldsymbol{u}$ in problem (6) are discrete so that Theorem 1 does not apply directly to the scenario solution $\boldsymbol{h}_N^\star$, $\boldsymbol{u}_N^\star$. However, we can easily generalize Theorem 1 to our setting by considering $|U|^M$ instances of problem (6) (here, $|U|$ denotes the cardinality of the discrete set U), one for each possible value $\boldsymbol{u}^j$, $j=1,\dots,|U|^M$, of $\boldsymbol{u}\in U^M$:

$$\begin{aligned}&\min_{\boldsymbol{h}\in\mathbb{R}^{2(M+1)}} \sum_{k=0}^{M} h_k W h_k^\top \\ &\text{subject to:} \\ &\mathbb{P}_{(x_0,\boldsymbol{d})}\{|\mathcal{A}x_0+\mathcal{B}_u\boldsymbol{u}^j+\mathcal{B}_d\boldsymbol{d}|\leq\boldsymbol{h}\wedge\boldsymbol{h}^H\geq\bar{\boldsymbol{H}}\}\geq 1-\varepsilon.\end{aligned} \tag{12}$$

Let us denote problem (12) as P_C^j. A scenario solution to P_C^j can be computed with the guarantees provided by Theorem 1 since the assumptions of the theorem are now satisfied. In particular, the solution $\boldsymbol{h}_{N,j}^\star$ to the scenario version

$$\begin{aligned}&\min_{\boldsymbol{h}\in\mathbb{R}^{2(M+1)}} \sum_{k=0}^{M} h_k W h_k^\top \\ &\text{subject to:} \\ &|\mathcal{A}x_0^{(i)}+\mathcal{B}_u\boldsymbol{u}^j+\mathcal{B}_d\boldsymbol{d}^{(i)}|\leq\boldsymbol{h},\ i=1,\dots,N, \\ &\boldsymbol{h}^H\geq\bar{\boldsymbol{H}}\end{aligned}$$

of problem P_C^j in (12) satisfies

$$\mathbb{P}_{(x_0,\boldsymbol{d})}\{|\mathcal{A}x_0+\mathcal{B}_u\boldsymbol{u}^j+\mathcal{B}_d\boldsymbol{d}|\leq\boldsymbol{h}_{N,j}^\star\}\geq 1-\varepsilon \tag{13}$$

with probability at least $1-\beta$ if N is chosen according to (9) with $d=2(M+1)$. Now, the solution $\boldsymbol{h}_N^\star$, $\boldsymbol{u}_N^\star$ to (11) can be obtained as $\boldsymbol{h}_N^\star=\boldsymbol{h}_{N,j_N^\star}^\star$ and $\boldsymbol{u}_N^\star=\boldsymbol{u}^{j_N^\star}$ where

$$j_N^\star=\arg\min_{j\in\{1,2,\dots,|U|^M\}}\sum_{k=0}^{M} h_{N,j\,k}^\star W {h_{N,j\,k}^\star}^\top$$

and, hence, it satisfies

$$\mathbb{P}_{(x_0,\boldsymbol{d})}\{|\mathcal{A}x_0+\mathcal{B}_u\boldsymbol{u}_N^\star+\mathcal{B}_d\boldsymbol{d}|\leq\boldsymbol{h}_N^\star\}\geq 1-\varepsilon,$$

with probability at least $1-|U|^M\beta$, since conditions (13), $j=1,\dots,|U|^M$, hold jointly with such a probability.
This finally leads to the following statement.

Proposition 1: Choose a *confidence parameter* β. If N is selected so as to satisfy

$$\sum_{i=0}^{2(M+1)}\binom{N}{i}\varepsilon^i(1-\varepsilon)^{N-i}\leq\frac{\beta}{|U|^M}, \tag{14}$$

then, with probability at least $1-\beta$, the solutions $\boldsymbol{h}_N^\star$ and $\boldsymbol{u}_N^\star$ to (11) are feasible for the original chance-constrained mixed integer program (6). ■

Since N satisfying (14) depends logarithmically on $\frac{\beta}{|U|^M}$, then, N scales linearly with the time horizon length M (see (10) where d should be set equal to $2(M+1)$ and β should be replaced by $\frac{\beta}{|U|^M}$).

B. Receding horizon implementation with state filtering

Due to the presence of the stochastic input $\boldsymbol{d}$, a closed-loop solution would be more desirable for control purposes rather than an open-loop one. We here adopt a receding-horizon approach in which, at each time step ℓ, a new instance of problem (6) is solved over the shifted time window $[\ell, M+\ell]$, the first control action $u_\ell^\star$ is applied and then the procedure is repeated at $\ell+1$. Note that when solving problem (6) over the shifted time window $[\ell, M+\ell]$, the initial state x_0 becomes x_ℓ and the stochastic input realization $\boldsymbol{d}$ contains shifted samples of d_k, $k=\ell,\ell+1,\dots,\ell+M-1$. Some knowledge is acquired on the probability distributions of the shifted initial state x_ℓ and of the shifted stochastic input realization $\boldsymbol{d}$, if d_k is a correlated process, so that the a-posteriori probability distributions of state and stochastic input given the past output observations can be used to get feedback in the receding horizon implementation.

Filtering techniques can be adopted to determine the a-posteriori distribution of the current state and future stochastic input realizations given the past output observations. Interestingly, given the adopted scenario solution to the chance-constrained optimization program, we actually need N samples extracted from such a-posteriori distributions so that particle filtering techniques can be adopted, [35], [36]. Particle filtering are indeed of general applicability. If the stochastic input d_k is modeled as a colored Gaussian process obtained by filtering a white Gaussian process e_k with a linear system and the initial state x_0 is Gaussian, the state of the system can be enlarged to include the input d_k (see the numerical example in Section IV), and Kalman filtering can then be applied to get covariance and mean of the a-posteriori enlarged state distribution, see e.g. [37].

IV. NUMERICAL EXAMPLE

In this section we present some simulation results on a numerical instance of the problem to show the effectiveness of the proposed approach.

The considered RTU is equipped with a two-stage compressor, a multi-speed supply fan, and a modulating economiser. For the sake of simplicity, the economiser is considered fixed. The controlled system operating in some nominal condition and the adopted continuous time linear system model (2) has order $n=4$. Its state $\xi=[\xi_1\ \xi_2\ \xi_3\ \xi_4]^\top$ comprises the temperature (ξ_1) and the humidity (ξ_3), which are provided as output of the system $\zeta=[\zeta_1\ \ \zeta_2]^\top$. The control input $\upsilon=[\upsilon_1\ \upsilon_2]^\top$ includes the speed of the supply fan and the compressor, both taking three possible values: OFF, LOW, HIGH, which are coded here as -50, 0, 50. The disturbance ω is modeled as a Gaussian process. The matrices of system (2) are as follows

$$F = 10^{-4}\begin{bmatrix} -28 & -5.6 & 0 & 0 \\ 0 & -8.3 & 0 & 0 \\ 0 & 0 & -17 & 1 \\ 0 & 0 & 0 & -2.8 \end{bmatrix}$$

$$G_\upsilon = G_\omega = 10^{-4}\begin{bmatrix} -0.8 & -1.7 \\ 0 & 5.8 \\ -1.7 & 0.08 \\ 0 & 2.3 \end{bmatrix} \qquad \tilde{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The discretization time interval Δ_t is set equal to 5 minutes. Matrices of the discrete time model (3) can be derived via (4). Note that $B_u = B_d = B$ in this example.

The stochastic input d_k acting on (3) is modeled as the following filtered Gaussian process

$$d_{k+1} = A_d d_k + e_k, \tag{15}$$

where $A_d = \mathrm{diag}(a,a)$ with $a=0.9835$, and e_k is a white Gaussian process $\mathcal{N}(0_2, \sigma_e^2 I_2)$, 0_2 being the zero vector with two components, I_2 the identity matrix of order two, and $\sigma_e^2 = 0.6147$. Filter (15) is assumed to be initialized with the stationary distribution: $d_0 \sim \mathcal{N}(0_2, \sigma_d^2 I_2)$, where $\sigma_d^2 = \sigma_e^2/(1-a^2)$. Figure 1 plots a realization of the stochastic input for reference.

The desired bounds on temperature and (relative) humidity are $\bar{T} = 0.5$°C and $\bar{H} = 5\%$. The weighting matrix W entering the definition of the chance-constrained problem (6) is set equal to $W=\mathrm{diag}(10,1)$ so as to give priority to the temperature control. The discrete control horizon length is set equal to $M=3$. A receding horizon implementation using Kalman filtering is adopted.

Since the dynamic of the stochastic input d_k is known, we can augment the state of the system including d_k. The augmented state is defined as $\bar{x}_k = [x_k^\top\ d_k^\top]^\top$.

From (3) and (15), we get the following equivalent system

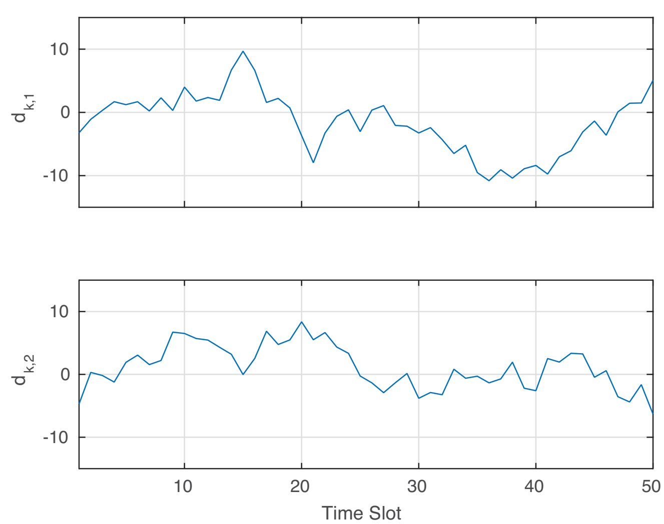

Fig. 1. A stochastic input realization.

where the original state x_k is replaced by the augmented state $\bar{x}_k$ and the stochastic input dynamic is included:

$$\begin{cases} \bar{x}_{k+1} = \bar{A}\bar{x}_k + \bar{B}u_k + \bar{S}e_k \\ y_k = \bar{C}\bar{x}_k \end{cases} \tag{16}$$

e_k being the white Gaussian noise feeding (15). Matrices $\bar{A}$, $\bar{B}$, $\bar{S}$ and $\bar{C}$ are given by:

$$\bar{A} = \begin{bmatrix} A & B \\ 0 & A_d \end{bmatrix},\ \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},\ \bar{S} = \begin{bmatrix} 0 \\ I_2 \end{bmatrix},\ \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix} \tag{17}$$

with 0 representing a zero matrix of appropriate dimensions. A reduced order filter can then be derived to obtain an estimate of the components, say $\tilde{x}_k$ of the state $\bar{x}_k$ that are not available as measurements. If the initial state is Gaussian and the gain of the filter is set equal to the Kalman gain, the obtained estimate of $\tilde{x}_k$ is the mean of its a-posteriori Gaussian distribution given the output observations up to time k, and the variance of such a distribution can be obtained via the Riccati equations used to set the Kalman gain, see [38] for details of the implementation. The realizations adopted at time k for the scenario solution to the chance-constrained optimization are then obtained by sampling from this a-posterior distribution. In particular, the realizations of $\boldsymbol{d}$ are generated by initializing system (15) with the extracted samples of the d_k component of $\bar{x}_k^{(i)}$.

In the following results, the initial state $x_0 = [x_{1,0}\, x_{2,0}\, x_{3,0}\, x_{4,0}]^\top$ of the system has a Gaussian distribution with all independent components with zero mean and standard deviation 0.17 for $x_{1,0}$ and 1.7 for the others, and $\varepsilon = 0.1$. In the scenario implementation, we set $\beta = 10^{-6}$. For comparative purposes, we consider the average cost function

$$J(\boldsymbol{u}) = \mathbb{E}_{(x_0,\boldsymbol{d})}[\boldsymbol{y}^\top \mathcal{W} \boldsymbol{y}]$$

and the (standard) MPC controller obtained by minimizing $J(\boldsymbol{u})$ and implementing the solution in a receding horizon fashion, integrating state filtering via Kalman filtering as in the proposed chance-constrained approach.

For comparative purposes, the two control strategies are applied using the same initial state and the same realization for the stochastic input, over a time horizon of 250 minutes ($L = 50$ sampled times). Figures 2 and 3 represent the output and control input (u^C for the compressor and u^F for the fan) obtained by applying the proposed approach and the standard MPC approach, respectively. Note that in the case of the approach proposed in this paper, humidity stay close to the boundary of the desired $[-\bar{H}, \bar{H}]$ range to keep the temperature closer to 0 and hence inside $[-\bar{T}, \bar{T}]$, whereas in the standard MPC approach the humidity is better centered in the range $[-\bar{H}, \bar{H}]$ but the temperature has higher fluctuations which brings it outside $[-\bar{T}, \bar{T}]$.

We applied $N_v = 1981$ times the two control laws to different realizations of the initial state x_0 and stochastic input $\{d_k,\ k = 0, 1, \ldots, L-1\}$ and computed the expected average temperature violation as

$$\frac{1}{N_v}\sum_{i=1}^{N_v}\left[\frac{1}{L}\sum_{k=0}^{L}\left(1 - \mathbf{I}_{[-\bar{T},\bar{T}]}(y_k^{T^{(i)}})\right)\right]$$

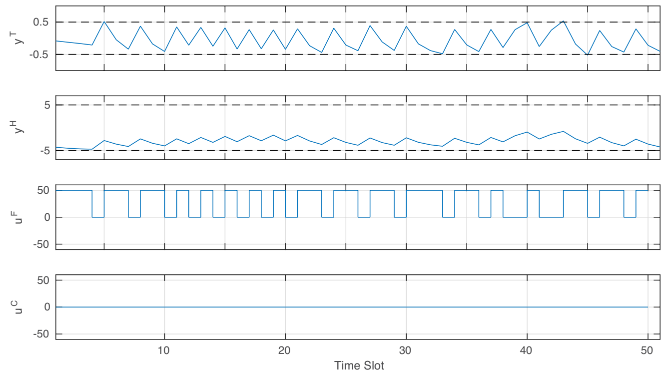

Fig. 2. Proposed chance-constrained approach.

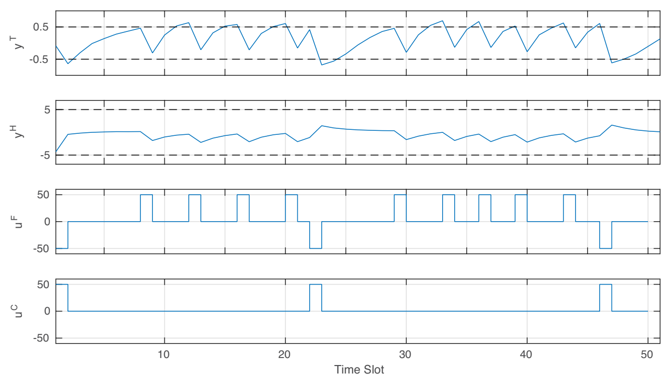

Fig. 3. Standard average approach

where $\mathbf{I}_{[\ell_1,\ell_2]}(\cdot)$ is the indicator function of the interval $[\ell_1, \ell_2]$. Results are shown in Figure 4 plotting the expected average temperature violations as a function of the thresholds $\bar{T}$ in the range $[0.3, 0.6]$ for the two approaches. Note that the curve obtained with the standard MPC approach is above that obtained with the proposed approach for all threshold values $\bar{T} \in [0.3, 0.6]$.

V. CONCLUSION

In this paper we introduced a novel control design strategy that is suitable for a heat, ventilation and air conditioning system. Due to the multi-stage nature of its actuators, the system is characterized by limited control capabilities, which calls for a prioritization of control goals, when aiming at the regulation of multiple variable at some desired set-point. The main features of the proposed approach can be summarized as follows:

- *quantized nature of the control input is accounted for*: the control design problem is formulated as an optimization problem over a finite-horizon with respect to the discrete control inputs;
- *probabilistic prioritized constraints on the state are incorporated in the design*: constraints on variables affected by possibly unbounded stochastic inputs are introduced, their feasibility is guaranteed by adding optimizations variables that relax the constraints if needed, and the constraint on the variable with lower priority is

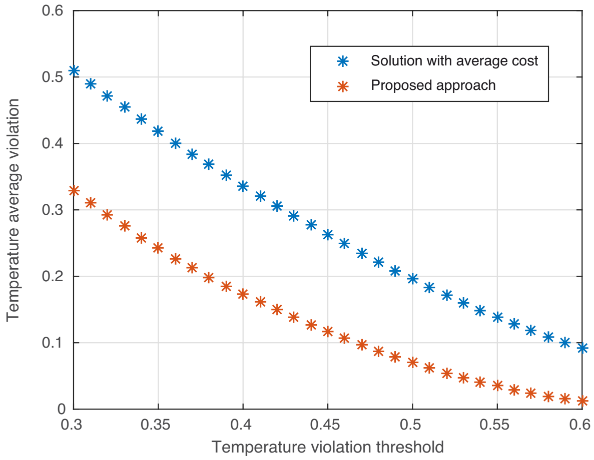

Fig. 4. Average violation for the temperature.

made loose so as to favor the one on the variable with higher priority;

- *receding horizon implementation with state filtering*: the finite-horizon solution is recomputed based on the updated information on the uncertainty obtained via state filtering, and only the first control sample is applied. This allows to obtain a closed-loop controller that can better counteract uncertainty.

The effectiveness of the proposed approach was shown on a numerical case study. Real experiments should be run to better assess its performance and practical impact. Note that in this paper, we focus on control of the HVAC system in a given operating condition. Further work is needed to account for changes in the operating condition. A possible solution is to adopt an adaptive switching mechanism based on state estimation, [39], [40], [41]. This solution has been explored in [38] with reference to a particular setting where the same model of the stochastic input is adopted for the systems associated to different operating conditions. Results are preliminary but appear promising. Alternatively, one could adopt a Linear Parametric Varying (LPV) model (see e.g. [42]) for the HVAC system that varies with continuity in a family of linear systems. This would call for a gain scheduling control solution, which should, however, be able to cope with probabilistic prioritized constrained and quantized control inputs.

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