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Flavour Problems

and New Physics at TeV Scale

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Abstract

There is no explanation in the Standard Model for the replication of fermion families, neither for the mass hierarchy between them nor for the structure of the Yukawa coupling matrices, which remain arbitrary. The key towards understanding the fermion masses and mixing pattern can be in symmetry principles. Masses of fermions can actually have a dynamical origin, following the spontaneous symmetry breaking of gauge “horizontal" symmetries unifying families, differently acting on left and right species. $U(3)$ or $SU(3)$ “horizontal" family symmetries seem an intuitive hypothesis to be considered. Fermion masses should be induced by higher order operators containing flavon scalars emerging from renormalizable interactions via ‘universal seesaw’ mechanism after integrating out some heavy fields, scalars or verctor-like fermions. Then in this case the fermion mass hierarchy and mixing among families can be related to the pattern of spontaneous breaking of the gauge $SU(3)$ symmetry. The corresponding gauge bosons have flavor-nondiagonal couplings to fermions which in principle can induce flavour changing phenomena. In this case strong lower limits on the flavor symmetry breaking scales are expected. However, for special choices of horizontal symmetries there is a natural suppression of flavour changing effects due to a custodial symmetry. So gauge bosons can have mass in the TeV range, without contradicting the existing experimental limits.

However an unexpected anomaly shows up in quark mixing. After the recent high precision determinations of V_{us} and V_{ud}, the first row of the CKM matrix is about 4σ deviated from unitarity.

The existence of the gauge symmetry $SU(3)_\ell$ acting between lepton families can recover unitarity if the symmetry is broken at a scale of about 6 TeV. In fact the gauge bosons of this symmetry contribute to muon decay in interference with the Standard Model, so that the Fermi constant is slightly smaller than the muon decay constant and unitarity is restored.

Alternatively, extra vector-like quarks can be thought as a solution to the CKM unitarity problem. The extra species should exhibit a large mixing with the first family in order to recover unitarity, then their mass should be no more than 6 TeV or so. The implications of the existence of so large mixing must be examined, in order to understand if it can actually exist without contradiction with experimental results on flavour changing neutral current processes and Standard Model observables. In principle an extra weak isodoublet can solve all the discrepancies between independent determinations of the CKM elements in the first row. However not all the discrepancies can be entirely recovered without contradicting experimental constraints. Then the existence of two or more

vector-like doublets or a vector-like isodoublet with a down-type or up-type isosinglet can be considered. In these scenarios unitarity can be resettled and flavour changing can be avoided by setting to zero some couplings of extra species with Standard Model families.

If the anomalies in the determination of CKM mixing angles are confirmed by future experiments with greater precision, there might be strong indication towards the existence of physics beyond the Standard Model at the TeV scale, such as flavour changing gauge bosons and vector-like fermions with masses of few TeV. This new physics can be testable at next runs of high luminosity LHC or, more effectively, in future accelerators.

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Chapter 1

Introduction

1.1 The Standard Model and Flavour Physics

Particles and interactions can be described by quantum field theory. A field theory can be derived, through a variational principle, from an action integral involving a Lagrangian density $\mathcal{L}$. The Lagrangian density holds symmetry properties. Symmetries are mathematically described by the group theory. Symmetries play a fundamental role, since conserved quantities and conserved currents follow from the invariance of $\mathcal{L}$ under symmetry transformations (Noether's theorem). Furthermore, a global symmetry can be turned into a space-time dependent symmetry, called local or gauge symmetry, through the insertion of massless gauge fields (one for each group generator, transforming as members of the adjoint representation) inside the gauge-covariant derivative D_μ (that is "to gauge the symmetry") and their lagrangian kinetic term thus generating interactions.

The Standard Model (SM) is a theory describing the fundamental interactions among elementary particles as gauge interactions. The Lagrangian of the Standard Model is invariant under a transformation of the gauge symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Strong interactions arise from the gauging of $SU(3)_C$. Electroweak interactions arise from the gauging of $SU(2)_L \times U(1)_Y$ and its spontaneous symmetry breakdown to $U(1)_{em}$, caused by one $SU(2)_L$ doublet of Higgs scalar fields, with non-zero vacuum expectation value (VEV) [81, 2, 3]. Fermions are represented as Weyl spinors and the electroweak (EW) part $SU(2)_L \times U(1)_Y$ is chiral with respect to fermion multiplets. The fermion content of the SM with corresponding transformation properties of the fields under $SU(3)_C \times SU(2)_L \times U(1)_Y$ is:

$$q_L^{(f)} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}^{(f)} \sim (3,2)_{1/6} \qquad u_R^{(f)} \sim (3,1)_{2/3} \qquad d_R^{(f)} \sim (3,1)_{-1/3}$$
$$l_L^{(f)} = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}^{(f)} \sim (1,2)_{-1/2} \qquad e_R^{(f)} \sim (1,1)_{-1} \tag{1.1}$$

where f is a family index. There is no explanation in the SM for the replication of the representations in (1.1), a simplified theory containing only one family is completely self-contained. However, there is strong experimental evidence that there are three fermion families in identical representations of the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$. Antifermions have opposite gauge charges and opposite chiralities with repect to what we call fermions. They are described by the field operators $\psi^c_{R,L} = C\overline{\psi_{L,R}}^T$. Then the $SU(3)_C \times SU(2)_L \times U(1)_Y$ transformation properties of the antiquark and antileptons fields are:

$$q^c_R \sim (\bar{3},\bar{2})_{-1/6} \qquad u^c_L \sim (\bar{3},1)_{-2/3} \qquad d^c_L \sim (\bar{3},1)_{1/3}$$
$$l^c_R \sim (1,\bar{2})_{1/2} \qquad e^c_L \sim (1,1)_1 \tag{1.2}$$

where the index of family has been omitted. The scalar content of the theory is made by a single Higgs doublet:

$$\phi \sim (1,2)_{1/2}\,, \qquad \langle\phi^0\rangle = \begin{pmatrix} 0 \\ v_w \end{pmatrix} \tag{1.3}$$

The required gauge bosons in the SM are the octet of gluons G^a_μ, the three $SU(2)_L$ bosons $W^a_{\mu\nu}$ and the $U(1)_Y$ boson B^μ. The gauge-invariant renormalizable Lagrangian of the SM can be written:

$$\mathcal{L} = \mathcal{L}_b + \mathcal{L}_F + \mathcal{L}_\phi + \mathcal{L}_{\text{Yuk}} \tag{1.4}$$

Here $\mathcal{L}_b$ and $\mathcal{L}_F$ contain the derivative terms and interactions of fermions and gauge bosons, $\mathcal{L}_\phi$ describes the Higgs boson, $\mathcal{L}_{\text{Yuk}}$ contains Yukawa interactions.

$\mathcal{L}_b$ contains the strengths of the gauge boson fields: $\mathcal{L}_b = -\frac{1}{4}W^{\mu\nu}_a W^a_{\mu\nu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu}$ where $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu$ and $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc}G^b_\mu G^c_\nu$. $\mathcal{L}_F$ describes free propagation and gauge interactions of fermions. The QCD Lagrangian containing interactions of fermions with gluons in $\mathcal{L}_F$ is:

$$\overline{q_{Li}}\gamma^\mu(g_s\frac{\lambda_a}{2}G^a_\mu)q_{Li} + \overline{u_{Ri}}\gamma^\mu(g_s\frac{\lambda_a}{2}G^a_\mu)u_{Ri} + \overline{d_{Ri}}\gamma^\mu(g_s\frac{\lambda_a}{2}G^a_\mu)d_{Ri} \tag{1.5}$$

where λ_a are the $SU(3)$ generators (the Gell-Mann matrices). With the assignments (1.1), the electroweak piece is:

$$\begin{aligned}\mathcal{L}_{\text{ew}} =&\overline{q_{Li}}i\gamma^\mu(\partial_\mu - ig_s\frac{\lambda_a}{2}G^a_\mu - ig\frac{\tau_a}{2}\cdot W^a_\mu - ig'\frac{1}{6}B_\mu)q_{Li}+\\ &\overline{l_{Li}}i\gamma^\mu(\partial_\mu - ig\frac{\tau_a}{2}\cdot W^a_\mu + ig'\frac{1}{2}B_\mu)l_{Li}+\\ &\overline{u_{Ri}}i\gamma^\mu(\partial_\mu - ig_s\frac{\lambda_a}{2}G^a_\mu - ig'\frac{2}{3}B_\mu)u_{Ri} + \overline{d_{Ri}}i\gamma^\mu(\partial_\mu - ig_s\frac{\lambda_a}{2}G^a_\mu + ig'\frac{1}{3}B_\mu)d_{Ri}+\\ &\overline{e_{Ri}}i\gamma^\mu(\partial_\mu + ig'B_\mu)e_{Ri}\end{aligned} \tag{1.6}$$

where i here is simply a family index and τ_a are the $SU(2)$ generators (the Pauli matrices). However the electroweak symmetry is spontaneously broken, three gauge bosons acquire masses from the interaction with the Higgs and $U(1)_{\text{em}}$ survives SSB. Since $J^{em}_\mu = J^3_\mu + J^Y_\mu$:

$$Q = t_3 + Y \tag{1.7}$$

where t_3 is the eigenvalue of the third generator of $SU(2)_L$. Defining the Weinberg angle by $\tan\theta_W = \frac{g'}{g}$, the photon is the massless gauge boson:

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu \tag{1.8}$$

and the electromagnetic coupling constant is $e = g\sin\theta_W$ and the QED interactions are described by the Lagrangian density:

$$-eQ\overline{f_i}\gamma_\mu f_i A^\mu \tag{1.9}$$

The copies of the same representation of the unbroken $SU(3)_C\times U(1)_{\text{em}}$ are called flavours. At this point the first remarks are in order:

- the Lagrangians in (1.5) and (1.9) are vector-like and C, P and CP invariant;
- it is clear from (1.5) and (1.9) thet both the electromagnetic and strong interactions are not only flavour diagonal but also flavour universal.

$\mathcal{L}_\phi$ contains the Higgs potential:

$$\mathcal{L}_\phi = (\partial_\mu\phi)^\dagger(\partial^\mu\phi) - V(\varphi) \tag{1.10}$$

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \tag{1.11}$$

where the covariant derivative is $D_\mu\phi = (\partial_\mu - ig\frac{\tau_a}{2} - i\frac{1}{2}g'B_\mu)\phi$. The $SU(2)_L \times U(1)_Y$ symmetric Yukawa couplings are:

$$\mathcal{L}_{\text{Yuk}} = Y_u^{ij}\tilde{\phi}\overline{q_{Li}}u_{Rj} + Y_d^{ij}\phi\overline{q_{Li}}d_{Rj} + Y_e^{ij}\phi\overline{\ell_{Li}}e_{Rj} + \text{h.c.} \tag{1.12}$$

where $i, j = 1, 2, 3$, $Y_{u,d,e}$ are the matrices of the Yukawa couplings and $\tilde{\phi} = i\tau_2\phi^*$. It is worth noting that in the absence of Yukawa matrices $Y_{u,d,e} = 0$ the SM Lagrangian in (1.4) acquires a maximal global chiral "horizontal symmetry"

$$U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e \tag{1.13}$$

under which fermion species q_L, u_R, d_R, ℓ_L, e_R transform as triplets of independent $U(3)$ groups. The Yukawa interactions break the global symmetry to $U(1)^5$ (baryon number, family lepton number, hypercharge), that is Yukawa couplings break the $SU(3)^5$ symmetry, so they are flavour violating parameters.

The nonzero VEV of the Higgs gives mass to three gauge bosons and to fermions too. In fact, without spontaneous symmetry breaking, the chiral fermion content of the SM has the remarkable feature that the fermion masses emerge only after spontaneous breaking of $SU(2)\times U(1)$ since there are no $SU(2)\times U(1)$ invariant bilinear in the fermion fields. Then both fermion masses and gauge boson masses are proportional to the same VEV $\langle\phi^0\rangle = v_{\text{w}} = 174$ GeV, since without spontaneous symmetry breaking masses are forbidden by the chiral nature of fermions and gauge symmetry. After electroweak SSB fermions acquire masses through the Yukawa couplings:

$$m^{(u,d,e)ij} = Y^{ij}_{u,d,e}v_w \tag{1.14}$$

Since gauge invariance does not constrain the flavour structure of the Yukawa interactions, there is no reason for which the mass matrices $m^{(u,d,e)}$ must be diagonal. The Yukawa coupling matrices and the fermion mass matrices are not necessarily hermitian and they can be diagonalized with positive eigenvalues by biunitary tranformations:

$$\mathbf{m}_{u,d,e} = V_L^{(u,d,e)\dagger}\mathbf{m}^{(u,d,e)}V_R^{(u,d,e)} \tag{1.15}$$

where $V_{L,R}^{(u,d,e)} = V_{L,R}^{(u,d,e)\dagger}$ are unitary matrices and $\mathbf{m}_{u,d,e}$ is the diagonal matrix of the eigenvalues, which are the masses of leptons and quarks, e.g. $\mathbf{m}_d =$

diag(m_d, m_s, m_b) with $m_{d,s,b} = y_{d,s,b} v_w$. The mass spectrum of fermions is [111]:

$$
\begin{aligned}
m_e &= 0.5109989461(31)\,\text{MeV} \quad & m_\mu &= 105.6583745(24)\,\text{MeV} \\
m_\tau &= 1.77686(12)\,\text{GeV} & & \quad (1.16)\\
m_d(2\text{GeV}) &= 4.67^{+0.48}_{-0.17}\,\text{MeV} & m_s(2\text{GeV}) &= 93.12(69)\,\text{MeV}\,[107] \\
m_b(m_b) &= 4.18(3)\,\text{GeV} & & \quad (1.17)\\
m_u(2\text{GeV}) &= 2.16^{+0.49}_{-0.26}\,\text{MeV} & m_c(m_c) &= 1.27(2)\,\text{GeV} \\
m_t &= 172.9(4)\,\text{GeV} & & \quad (1.18)
\end{aligned}
$$

It should be noted that since there is only one Higgs doublet, the Yukawa coupling matrices Y_f are proportional to the mass matrices $\mathbf{m}_f$ and the same transformation diagonalizes both mass matrices and Yukawa couplings, so:

- Yukawa couplings are not flavour universal, but they are flavour-diagonal in the mass basis and thus flavour-conserving.

This means that, unless higher order loop diagrams are used, there is no flavour changing process mediated by Higgses. It is useful focusing now on quarks. Gauge eigenstates of Eq. (1.6) in terms of mass eigenstates are from (1.15):

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L = V_L^{(d)} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \qquad \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L = V_L^{(u)} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \tag{1.19}$$

From (1.6), the charged current weak interactions of quarks are described by the Lagrangian density:

$$\frac{g}{\sqrt{2}} \overline{u_{Li}} \gamma^\mu d_{Li} W_\mu^+ + \text{h.c.} = \tag{1.20}$$

$$= \frac{g}{\sqrt{2}} \begin{pmatrix} u & c & t \end{pmatrix}_L V_L^{(u)\dagger} \gamma^\mu V_L^{(d)} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^\dagger + \text{h.c.} = \tag{1.21}$$

$$= \frac{g}{\sqrt{2}} \begin{pmatrix} u & c & t \end{pmatrix}_L \gamma^\mu V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^\dagger + \text{h.c.} \tag{1.22}$$

where $V_{\text{CKM}} = V_L^{(u)\dagger} V_L^{(d)}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Since $V_L^{(u)}$ and $V_L^{(d)}$ are generally different, the CKM matrix is non-trivial:

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1.23}$$

which in the two families approximation becomes the Cabibbo rotation. Then the "left" matrices $V_L^{(f)}$ determine the mixing matrices in charged currents, while the "right" matrices $V_R^{(f)}$ have no physical meaning in the SM context since up-right quarks do not couple with down-right quarks. In the context of the SM, V_{CKM} is unitary. Then it is worth underline that:

- charged current interactions are not flavour diagonal, the universality of the coupling (the couplings are equal for all the families) is hidden in the unitarity of V_{CKM}.

The weak neutral current which couples with the massive boson $Z = \cos\theta_W W^3 - \sin\theta_W B$ is:

$$\mathcal{L}_{\text{nc}} = \frac{g}{\cos\theta_W}\left(g_L^f \overline{f_L}\gamma^\mu f_L + g_R^f \overline{f_R}\gamma^\mu f_R\right) Z_\mu \tag{1.24}$$

with

$$g_L^f = t_3(f) - Q(f)\sin^2\theta_W \tag{1.25}$$
$$g_R^f = -Q(f)\sin^2\theta_W \tag{1.26}$$

So Z couplings are proportional to the unit matrix in the flavor space and when the quark gauge eigenstates are transformed through (1.19) into quark mass eigenstates, the unitarity of $V_L^{(u)}$ and $V_L^{(d)}$ implies that the couplings with Z have the same form in the new fields. So, unlike (1.22):

- the weak neutral currents are flavour-conserving and flavour universal.

this is a consequence of the fact that all fermions of a given charge and helicity have the same value of T_3, the third component of weak isospin.

Therefore the SM exhibits a remarkable feature: since no flavor mixing emerges in neutral currents coupled to Z boson and Higgs boson, there are no flavor-changing neutral currents (FCNC) at tree level. FCNC phenomena only emerge from radiative corrections [5, 6]. FCNC should also be proportional to (and then suppressed by) the off-diagonal elements of the mixing matrix. Moreover flavour changing phenomena are subjected to the Glashow-Iliopoulos-Maiani (GIM) suppression, which makes FCNC proportional to mass-squared differences between quarks. This is actually the mechanism who led to the prediction of c-quark from the suppression of the observed branching ratio of the decay $\text{Br}(K_L \to \mu^+\mu^-)$. In fact, the expected branching ratio from the box diagram as in Fig. 1.1 with u-quark alone is still too high with respect to the observed one. However, by

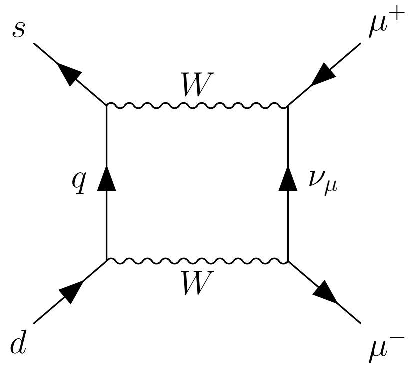

Figure 1.1: A box diagram for the decay $K_L \to \mu^+\mu^-$, $q = u, c, t$.

adding the c-quark, the two amplitudes from the two box diagram with u and c-quark running in the loop sum together. The amplitudes are proportional respectively to $V_{ud}V_{us}^*$ and $V_{cd}V_{cs}^*$. Then, if the masses of up-type quarks were equal, it would be that the total amplitude would be proportional to $V_{ud}V_{us}^* + V_{cd}V_{cs}^*$ which vanishes in the two generation limit.

As regards leptons, in the context of the SM neutrinos are massless, then mass eigentstates and gauge eigenstates are not distinguishable. However we know from oscillations that neutrinos are massive and that there exist the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix U_{PMNS}.

It is also clear from the Lagrangian in (1.22) that the W interactions are parity violating, since only left-handed particles participates in charged weak currents, as well the Z interactions are chiral and parity violating, as it emerges from (1.24) and (1.26). However it is necessary to investigate under which conditions CP symmetry can be violated in the SM. Under CP transformation, gauge bosons behave as:

$$\mathcal{CP}W_\mu^{\mp}(x^\mu)\mathcal{CP}^\dagger = -W^{\pm\mu}(x_\mu)e^{\mp i\phi_w} \tag{1.27}$$

$$\mathcal{CP}Z_\mu(x^\mu)\mathcal{CP}^\dagger = -Z^\mu(x_\mu) \tag{1.28}$$

with ϕ_w an arbitrary phase. Then, for neutral current weak interactions:

$$\mathcal{CP}(\overline{f_{L/R}}\gamma^\mu f_{L/R}Z_\mu)\mathcal{CP}^\dagger = \overline{f_{L/R}}\gamma_\mu f_{L/R}Z^\mu \tag{1.29}$$

Then Z boson mediated interactions seem CP invariant. However, as regards charged currents interactions:

$$\mathcal{CP}\overline{u_{L\alpha}}\gamma^\mu[V_{\text{CKM}}]_{\alpha\beta}d_{L\beta}W_\mu^+\mathcal{CP}^\dagger = \overline{d_{L\beta}}\gamma_\mu[V_{\text{CKM}}]_{\alpha\beta}u_{L\alpha}W^{-\mu}e^{i(\phi_w+\phi_\alpha-\phi_\beta)} \tag{1.30}$$

where ϕ_α, ϕ_β are two arbitrary phases, to be compared with the hermitian conjugated term:

$$\overline{d_{L\beta}}\gamma^\mu[V_{\text{CKM}}]^*_{\alpha\beta}u_{L\alpha}W^{-\mu} \tag{1.31}$$

Then CP invariance would hold only if $(\varphi_W, \varphi_i, \varphi_j)$ exist such that:

$$(V_{CKM})_{\alpha\beta} = e^{-i(\varphi_W+\varphi_\alpha-\varphi_\beta)}(V_{CKM})^*_{\alpha\beta} \tag{1.32}$$

The Higgs field transforms as:

$$\mathcal{CP}\phi^\pm(x_\mu)\mathcal{CP}^\dagger = \phi^\mp(x^\mu)e^{\pm i\phi_w} \tag{1.33}$$

then as regards Yukawa interactions in $\mathcal{L}_Y$:

$$\mathcal{CP}y_u\phi^+\overline{u_{R\alpha}}[V_{\text{CKM}}]_{\alpha\beta}d_{L\beta}\mathcal{CP}^\dagger = e^{i(\phi_w+\phi_\alpha-\phi_\beta)}y_u\phi^-\overline{d_{L\beta}}[V_{\text{CKM}}]_{\alpha\beta}u_{R\alpha} \tag{1.34}$$

then the same result is obtained. Therefore CP violation is strictly related to flavour changing: the CKM matrix is the only possible source of CP violation in the Standard Model.

Indeed, one has the freedom to rephase the quark fields, and thus arbitrarly change, and eliminate, the phases of $2N_f-1$ matrix elements of V_{CKM} (which thus are meaningless phases) where N_f is the number of families. Also if two quarks with same charge were degenarate, one angle and one phase could be removed. Then CP violation can occur only if one rephasing-invariant of the CKM matrix cannot be made real by any choice of the arbitrary phases. A unitary matrices of dimension N_f can be parametrized by $N_f^2-(2N_f-1)$ parameters, where $\frac{N_f(N_f-1)}{2}$ can be identified with Euler angles, and so $\frac{(N_f-1)(N_f-2)}{2}$ are physical phases. Thus it can be noted that, in case of two generations, the mixing matrix is a 2×2 unitary matrix and all phases can be absorbed by rephasing of the fields, then there is no physical phase and so no CP violation. If instead $N_f = 3$ then there is one physically meaningful phase in the 3×3 V_{CKM} matrix which then can generate CP violation. In particular it is usually defined the Jarlskog invariant:

$$J = \text{Im}(V_{us}V_{cb}V^*_{ub}V^*_{cs}) \tag{1.35}$$

From unitarity, for every i, j, k, l the imginary part of the quartets $V_{ij}V_{kl}V^*_{il}V^*_{kj}$ is equal to J up to their sign. Then in order to have CP violation it is needed that:

$$J \neq 0 \tag{1.36}$$

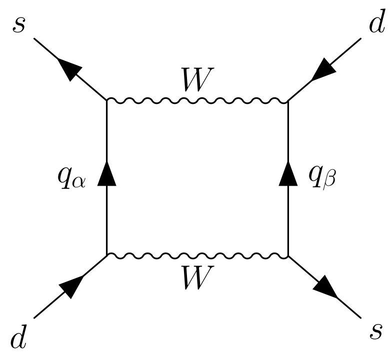

Figure 1.2: A box diagram for $K^0 \leftrightarrow \bar{K}^0$ mixing, $q_\alpha = u, c, t$.

The CKM matrix is usually parameterized as:

$$V_{\text{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{1.37}$$

c_{ij} and s_{ij} stand for $\cos\theta_{ij}$ and $\sin\theta_{ij}$, respectively. The Wolfenstein paramerization instead expands the matrix elements in the parameter λ:

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \tag{1.38}$$

The SM fit gives $\lambda = 0.22453(44)$.

As said before, though in the SM there are no flavour-changing neutral currents at tree level, they can still be induced by higher order loop diagrams although they are further suppressed by the magnitude of CKM elements and by GIM suppression. Moreover, with three generations of fermions, there can be a CP-violating phase in the mixing matrix, which would imply the existence of CP-violating measurable quantities. All these effects are testable for example in neutral meson systems: K^0-$\bar{K}^0$, B^0-$\bar{B}^0$, B^0_s-$\bar{B}^0_s$, D^0-$\bar{D}^0$, (see for example Fig. 1.2). At present, the majority of experimental data on flavor changing and CP violating processes are in good agreement with the SM predictions. In extensions of the SM, FCNC are likely to appear at tree level. Then very stringent constraints may arise on the energy scale of new physics (NP).

1.2 Beyond the Standard Model

There are many reasons to think that the SM is not the complete theory. The SM contains no renormalizable couplings that could generate the neutrino masses. There is no explanation for dark matter, dark energy. CP violation predicted by the SM is not enough to explain baryogenesis, only mentioning some of the open problems. However the SM has been extremely successfull in describing experimental data. Then it should be considered a correct theory at presently available energies which is although an effective field theory of a more fundamental theroy showing up at higher energy scale. For example, the SM cannot be valid at energies above $M_{\rm Planck} \sim 10^{19}$ GeV. Also neutrino masses in the seesaw scenario would require some new physics at about $\sim 10^{15}$ GeV. As speaking of new physics at some energy scale, the value of the Higgs mass turns out to require a huge fine tuning of the Higgs potential, which is called the hierarchy problem.

The Higgs potential has the form:

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda_\phi(\phi^\dagger\phi)^2 \tag{1.39}$$

Suppose the bare mass of the Higgs is μ_0. The loop contributions to μ_0^2 quadratically depend on the cutoff of the theory Λ:

$$\delta\mu_0^2 \sim f\Lambda^2 \tag{1.40}$$

where f stands for dimensionless couplings. The EW breaking scale is $v_w^2 = \mu^2/(2\lambda_\phi)$ and we know from W, Z masses that $v_w = 174$ GeV, while the Higgs mass is $\mu = 2\sqrt{\lambda_\phi}v_w = 125$ GeV, that is $\lambda_\phi = 0.13$. These values are the physical measured quantities which include the quadratically divergent loop corrections to the bare mass: $\mu^2 = \mu_0^2 + \delta\mu_0^2$. Λ is the energy scale at which the low energy effective field theory is cut-off. Then Λ is expected to be the mass scale of the new physics showing up at that energy. It is believed that such higher energy scale exists. For example Λ could be the Planck scale $M_{\rm Pl} \sim 10^{19}$ GeV, or the Grand Unification scale $M_{\rm GUT} \sim 10^{15}$ GeV, but a huge ratio Λ/v_w gives rise to the so called hierarchy problem. In fact, if $\Lambda \gg$ TeV, then from (1.40) $\delta\mu_0^2 \gg \mu_0^2$. That is, in order to obtain the physical mass squared $\mu^2 \sim v_w$, an extreme fine-tuning is needed for the bare parameters of the potential to obtain the cancellation between μ_0^2 and $\delta\mu_0^2$ which seems highly unnatural. Then new physics should be around the TeV scale in order to avoid the huge fine-tuning.

The hierarchy problem regards specifically elementary scalars. As regards fermions and gauge bosons, their masses are protected by symmetries. In fact, in the limit of the masses set to zero, the Lagrangian acquires new symmetries which protect the masses from unsuppressed loop corrections, chiral symmetry and unbroken gauge symmetry respectively for fermions and gauge bosons. The effect is that quantum corrections are proportional to the mass itself and only logarithmically dependent on the cutoff: $\Delta m_f \propto m_f \log(\Lambda/m_f)$ and $\Delta M_W^2 \propto M_W^2 \log(\Lambda/M_W)$. In fact, the hierarchy problem is a naturalness problem. It is natural to expect that dimensionless ratios between parameters of the theory are all of order unity. A quantity can be expected to be small only if the underlying theory becomes more symmetric as that quantity tends to zero, which is 't Hooft's principle of naturalness [8].

Then it would be helpful to find a more fundamental theory at some lower scale, in particular the hierarchy problem and dark matter point towards new physics at a scale of few TeV.

However, as already stated, experimental data on flavor changing and CP violating processes are in good agreement with the SM predictions, and in the SM FCNC are strongly suppressed. Then if new physics shows no suppression for FCNC, its scale must be above $\sim$ 10 PeV. As an example, if the effective operator of the neutral K-mesons system is written as:

$$\frac{1}{\Lambda^2}(\overline{s_L}\gamma^\mu d_L)^2 \tag{1.41}$$

then the scale of the CP violating operator contribution in the SM is $\Lambda = 1.5 \cdot 10^4$ TeV, and for the CP conserving part about $1.4 \cdot 10^3$ TeV. Nevertheless flavour physics can be the door showing new physics. It already happened in the past, as described in the previous section for the prediction of the c-quark, as well as for the prediction of the third generation after the measurement of the CP violating parameter ϵ_K, or for neutrino flavour oscillations implying neutrino masses. Moreover mixing between families is the only source of CP violation in the SM, but the CKM phase alone cannot account for baryogenesis. Besides, it clearly emerges that flavour physics shows many mysterious features.

- Family replication is totally unexplained, neither a deep reason is known for having three families.
- Even neglecting neutrino masses, the mass spectrum of fermions (and so eigenvalues of Yukawa matrices) spreads for more than five orders of magnitude.
- The CKM matrix shows a hierarchical pattern with rather small mixing angles between the different families and with the largest mixing between

adjacent generations. Yukawa matrices remain arbitrary in the SM. In a sense, the SM is technically natural since it can tolerate any Yukawa matrices Y_f^{ij}, but it tells nothing about their structures.

- As regards the lepton sector, the origin of neutrino masses is not included in the SM. Moreover the there is no hierarchy in neutrino mixings and mixing angles are quite large.

For all these reasons it seems reasonable to investigate the origin of families and flavour changing phenomena.

If SM is an effective theory, then other operators of dimension higher than four may emerge in the theory. The first evident example is that the SM contains no renormalizable couplings which could generate neutrino masses. The lowest order couplings relevant for the neutrino masses are dimension-5:

$$\frac{Y_\nu^{ij}}{\mathcal{M}} \phi\phi\ell_{Li}^T C\ell_{Lj} + \text{h.c.} \tag{1.42}$$

where C is the charge conjugation matrix. Operator (1.42) can be induced by exchange of some heavy right-handed neutrinos, which is a heavy neutral fermion with mass related to $\mathcal{M} \gg v_w$ (seesaw mechanism).

Then there can be dimension-6 operators such as:

$$\frac{c_{ij}}{\Lambda^2}(\overline{q_{Li}}\gamma_\mu q_{Lj})^2 + \frac{k_{ij\alpha\beta}}{\Lambda^2}\overline{q_{Li}}\gamma_\mu q_{Lj}\overline{\ell_{L\alpha}}\gamma^\mu \ell_{L\beta} + ... \tag{1.43}$$

which for example affect neutral mesons decays and mixing, as in (1.41), strictly constrained by experimental data. Then coefficients such as c, k cannot be all generic $O(1)$ parameters. If there is new physics at the TeV scale, then its flavour structure must be highly non-generic.

Also fermion masses can be induced by higher order operators. In the context of a UV-complete picture higher order operators can be induced through renormalizable interactions, as a result of integrating out heavy fermion. One can introduce also a vector-like set of charged heavy fermions, having the same quantum numbers as the usual quark and lepton species. Vector-like fermions can exist in string theories and they also emerge in GUT theories larger than $SU(5)$. In analogy to the mechanism for neutrinos, this mechanism is also known as "universal seesaw".

As remarked before, Yukawa interactions break the global symmetry $SU(3)^5$. The Yukawa couplings can does be regarded as spurions, transforming in the mixed representation of $Y_u \sim (3, \bar{3}, 1)$, $Y_d \sim (3, 1, \bar{3})$ in the case of the quark symmetry $SU(3)_q \times SU(3)_u \times SU(3)_d$. This means that Yukawa matrices are considered as fields with these transformation properties in order to construct terms of the Lagrangian, both effective operators and new renormalizable terms, formally invariant under the global symmetry. In this way the principle of *minimal flavour violation* (MFV) can be realized [9, 10, 11], which provide a suppression for flavour changing processes at low energies.

A useful tool which can be used in order to build a model for fermion masses and mixings can be mass matrices textures. Relations between the fermion masses and CKM angles can be obtained by considering mass matrices with reduced number of free parameters. In particular, certain elements in the Yukawa constant matrices can be set to zero (so called "zero textures"). A predictive and useful texture was suggested by Fritzsch [12, 13, 14]:

$$Y_f = \begin{pmatrix} 0 & A_f & 0 \\ A'_f & 0 & B_f \\ 0 & B'_f & C_f \end{pmatrix} \tag{1.44}$$

It implies that the fermion mass generation starts from the third family (C_f is assumed to be the largest entry) and proceeds to lighter families through the mixing terms.

1.2.1 SUSY + GUT

One attractive beyond the SM (BSM) scenario is related to the concepts of supersymmetry (SUSY) and grand unification theories (GUT). Supersymmetry is a symmetry relating bosons and fermions, which has to be broken at some mass scale. Then also the Higgs boson is related to its fermionic partner. The chiral symmetry of the fermionic partner also protect the Higgs mass from unsuppressed quantum corrections. Present data exclude the non-supersymmetric $SU(5)$ [15, 16], in supersymmetric $SU(5)$ instead the running gauge constants given at the energy scale $\mu = M_Z$ in the evolution to higher scales join at the scale $M \sim 10^{16}$ GeV [17]. Hence, at this scale the $SU(3) \times SU(2) \times U(1)$ symmetry can be consistently embedded into $SU(5)$ while softly broken (at the scale $m_S \sim 1TeV$) supersymmetry can solve the hierarchy problem in the context of GUT. $SU(5)$ grand unified theory unifies ℓ_L and d^c_L fragments of each family in $\bar{5}$-plets and e^c_L, u^c_L and Q_L fragments in 10-plets:

$$\bar{5}_L = (\ell, d^c)_L \qquad\qquad 10_L = (e^c, u^c, Q)_L \tag{1.45}$$

The relevant terms for fermion masses are:

$$\lambda_u^{ij} 10_i H 10_j + \lambda_d^{ij} 10_i \bar{H} \bar{5}_j \tag{1.46}$$

The $SU(5)$ model brings to $b\tau$ Yukawa unification, which after accounting for the RG running leads to the right ratio for masses of bottom and tau and to a large value for the top mass. However the other predictions on the Yukawas $\lambda_s = \lambda\mu$, $\lambda_d = \lambda_e$ are wrong. Moreover there is no explanation neither for the fermion mass hierarchy nor for the CKM mixing pattern. Then SUSY GUT alone cannot account for the structure of Yukawa matrices. For example, as described below, a symmetry between families can be added.

$SU(5)$ is only technically natural in solving the hierarchy problem, due to non-renormalization theorem. However, there are extensions of $SU(5)$ which can give more natural solutions and also can shed some more light on the origin of fermion masses. These are related to extension of gauge symmetry $SU(5)$ to $SO(10)$ [18] or $SU(6)$ [19].

In particular, in $SO(10)$ all fermions of one family are unified in a spinorial representation $16 = (q, l, u^c, d^c, e^c, N^c)_L$, where N^c is the singlet part associated with the "right-handed" neutrino involved in seesaw mechanism. It can be broken down to the SM also via a Pati-Salam symmetry $SU(4) \times SU(2) \times SU(2)$ [20].

Alternatively the SUSY $SU(6)$ model can be considered. A special feature of the $SU(6)$ theory is that the Higgs sector does not contain a specific set for electroweak symmetry breaking only. The scalar content is the minimal Higgs content needed for the local $SU(6)$ symmetry breaking down to $SU(3) \times SU(2) \times U(1)$. Higgs doublets emerge as Goldstone modes of the accidental global symmetry. The couplings relevant for fermion masses have to explicitly violate the global symmetry, otherwise fermions remain massless after the GUT symmetry breaking. This constraint leads to possibilities to explain fermion mass hierarchies and mixing pattern even in completely 'democratic' approach, without the horizontal symmetry. In particular, except top quark, fermion masses can emerge only through higher order operators [23]. Some symmetry reason is needed in order to avoid terms breaking the global symmetry in the Higgs potential. In this case discrete symmetry turns to be useful [24, 25].

In generic SUSY or SUSY GUT context, there emerges new flavor-changing problem associated to the fact that supersymmetry in itself does not require any relation between the fermion Yukawa terms and soft supersymmetry breaking terms. For example, contributions in neutral Kaon mixing $K^0 - \bar{K}^0$ emerge via gluino boxes. In general case, the flavor changing effects can be too strong with respect to experimental limits unless the soft supersymmetry breaking scale is larger that 100 TeV or so, which makes supersymmetry unnatural in the sense of flavor conservation in neutral transitions. However, there are possibilities of realizing minimal flavor violation (MFV) which align the SSB terms with the Yukawa terms [21] by making use of inter-family $SU(3)$ symmetries discussed in the last part of this introduction.

1.2.2 Other models Beyond the Standard Model

Some different mechanisms which were proposed in order to address SM problems are listed here.

Radiative mechanism Masses of the light fermions could arise as a radiative effect from the tree-level mass of the heavy family. Namely, if due to some reasons only fermions of the third family have tree-level masses, masses of the 2nd family can emerge at the 1-loop level and the 1st family can become massive only at 2-loops. Then the inter-family hierarchy is generated. Radiative models however provide a rather qualitative explanation to the fermion mass hierarchy and generally fail in predictivity. In particular, they cannot reproduce zero textures for mass matrices. Moreover, it is very difficult to obtain a quantitatively correct picture and also to avoid dangerous flavour changing phenomena. A consistent and predictive readiative approach was suggested in Ref. [22], where fermion mass hierarchy is first radiatively generated in the 'hidden' sector of the heavy vectorlike fermions and the transferred in an inverted way to the usual quarks and leptons by means of the "universal seesaw" mechanism. However, these models cannot be valid in the SUSY GUT context: at scales much larger than m_S the loop corrections will be suppressed by supersymmetry. Nevertheless, the message lasts that the lighter fermion masses could be due to higher order operators. In gauge theories the vanishing of certain mass terms at the tree-level can occur as a consequence of the representation content of the fields in the theory, or due to some inter-family symmetry. In radiative scenario these mass terms then can emerge in the effective action as operators of dimension $d > 4$ (i.e. involving more than one scalar leg). Within SUSY frames one could think of some tree level mechanism that could generate the relevant effective operators with successively increasing dimension, and thus explain the observed mass hierarchy.

Composite Higgs In composite Higgs models the scalar fields are no more elementary particles, rather the Higgs boson is a bound state of more fundamental fermions strongly interacting at the weak scale because of a dynamics which is confining at a certain scale. In particular, the strongly coupled sector can exhibit an approximate global symmetry and the Higgs boson can be identified as a pseudo-Nambu-Goldstone boson arising from the breaking of that approximate global symmetry. In this way the strong dynamics can naturally produce a composite Higgs doublet which is lighter than the the generic scale of composites [26, 27]. Then partial compositeness is a way to address the generation of Yukawa couplings in this scenario [28]. Elementary fermions couple to the strong sector by linearly mixing with composite fermionic states. The lighter mass eigenstates are the SM fermions, which are then partially composite, a linear combination of elementary and composite fields. The Yukawa couplings of the composites can be generic $O(1)$ numbers without any structure (anarchic flavor) while the mixings between the elementary and the composite states acquire hierarchical structure due to large anomalous dimensions of the composite states. Flavor violations in this anarchic scenario are protected by the RS-GIM mechanism [29], however the RS-GIM mechanism with completely anarchic Yukawa couplings is not sufficient to avoid the flavor constraints [30]. Alternatively a flavor symmetry can be imposed on the composite sector in order to obtain a minimal flavor violation (MFV) scenario [31, 32].

Extra dimensions In this scenario the variation of the fundamental energy scale along the extra dimension leads to a solution of the hierarchy problem (for a review see for example [33]). In particular, warped extra dimensions were proposed by Randall and Sundrum (RS) [34]. This model is dual to the composite Higgs scenario (AdS/CFT correspondance). The scalar is an extra dimensional component of the gauge field, which correspnds to the identification of the Higgs with a Goldstone boson of a spontaneously broken global symmetry. The Higgs field is localized at the infrared (IR) boundary. The weak scale is small because the warp factor in the IR provides a huge suppression, while the graviton is ultraviolet (UV) localized and the Planck scale is not suppressed. Also SM fermions can propagate in the five-dimensional spacetime [35]. Then exponentially suppressed Yukawa couplings can be obtained by localizing the fermion zero-modes towards the UV boundary, except for the topo quarks, which should be localized towards the IR-boundary.

1.2.3 Horizontal symmetries

However the key for understanding the fermion mass and mixing pattern may lie in symmetry principles. It is natural to explain small couplings by looking for symmetries arising when those couplings are set equal to zero. Then it seems appealing to link the hierarchy in fermion mass ratios to some symmetry. The fermion masses should be kept to be zero if the exact symmetry holds. Mass terms can be generated only at higher order, in symmetry breaking interactions. Then, in order to forbid mass terms, left and right-handed components should transform differently under the unbroken symmetry, that means, the symmetry cannot be a vector-like symmetry.

In Ref. [36] an abelian symmetry was considered in order to explain the quark mass matrix. Different values of the charge R of the $U(1)$ symmetry are assigned to each quark species. All SM fermions are massless in the limit of exact symmetry, since left and right-handed species have different charges.

The existence of several heavy vector-like fermions is assumed, which acquire masses from the non-zero VEV $\langle\phi_0\rangle$ of a scalar field neutral under the new $U(1)$ symmetry. The symmetry breaking which allows (together with EW breaking) non-zero masses for SM fermions is due to a $U(1)$ charged scalar instead, whose VEV $\langle\phi_1\rangle$ is lower than the mass scale of heavy fermions. The usual Higgs boson breaks weak $SU(2)$ but cannot generate fermion masses if the abelian symmetry is unbroken since, for example, it can be neutral under $U(1)$ symmetry. Masses arise from higher order operators induced via the exchange of heavy fermions having different values of the new charge R interacting through the insertion of the charged scalar field:

$$m_{ij} = g_{ij} \left(\frac{\langle\phi_1\rangle}{M} \right)^{n_{ij}} v_w \tag{1.47}$$

where g_{ij} are order $O(1)$ numbers, M is the mass scale of heavy fermions, v_w is the electroweak scale, n_{ij} is the difference in R quantum numbers between the corresponding left and right-handed components. Then all Yukawa couplings can be of order $O(1)$, and the hierarchical structure of fermion masses is due to the number of insertions of scalars needed to compensate the difference in R quantum numbers between right and left-handed fermions. In fact, in this way large quark mass ratios are obtained and then the order of magnitude of the weak mixing angles can be related to these mass ratios.

Alternatively, a non-abelian gauge horizontal flavor symmetry G_H between the fermion families can be considered [37, 38, 39, 40, 41, 42, 43]. Fermion masses cannot emerge if G_H symmetry is unbroken. Thus, G_H cannot be a vector-like symmetry but it should have a chiral character transforming the LH and RH particle species in different representations. Then the form of the Yukawa matrices $Y_{u,d,e}$ in (1.12) can be determined by the G_H symmetry breaking pattern, i.e. by the VEV structure of the horizontal scalar fields (known as flavons) which spontaneously break G_H. In this picture the fermion masses emerge from higher order operators involving, besides the Higgs doublet ϕ, also flavon scalars which transfer their VEV structure to the Yukawa matrices $Y_{u,d,e}$. Since the fermion mass generation is possible only after the spontaneous breaking, then the fermion mass hierarchy can directly reflect the hierarchy between the scales of this breaking (hypothesis of horizontal hierarchies [39, 40]). These effective higher order operators (“projective" operators) in the UV-complete renormalizable theory can be obtained via integrating out some extra heavy fields, scalars [37, 38] or verctor-like fermions [39, 40, 41, 42, 43]. The symmetry $SU(3)_H$ seems an attractive choice, unifying all the families. Then the family symmetry can be considered in the context of Grand Unified theories, for example $SU(5) \times SU(3)_H$ [40], with the LH fermions of the three families transforming as triplets and the RH ones as anti-triplets. In this case the VEV pattern can lead directly to the Fritzsch texture.

In the context of supersymmetry, such horizontal symmetries can lead to interesting relations between the mass spectra of fermions and their superpartners and naturally realize the minimal flavor violation scenarios [9, 10, 11, 21].

Within the same lines one can consider the models with a reduced horizontal symmetry $SU(2)_H$ acting bewteen first two families and the third family getting mass from the direct Yukawa couplings.

1.3 Outline

In chapter 2 of this thesis $SU(3)_q \times SU(3)_u \times SU(3)_d \times SU(3)_\ell \times SU(3)_e$ symmetry between families is considered as gauge symmetry extending the SM. It is shown how the mass spectrum and mixing patterns can be naturally generated. Then it is argued that the energy scale of the symmetry breaking, and so of the masses of new gauge bosons, can be as light as few TeV, thanks to a custodial symmetry [45]. FCNC and other experimental constraints are analyzed for this purpose. It is also shown that family symmetry between left-handed leptons can be low enough (few TeV) to be a solution to the 4σ discrepancy from unitarity of the first row of CKM matrix [46].

This CKM unitarity problem is analyzed in details in chapter 3, together with the possibility to cure the anomaly by allowing the Fermi constant to be different from the muon decay constant. This scenario can be realized considering a new effective operator interfering with the SM muon decay as the one generated by flavour changing gauge bosons. Also the neutron lifetime problem, that is about 4σ discrepancy between the neutron lifetimes measured in beam and trap experiments, is analyzed in the light of the new determinations of the CKM matrix elements.

In chapter 4 mixing of ordinary quarks with vector-like quarks is tested as one possible solution to the CKM unitarity problem. Again the mass scale of these particles should be of few TeV, since a quite large mixing with SM fermions is needed in order to recover unitarity. Then the feasibility of this scenario is analyzed by imposing the experimental constraints from flavour changing processes and electroweak observables. Vector-like fermions exist in several scenarios beyond the SM, as GUT theories or string theories, but also in the context of flavour physics as UV content of effective operators generating the fermion masses, e.g. vectorlike fermions emerge in the scenario of family symmetries, as also shown in this thesis.

Chapter 2

Family symmetries

The inter-family hierarchy between quark species exhibits an approximate scaling low which can be parametrized by a small parameter $\epsilon \sim 1/20$:

$$\begin{aligned} m_d : m_s : m_b &\sim \epsilon^2 : \epsilon : 1 \\ m_u : m_c : m_t &\sim \epsilon^4 : \epsilon^2 : 1 \end{aligned} \tag{2.1}$$

Furthermore, one can observe that the CKM angles of quark mixing in V_{CKM} can be parametrized with the same parameter ϵ too:

$$\sin\theta_{12} \sim \sqrt{\epsilon} \sim 4\epsilon; \quad \sin\theta_{23} \sim \epsilon; \quad \sin\theta_{13} \sim \epsilon^2 \tag{2.2}$$

In fact, there are intriguing relations between masses and mixing angles, such as the phenomenologically successful relation for the Cabibbo angle $\sin\theta_{12} = \sqrt{m_d/m_s}$ [12]. Also the hierarchy among charged leptons can be parametrized with ϵ, with some factor k of order $O(1)$:

$$m_e : m_\mu : m_\tau \sim k^{-1}\epsilon^2 : k\epsilon : k \tag{2.3}$$

with $k \simeq 3$, or alternatively with two scaling parameters ϵ and $\tilde{\epsilon}$:

$$m_e : m_\mu : m_\tau \sim \tilde{\epsilon}\epsilon : \epsilon : 1 \tag{2.4}$$

The neutrino mixing angles are much larger than mixing angles of quarks and presumably there is no significant hierarchy between the neutrino masses.

The correlations between the fermion mass spectrum and mixing pattern may suggest an intrinsic connection with the features of the new physics. Since, as already stated, it is natural to explain small couplings by symmetry principles, the hierarchy in fermion mass ratios may be associated to the additional symmetry showing up when Yukawa couplings are set to zero, remembering that left and right-handed components should transform differently under the unbroken symmetry in order to forbid mass terms.

In the limit of vanishing Yukawa couplings, $Y_f \to 0$, the SM acquires a maximal global chiral symmetry

$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d \tag{2.5}$$

under which fermion species transform as triplets of independent $U(3)$ groups respectively as $\ell_L \sim 3_\ell$, $e_R \sim 3_e$, $q_L \sim 3_q$, $u_R \sim 3_u$, $d_R \sim 3_d$. The Yukawa couplings (1.12) can be induced by the VEVs of flavons in mixed representations of these symmetry groups. One can consider the higher order operators such as for leptons

$$\frac{X_e}{M}\phi\, \overline{\ell_L} e_R + \text{h.c.} \tag{2.6}$$

where $X_e \sim (3_\ell, \bar{3}_e)$ is a flavon in mixed representation of $U(3)_\ell \times U(3)_e$ which can be also viewed as composite tensor product $3_\ell \times \bar{3}_e$ of scalars in fundamental representations of $U(3)_\ell$ and $U(3)_e$.

In the SM extensions the maximal flavor symmetry (2.5) reduces to a smaller symmetry. E.g. in the context of $SU(5)$ grand unified theory (GUT) which unifies ℓ_L and d^c_L fragments of each family in $\bar{5}$-plets and e^c_L, u^c_L and Q_L fragments in 10-plets the maximal symmetry reduces to two factors $U(3)_\ell \times U(3)_e$:

$$\bar{5}_L = (\ell, d^c)_L \sim (3_\ell, 1), \qquad 10_L = (e^c, u^c, Q)_L \sim (1, \bar{3}_e) \tag{2.7}$$

In the context of $SO(10)$ GUT all fermions of one family including the RH neutrino N_R reside in the spinor multiplet $16_L = (\bar{5} + 10 + 1)_L$. Hence, there can be only one chiral symmetry $U(3)$ between three families of 16-plets, with all LH fermions ℓ_L, Q_L transforming as triplets and the RH ones N_R, e_R, u_R, d_R as anti-triplets [47, 48, 49, 50].

It is interesting to consider the non-abelian $SU(3)$ factors of this maximal flavor symmetry (2.5), or its GUT-restricted versions, as a gauge symmetry G_H (gauging of chiral $U(1)$ factors is problematic because of anomalies, although there are models in which anomalous gauge symmetry $U(1)_A$ is used as a flavor symmetry [51, 52, 53, 54, 55, 56]. The abelian $U(1)$ factors remain as global symmetries. Discovery of the flavor gauge bosons and/or related FCNC effects would point towards new BSM physics of flavor. However, a direct discovery at future accelerators can be realistic only if the scale of G_H symmetry breaking is rather low, in the range of few TeV.

Therefore, the following questions arise: (i) for which choice of symmetry group G_H one can relate the fermion mass hierarchy to its breaking pattern, and (ii) which is the minimal scale of G_H symmetry allowed by present experimental limits – namely, can this scale be low enough to have G_H flavor bosons potentially within the experimental reach?

In the context of GUT theories new gauge interactions require the mass scale of gauge bosons to be much higher than few TeV. Also in $SU(5)$ scenario, gauge bosons associated to $SU(3)_\ell \times SU(3)_e$, with fermion fields in the representations indicated in (2.7), would give rise to unsuppressed contributions to flavour changing interactions between quarks and leptons like leptonic and semileptonic mesons decays. For example, there would be a tree level contribution to K mesons decays $K^- \to \pi^-\mu^-e^+$, $K_L^0 \to e^\pm\mu^\mp$. Experimental limits on these decays are $\mathrm{Br}(K^- \to \pi^-\mu^-e^+) < 1.3 \cdot 10^{-11}$, $\mathrm{Br}(K_L^0 \to e^\pm\mu^\mp) < 4.7 \cdot 10^{-12}$ both at 90% C.L. [111]. These branching ratios can be roughly estimated e.g. by exploiting the measured $\mathrm{Br}(K^+ \to \pi^0\mu^+\nu_\mu) = 3.352 \pm 0.033\%$ and $\mathrm{Br}(K^+ \to \mu^+\nu_\mu) = 63.56 \pm 0.11\%$ respectively [111], then assuming the gauge coupling constant of $O(1)$, the limits constraint the mass scales of gauge bosons to be higher than 100 TeV and 300 TeV respectively.

In this thesis horizontal symmetries are analyzed in the context of the SM gauge group. This chapter focuses in particular on the horizontal symmetry involving the lepton sector $U(3)_\ell \times U(3)_e$ whose non-abelian part $SU(3)_\ell \times SU(3)_e$ is gauged. The lepton mass hierarchy $m_\tau \gg m_\mu \gg m_e$ can be directly related to the hierarchy of $U(3)_e$ symmetry breaking scales, as well as large mixing of neutrinos reflect $U(3)_\ell$ breaking pattern.

In order to understand the mechanisms working in these symmetries, in the first section a model with only $SU(3)_e$ as a gauge symmetry is analyzed. Then it emerges clearly that lepton flavor violating (LFV) phenomena induced by gauge bosons can be strongly suppressed because of custodial symmetry. In fact, the mass scale of $SU(3)_e$ gauge bosons is allowed to be as low as 2 TeV thanks to the custodial property of $SU(2)_e$ subgroup, without contradicting the present experimental limits on the LFV processes [1]. Then also the gauge symmetry $SU(3)_\ell$ is considered. It comes out that $SU(3)_\ell$ gauge bosons can be a solution to the CKM unitarity problem, if their mass scale is about 6 - 7 TeV, which again can be reached thanks to custodial properties.

[1]One could consider also the case of vector-like horizontal symmetry $SU(3)_V$ under which both ℓ_L and e_R (and RH neutrinos N_R) all transform as a triplet, or its $SU(2)$ subgroup. Such a symmetry is anomaly-free and it also has a custodial property for suppression of flavor-changing [57] discussed in this section. However, it allows a degenerate spectrum between the fermion families in the exact symmetry limit, and thus does not meet the paradigm of HHH.

2.1 $SU(3)_e$

2.1.1 Lepton masses

In this section a simple model is discussed with a gauge symmetry $G_H = SU(3)_e$ transforming the RH leptons as a triplet $e_{R\alpha} = (e_1, e_2, e_3)_R^T$, while regarding LH leptons $\ell_{Li} = \ell_{1,2,3}$ $i = 1,2,3$ is just a family number, assuming for simplicity that $SU(3)_\ell$ is not a gauge symmetry, or broken at some higher scale without having substantial hierarchy between its breaking scales. The LH and RH lepton fields are in the following representations:

$$\ell_{Li} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \sim (2,-1,1), \qquad e_{R\alpha} \sim (1,-2,3_e) \tag{2.8}$$

where the multiplet content with respect to the EW $SU(2)\times U(1)$ and horizontal $SU(3)_e$ is indicated. This set of fermions is not anomaly free. The ways of the anomaly cancellation will be discussed in section 2.1.4.

There is only one Higgs doublet ϕ with the standard Higgs potential $V(\phi) = \lambda(|\phi|^2 - v_{\mathrm{w}}^2)^2$. However, the Yukawa couplings of ϕ with the fermions ℓ_{Li} and $e_{R\alpha}$ are forbidden by $SU(3)_e$ symmetry. So, for generating the lepton masses this symmetry should be broken.

For breaking $SU(3)_e$ three flavon fields are introduced ξ_n^α, $n = 1,2,3$, each transforming as $SU(3)_e$ (anti)triplet. Then, as regards charged leptons, masses emerge from the gauge invariant dimension 5 operators

$$\sum_n \frac{g_{in}\xi_n^\alpha}{M}\,\phi\,\overline{\ell_{Li}}e_{R\alpha} + \text{h.c.} \tag{2.9}$$

where g_{in} are order one constants (see upper diagram of Fig. 2.1). For having an UV-complete theory, one can consider these operators as induced form renormalizable terms via seesaw-like mechanism [39, 40]. E.g. one can integrate out from the following Yukawa Lagrangian

$$h\phi\overline{L_\alpha}e_{R\alpha} + M\overline{R_\alpha}L_\alpha + \sum_n g_{in}\xi_n^\alpha\overline{\ell_{Li}}R_\alpha \; + \; \text{h.c.} \tag{2.10}$$

the extra heavy vector-like lepton doublets

$$L_\alpha, R_\alpha = \begin{pmatrix} N_\alpha \\ E_\alpha \end{pmatrix}_{L,R} \sim (2,-1,3_e) \tag{2.11}$$

with a large Dirac mass M (see Fig. 2.1, lower diagram).

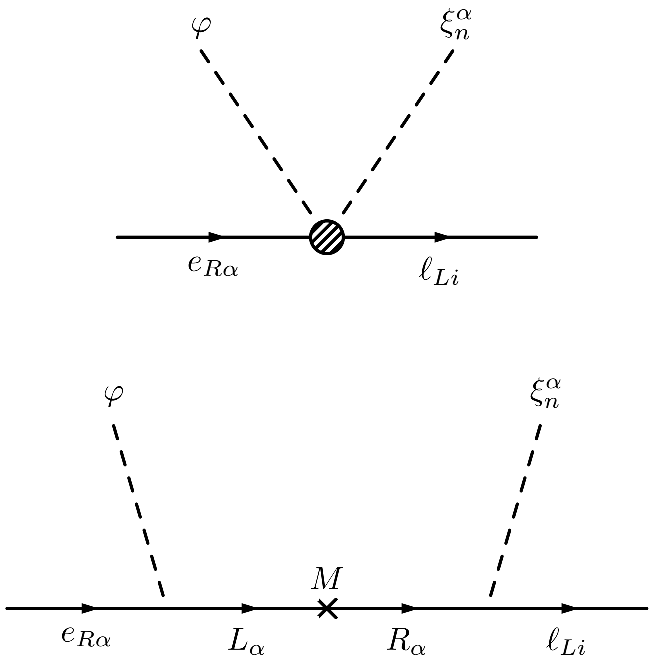

Figure 2.1: Upper diagram represents operator (2.9) and lower diagram shows how it can be obtained via seesaw exchange of heavy vector-like fermions (2.11).

The scales of symmetry breaking can be defined as:

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \tag{2.12}$$

with $v_3 > v_2$, so that $SU(3)_e$ is broken into its subgroup $SU(2)_e$ at a scale u_3 and completely broken at a scale u_2. Operator (2.9) has a global symmetry $U(3)_e = SU(3)_e \times U(1)_e$, where the abelian part $U(1)_e$ is related to the phase change of fermions $e_{R\alpha}$ and flavons ξ_n^α. In order to generate non-zero masses of all three leptons e, μ, τ, this global symmetry must be fully broken. This means that all three flavons ξ_n should have the non-zero VEVs with disoriented directions. In other words, the VEVs $\langle \xi_n^\alpha \rangle$ should form a rank-3 matrix, which is generically non-diagonal. Then, without loss of generality, one can choose superpositions of these fields $\xi_n \to U_{nm}\xi_m$ so that their VEVs are orthogonal and hence the matrix

$\langle\xi_n^\alpha\rangle$ becomes diagonal, $\langle\xi_n^\alpha\rangle = v_n\delta_n^\alpha$, or in explicit form

$$\langle\xi_1\rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}, \qquad \langle\xi_2\rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix}, \qquad \langle\xi_3\rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix} \tag{2.13}$$

ordered as $v_3 > v_2 > v_1$ reflecting the steps of the global symmetry breaking $U(3)_e \to U(2)_e \to U(1)_e \to$ nothing. By substituting these VEVs in operator (2.9), it reduces to the SM Yukawa couplings

$$Y_e^{i\alpha}\,\phi\,\overline{\ell_{Li}}e_{R\alpha} + \text{h.c.}, \quad Y_e^{i\alpha} = \sum_n \frac{g_{in}\langle\xi_n^\alpha\rangle}{M} = g_{i\alpha}\frac{v_\alpha}{M} \tag{2.14}$$

Without loss of generality, ℓ_{Li} states can be turned to the basis in which matrix g_{in} has a triangular form and the diagonal elements g_{33}, g_{22}, g_{11} are real. Then:

$$\begin{aligned} Y_e &= \frac{1}{M}\begin{pmatrix} g_{11}v_1 & 0 & 0 \\ g_{21}v_1 & g_{22}v_2 & 0 \\ g_{31}v_1 & g_{32}v_2 & g_{33}v_3 \end{pmatrix} \\ &= \frac{v_3}{M}\begin{pmatrix} g_{11}\tilde{\varepsilon}\varepsilon & 0 & 0 \\ g_{21}\tilde{\varepsilon}\varepsilon & g_{22}\varepsilon & 0 \\ g_{31}\tilde{\varepsilon}\varepsilon & g_{32}\varepsilon & g_{33} \end{pmatrix} \end{aligned} \tag{2.15}$$

where $v_2/v_3 = \varepsilon$ and $v_1/v_2 = \tilde{\varepsilon}$. The Yukawa matrix Y_e (and the mass matrix $M_e = Y_e v_\text{w}$) can be diagonalized via a bi-unitary transformation

$$Y_e \to V_L^\dagger Y_e V_R = \text{diag}\big(y_e, y_\mu, y_\tau\big) \tag{2.16}$$

Hence, modulo ~ 1 ratios of the Yukawa constants, the mass hierarchy $m_\tau : m_\mu : m_e$ corresponds to the hierarchy between the scales $v_3 : v_2 : v_1$. Namely, neglecting the small $\sim \varepsilon^2$ corrections, for the charges lepton masses it is obtained:

$$m_\tau = \frac{g_{33}v_\text{w}}{M}v_3\,, \qquad m_\mu = \frac{g_{22}v_\text{w}}{M}v_2\,, \qquad m_e = \frac{g_{11}v_\text{w}}{M}v_1 \tag{2.17}$$

It is worth discuss whether such a hierarchy of the VEVs can be natural. Since three flavons have identical quantum numbers, their scalar potential has a generic form:

$$\mathcal{V}(\xi) = \lambda_n\Big(\big|\xi_n\big|^2 - \frac{\mu_n^2}{2\lambda_n}\Big)^2 + \lambda_{klnm}\xi_k^\dagger\xi_l\xi_n^\dagger\xi_m + \left(\mu\xi_1\xi_2\xi_3 + \text{h.c.}\right) \tag{2.18}$$

All constants λ can be assumed of order 1, fluctuating say in the range $\lambda \sim 0.1$-1, and the mass terms μ_n^2, positive or negative, are of the same order, fluctuating say in the range of several TeV. The last (trilinear) coupling $\mu\epsilon_{\alpha\beta\gamma}\xi_1^\alpha\xi_2^\beta\xi_3^\gamma$ has a dimensional constant μ which is however allowed (by 't Hooft's naturalness principle) to be arbitrarily small since in the limit $\mu \to 0$ the Lagrangian acquires global $U(1)_e$ symmetry respected also by the Yukawa terms (2.9). In fact, this latter coupling softly breaks $U(1)_e$ and thus reduces the global symmetry $U(3)_e$ to $SU(3)_e$.

For full breaking of gauge $SU(3)_e$ symmetry, just two flavons with non-aligned VEVs are sufficient. An order of magnitude hierarchy between the scales v_2 and v_3, $v_2/v_3 \sim m_\mu/m_\tau$, can emerge due to some moderate conspiracy of parameters admitting a natural "spread", say within an order of magnitude, between the mass terms and coupling constants of ξ_2 and ξ_3 in (2.18). But large hierarchy $v_1/v_3 \sim m_e/m_\tau$ at first sight requires a strong fine tuning. However, small v_1 can be obtained naturally considering the case in which the VEV matrix $\langle\xi_n^\alpha\rangle$ has rank 2 in the limit $\mu \to 0$. This means that only two flavons, ξ_2 and ξ_3, get the VEVs oriented as in (2.13), because of their negative mass squared terms in (2.18), while the third flavon ξ_1 has a positive mass squared term, i.e. $\mu_1^2 < 0$, and in the limit $\mu = 0$ it remains VEVless. The ratio of VEVs $v_3/v_2 = (\mu_3/\mu_2)\sqrt{\lambda_2/\lambda_3}$ can be of one order of magnitude due to natural fluctuation of the involved parameters. Then for $\mu \neq 0$ the last term in (2.18) explicitly breaks global $U(1)_e$ symmetry and induces non-zero VEV $\langle\xi_1\rangle$:

$$v_1 = \frac{\mu v_2 v_3}{\mu_1^2} \tag{2.19}$$

Thus, taking μ small enough, say $\mu < v_2$, one can naturally get $v_1 \ll v_2$.

The unitary matrix V_R in (2.16) connecting the initial flavor basis of the RH leptons to the mass basis,

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_R = V_R \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R = \begin{pmatrix} V_{1e} & V_{1\mu} & V_{1\tau} \\ V_{2e} & V_{2\mu} & V_{2\tau} \\ V_{3e} & V_{3\mu} & V_{3\tau} \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R \tag{2.20}$$

has no physical meaning for the EW interactions, but it is meaningful for the LFV interactions mediated by the gauge bosons of $SU(3)_e$. For the mass matrix $M_e = Y_e v_w$ in (2.15), modulo ε^2 corrections, it is obtained $V_{1e}, V_{2\mu}, V_{3\tau} = 1$ and for non-diagonal elements:

$$V_{3\mu} = -\frac{g_{32}}{g_{33}}\varepsilon, \quad V_{2e} = -\frac{g_{21}}{g_{22}}\tilde{\varepsilon}, \quad V_{3e} = -\frac{g_{31}}{g_{33}}\tilde{\varepsilon}\varepsilon \tag{2.21}$$

The other elements in matrix V_R can be obtained from its unitarity.

As for the mixing matrix V_L of the LH charged leptons in (2.16), it is very close to unit matrix, with non-diagonal elements $\sim \varepsilon^2$. Therefore, the neutrino mixing angles are determined by the form of the neutrino mass matrix. The neutrino masses are induced by the higher order operator [2]:

$$\frac{Y_\nu^{ij}}{\mathcal{M}}\,\phi\phi\,\ell_{Li}^T C \ell_{Lj}\;+\;\text{h.c.}\,. \tag{2.22}$$

where $\mathcal{M}$ is a new scale which in the context of seesaw mechanism can be related to the Majorana masses of RH neutrinos. In this first scenario the states ℓ_{Li} are not distinguished by any symmetry and the matrix Y_ν^{ij} is a generic non-diagonal matrix, supposedly with all elements of the same order. Thus, the unitary matrix V_ν which diagonalizes it, $V_\nu^T Y_\nu V_\nu = Y_\nu^{\text{diag}}$, contains large rotations and the neutrino mixing angles are expected to be large.

2.1.2 $SU(3)_e$ gauge bosons

With the gauging of $SU(3)_e$, new horizontal interactions are set:

$$g\mathcal{F}_a^\mu J_{a\mu} \tag{2.23}$$

where g is the $SU(3)_e$ gauge coupling, λ^a are the Gell-Mann matrices and $\mathcal{F}_a^\mu$ are the gauge bosons of $SU(3)_e$, associated with the currents

$$J_{a\mu} = \frac{1}{2}\overline{\mathbf{e}_R}\lambda_a\gamma_\mu\mathbf{e}_R \tag{2.24}$$

where $\mathbf{e}_R = (e_1, e_2, e_3)_R^T$ denotes the triplet of the RH leptons. The gauge bosons can be visualized in matrix form:

$$\frac{\lambda_a}{2}\mathcal{F}_a = \frac{1}{2}\begin{pmatrix} \mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_1 - i\mathcal{F}_2 & \mathcal{F}_4 - i\mathcal{F}_5 \\ \mathcal{F}_1 + i\mathcal{F}_2 & -\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_6 - i\mathcal{F}_7 \\ \mathcal{F}_4 + i\mathcal{F}_5 & \mathcal{F}_6 + i\mathcal{F}_7 & -\frac{2}{\sqrt{3}}\mathcal{F}_8 \end{pmatrix} \tag{2.25}$$

Clearly, the horizontal currents, in particular those related to non-diagonal generators $\lambda_{1,2,4,5,6,7}$, are generically FCNC. Nevertheless, as we shall see below, the processes mediated by flavor bosons exhibit no LFV in the initial eigenstates basis e_{R1}, e_{R2}, e_{R3} of the flavor diagonal generators λ_3 and λ_8.

At low energies the flavor bosons induce four-fermion (current $\times$ current) interactions:

$$\mathcal{L}_{\text{eff}} = -\frac{g^2}{2}\,J_a^\mu\,(M^2)_{ab}^{-1}\,J_{b\mu} \tag{2.26}$$

where M^2_{ab} is the (symmetric) mass matrix of gauge bosons $\mathcal{F}^\mu_a$. In the flavon VEV basis (2.13) this matrix is essentially diagonal apart of a non-diagonal 2×2 block related to $\mathcal{F}_3$ - $\mathcal{F}_8$ mixing. Namely, for the masses of gauge bosons $\mathcal{F}^\mu_{4,5}$, $\mathcal{F}^\mu_{6,7}$ and $\mathcal{F}^\mu_{1,2}$ respectively:

$$\begin{aligned} M^2_{4,5} &= \frac{g^2}{2}(v_3^2 + v_1^2), \quad M^2_{6,7} = \frac{g^2}{2}(v_3^2 + v_2^2), \\ M^2_{1,2} &= \frac{g^2}{2}(v_2^2 + v_1^2) \end{aligned} \tag{2.27}$$

while for the mass matrix of $\mathcal{F}^\mu_3$ - $\mathcal{F}^\mu_8$ system it is obtained that:

$$M^2_{38} = \frac{g^2}{2} \begin{pmatrix} v_2^2 + v_1^2 & \frac{1}{\sqrt{3}}(v_1^2 - v_2^2) \\ \frac{1}{\sqrt{3}}(v_1^2 - v_2^2) & \frac{1}{3}(4v_3^2 + v_1^2 + v_2^2) \end{pmatrix} \tag{2.28}$$

Obviously, the factor g^2 in operators (2.26) cancels and their strength is determined solely by $SU(3)_e$ symmetry breaking scales v_2 and v_3. In the following we neglect a small contribution $v_1^2/v_2^2 = \tilde{\varepsilon}^2 \ll 1$ in the gauge boson mass terms and in respective effective operators. Then

$$\frac{g^2}{2}(M^2_{38})^{-1} = \frac{1}{v_2^2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{4v_3^2} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 3 \end{pmatrix} \tag{2.29}$$

Hence, from all operators (2.26) one can single out the operator $\mathcal{L}_2$ which is cut off by the smaller scale v_2:

$$\mathcal{L}_2 = -\frac{1}{v_2^2} \sum_{a=1}^{3} (J^\mu_a)^2 = -\frac{1}{4v_2^2} \sum_{a=1}^{3} (\overline{\mathbf{e}_R}\, \lambda_a \gamma^\mu \mathbf{e}_R)^2 \tag{2.30}$$

which involves only e_{R1} and e_{R2} states. Using Fierz identities for $\lambda_{1,2,3}$ which in fact are the Pauli matrices, this operator can be rewritten as

$$\mathcal{L}_2 = -\frac{1}{v_2^2}\, (J^\mu_0)^2 = -\frac{1}{4v_2^2}\, (\overline{\mathbf{e}_R}\, \lambda_0 \gamma^\mu \mathbf{e}_R)^2 \tag{2.31}$$

where $\lambda_0 = \mathrm{diag}(1,1,0)$. The remaining operators in (2.26) are related to the scale v_3:

$$\mathcal{L}_3 = -\frac{1}{v_3^2} \left[(\tilde{J}^\mu_3)^2 + (J^\mu_4)^2 + (J^\mu_5)^2 \right] - \frac{1}{v_3^2 + v_2^2} \left[(J^\mu_6)^2 + (J^\mu_7)^2 \right] \tag{2.32}$$

where the current $\tilde{J}_3^\mu = \frac{1}{2}J_3^\mu + \frac{\sqrt{3}}{2}J_8^\mu$ has a form $\frac{1}{2}\overline{\mathbf{e}_R}\tilde{\lambda}_3\gamma^\mu \mathbf{e}_R$ with $\tilde{\lambda}_3 = \mathrm{diag}(1,0,-1)$. Hence, $\tilde{\lambda}_3$, λ_4 and λ_5 form a $SU(2)$ subalgebra of $SU(3)_e$, and using the Fierz identities for these matrices the low energy Lagrangian in (2.32) can be rewritten as:

$$\begin{aligned}\mathcal{L}_3 &= -\frac{1}{v_3^2}\left[(\tilde{J}_0^\mu)^2 + (J_6^\mu + iJ_7^\mu)(J_{6\mu} - iJ_{7\mu})\right] = \\ &-\frac{\varepsilon^2}{4v_2^2}\left[(\overline{\mathbf{e}_R}\,\tilde{\lambda}_0\gamma^\mu \mathbf{e}_R)^2 + 4(\overline{e_{R2}}\gamma_\mu e_{R2})(\overline{e_{R3}}\gamma^\mu e_{R3})\right] \end{aligned} \tag{2.33}$$

where $\tilde{\lambda}_0 = \mathrm{diag}(1,0,1)$. So, operators $\mathcal{L}_2$ and $\mathcal{L}_3$ do not induce any LFV transition between e_{R1}, e_{R2}, e_{R3} states.

In the basis of the mass eigenstates (e,μ,τ) all currents involved in operators (2.31) and (2.33), including those related to λ_0 and $\tilde{\lambda}_0$, should be rotated with the matrix V_R (2.20), or in explicit form

$$J_{a\mu} = \left(\overline{e},\overline{\mu},\overline{\tau}\right)_R \gamma_\mu \frac{V_R^\dagger \lambda_a V_R}{2}\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R \tag{2.34}$$

In particular, in (e,μ,τ) basis λ_0 in operator (2.31) is deformed to $\lambda_V = V_R^\dagger \lambda_0 V_R$, or explicitly

$$\begin{aligned}\lambda_V &= \begin{pmatrix} 1-|V_{3e}|^2 & -V_{3e}^*V_{3\mu} & -V_{3e}^*V_{3\tau} \\ -V_{3e}V_{3\mu}^* & 1-|V_{3\mu}|^2 & -V_{3\mu}^*V_{3\tau} \\ -V_{3e}V_{3\tau}^* & -V_{3\mu}V_{3\tau}^* & |V_{1\tau}|^2+|V_{2\tau}|^2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & \sim\tilde{\varepsilon}\varepsilon^2 & \sim\tilde{\varepsilon}\varepsilon \\ \sim\tilde{\varepsilon}\varepsilon^2 & 1 & \sim\varepsilon \\ \sim\tilde{\varepsilon}\varepsilon & \sim\varepsilon & \sim\varepsilon^2 \end{pmatrix}\end{aligned} \tag{2.35}$$

and $\tilde{\lambda}_V = V_R^\dagger \tilde{\lambda}_0 V_R$:

$$\begin{aligned}\tilde{\lambda}_V &= \begin{pmatrix} 1-|V_{2e}|^2 & -V_{2e}^*V_{2\mu} & -V_{2e}^*V_{2\tau} \\ -V_{2e}V_{2\mu}^* & |V_{1\mu}|^2+|V_{3\mu}|^2 & -V_{2\mu}^*V_{2\tau} \\ -V_{2e}V_{2\tau}^* & -V_{2\mu}V_{2\tau}^* & 1-|V_{2\tau}|^2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & \sim\tilde{\varepsilon} & \sim\tilde{\varepsilon}\varepsilon \\ \sim\tilde{\varepsilon} & \sim\varepsilon^2 & \sim\varepsilon \\ \sim\tilde{\varepsilon}\varepsilon & \sim\varepsilon & 1 \end{pmatrix}\end{aligned} \tag{2.36}$$

where the order of magnitude for the LFV entries is indicated in terms of the small parameters ε and $\tilde{\varepsilon}$.

2.1.3 How light can $SU(3)_e$ gauge bosons be

The interesting question now is about how small the scale v_2 can be without contradicting to existing experimental limits. The leading terms in (2.31) give rise to flavor conserving operators

$$-\frac{1}{4v_2^2}\left(\overline{e_R}\gamma^\nu e_R\right)^2 - \frac{1}{2v_2^2}\left(\overline{e_R}\gamma^\nu e_R\right)\left(\overline{\mu_R}\gamma_\nu \mu_R\right) \tag{2.37}$$

constrained by the compositeness limits $\Lambda^-_{RR}(eeee) > 10.2$ TeV and $\Lambda^-_{RR}(ee\mu\mu) >$ 9.1 TeV (95 % C.L.) as reported respectively in Refs. [59] and [60], where the compositeness scale is defined by the effective operators:

$$\begin{aligned} &\pm\frac{g^2}{(1+\delta_{ef})(\Lambda^\pm_{RR})^2}\overline{e_R}\gamma_\mu e_R \overline{f_R}\gamma^\mu f_R \\ &\frac{g^2}{4\pi} = 1 \end{aligned} \tag{2.38}$$

Translating the formal definitions of compositeness scales to the scale v_2, it is obtained:

$$\begin{aligned} v_2 &= (8\pi)^{-1/2}\Lambda^-_{RR}(eeee) > 2.0 \text{ TeV} \\ v_2 &= (8\pi)^{-1/2}\Lambda^-_{RR}(ee\mu\mu) > 1.8 \text{ TeV} \end{aligned} \tag{2.39}$$

Hence, these limits allow the scale of $SU(2)_e$ symmetry to be as small as $v_2 =$ 2 TeV. Using Eqs. (2.17), then one can evaluate also the scale of $SU(3)_3 \to SU(2)_e$ breaking $v_3 = v_2/\varepsilon$ and the cutoff scale M as

$$v_3 \simeq (g_{22}/g_{33}) \times 34 \text{ TeV}, \quad M \simeq g_{22} \times 3.3 \text{ PeV} \tag{2.40}$$

As for the flavor-changing phenomena induced by operators (2.31) and (2.33), they are suppressed by small parameters ε and $\tilde{\varepsilon}$ and agree with severe experimental limits on the LFV even for such a low scale of flavor symmetry. E.g. both $\mathcal{L}_2$ and $\mathcal{L}_3$ terms contribute to the following operator which induces the LFV decay $\mu \to ee\bar{e}$:

$$\begin{aligned} &\frac{4G_{\mu eee}}{\sqrt{2}}\left(\overline{e_R}\gamma_\nu \mu_R\right)\left(\overline{e_R}\gamma^\nu e_R\right) \\ &\frac{4G_{\mu eee}}{\sqrt{2}} = \frac{1}{2v_2^2}V^*_{3e}V_{3\mu} + \frac{1}{2v_3^2}V^*_{2e}V_{2\mu} \sim \frac{\varepsilon^2\tilde{\varepsilon}}{2v_2^2} \end{aligned} \tag{2.41}$$

Its amplitude can be normalized to the amplitude of the muon standard decay due to the weak interactions

$$-\frac{4G_F}{\sqrt{2}}(\overline{e_L}\gamma_\rho\nu_e)\,(\overline{\nu_\mu}\gamma^\rho\mu_L) \tag{2.42}$$

where $4G_F/\sqrt{2} = 1/v_{\rm w}^2$, $v_{\rm w} = 174$ GeV. Then for the decay rate it can be obtained:

$$\frac{\Gamma(\mu\to ee\bar{e})}{\Gamma(\mu\to e\bar{\nu}_e\nu_\mu)} = \frac{1}{2}\left|\frac{G_{\mu eee}}{G_F}\right|^2 \sim \frac{\tilde{\varepsilon}^2\varepsilon^4}{8}\left(\frac{v_{\rm w}}{v_2}\right)^4 \tag{2.43}$$

Hence, for $v_2 > 2$ TeV and $\varepsilon, \tilde{\varepsilon} \leq 1/20$ or so, this branching ratio is compatible with the existing experimental limit $\mathrm{Br}(\mu\to 3e)_{\rm exp} < 10^{-12}$ [111].

For τ lepton decay modes as $\tau\to\mu e\bar{e}$ and $\tau\to 3\mu$ leading contributions arise from operator (2.31). From (2.35) the relevant constants are $4G_{\tau\mu ee}\sqrt{2} = 4G_{\tau\mu\mu\mu}/\sqrt{2} = V_{3\mu}^* V_{3\tau}/2v_2^2$. Hence, the widths of these decays are suppressed by a factor $\sim \varepsilon^2/v_2^4$, and are compatible with the experimental limits [111]. The summary of predicted branching ratios of relevant LFV processes compared with experimental limits is given in Table 4.2, with the parameters ε, $\tilde{\varepsilon}$ normalized to a benchmark value 1/20.

Yet another LFV effect to be considered is muonium-antimuonium conversion $M(\bar{\mu}e)\to\overline{M}(\mu\bar{e})$ [61]. The relevant operator emerges from (2.33) and it reads

$$-\frac{4G_{M\overline{M}}}{\sqrt{2}}\left(\overline{\mu}_R\gamma^\nu e_R\right)^2, \qquad G_{M\overline{M}} = \frac{\varepsilon^2(V_{2e}V_{2\mu}^*)^2}{8\sqrt{2}v_2^2} \tag{2.44}$$

Thus the amplitude of $M-\overline{M}$ transition is doubly suppressed, by a factor $\sim \tilde{\varepsilon}^2\varepsilon^2 < 10^{-5}$ or so, and is much below the experimental limit $|G_{M\overline{M}}/G_F| < 3\times 10^{-3}$ [62].

One loop interactions of flavor bosons also contribute to the magnetic moment of the particles, with no suppression by mixing angles in V_R. The contribution to the parameter for the magnetic moment anomaly of the electron $a_e = \frac{1}{2}(g_e - 2)$ can be computed using formulas in Ref. [63]:

$$a_e = -\frac{m_e^2}{8\pi^2 v_2^2} = -8.3\times 10^{-16}\left(\frac{2\ \text{TeV}}{v_2}\right)^2 \tag{2.45}$$

which is about 3 orders of magnitude smaller than the present difference between the experimental [111] and theoretical [64] determinations of the electron anomalous magnetic moment, $a_e^{\rm exp} - a_e^{\rm SM} = (-7.0\pm 3.5)\times 10^{-13}$. Similarly, the contribution to the anomalous magnetic moment can be estimated from Ref. [63] $a_\mu = -\frac{m_\mu^2}{8\pi^2 v_2^2} = -3.5 \times 10^{-11}(2\ \text{TeV}/v_2)^2$, which is two orders of magnitude below the existing discrepancy between theory and experiment $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (2.7 \pm 0.8) \times 10^{-9}$ [111]. So, these contributions are irrelevant for both electron and muon. As regards the electron dipole moment, the experimental limit [111] gives a constraint at least 3 orders of magnitude more stringent with respect to new contributions to magnetic moment. However the electric dipole gets no contribution at one loop from the new horizontal interactions.

It should be remarked that potentially also flavons can mediate the LFV processes. From the effective operators (2.9), after substituting the VEV $\langle\phi\rangle = v_{\text{w}}$ the lepton Yukawa couplings with the flavon fields ξ_n become:

$$h_{in}\, \xi_n^\alpha \overline{\ell_{Li}} e_{R\alpha} \tag{2.46}$$

with

$$h_{in} = \frac{g_{in} v_{\text{w}}}{M} \tag{2.47}$$

which are generically flavor-changing. For example, in the basis (2.15) the Higgs mode of the flavon ξ_2 which is presumably the lightest, with the mass $\mu_2 \sim v_2$, induces the following effective operator:

$$-\frac{h_{32}h_{22}}{\mu_2^2}\,(\overline{\tau}\mu)\,(\overline{\mu}\mu)\,, \qquad \frac{h_{32}h_{22}}{\mu_2^2} \simeq \frac{m_\mu^2}{v_2^4} \tag{2.48}$$

where we have taken into account the relations (2.17). Thus, for $v_2 > 2$ TeV, the width of $\tau \to 3\mu$ decay induced by this operator is more than 12 orders of magnitude below the experimental limit. The width of $\mu \to 3e$ decay induced by analogous operator mediated by flavon ξ_1 is also suppressed by orders of magnitude.

2.1.4 Anomaly cancellation

For promoting the chiral non-abelian factors in (2.5) as $SU(3)_e$, etc. as gauge symmetries, one has to take care of anomaly cancellations. For more generality, the maximal symmetry

$$SU(3)_\ell \times SU(3)_e \times SU(3)_q \times SU(3)_u \times SU(3)_d \tag{2.49}$$

can be considered as gauge symmetry under which the different fermion species form the triplets of independent $SU(3)$ horizontal groups respectively as

$$\ell_L \sim 3_\ell, \qquad e_R \sim 3_e, \qquad Q_L \sim 3_q, \qquad u_R \sim 3_u, \qquad d_R \sim 3_d \tag{2.50}$$

Mode	Exp. Br.	Predicted Br.	
$\mu \to ee\bar{e}$	$< 1.0 \times 10^{-12}$	$\frac{1}{8}\left(\frac{v_{\rm w}}{v_2}\right)^4 \left\|V_{3e}^* V_{3\mu} + \varepsilon^2 V_{2e}^* V_{2\mu}\right\|^2$	$\leq 1.1 \times 10^{-13} \left(\left\| \frac{g_{31}^* g_{32}}{g_{33}^2} \right\| + \left\| \frac{g_{21}^*}{g_{22}} \right\| \right)^2 \tilde{\varepsilon}_{20}^2 \varepsilon_{20}^4 \left(\frac{2\,\rm TeV}{v_2}\right)^4$
$\tau \to \mu e\bar{e}$	$< 1.8 \times 10^{-8}$	$\frac{1}{4}\left(\frac{v_{\rm w}}{v_2}\right)^4 \left\|V_{3\mu}^* V_{3\tau}\right\|^2 \mathrm{Br}_{\tau\to\mu\nu_\tau\bar{\nu}_\mu}$	$= 6.2 \times 10^{-9} \left\|\frac{g_{32}}{g_{33}}\right\|^2 \varepsilon_{20}^2 \left(\frac{2\,\rm TeV}{v_2}\right)^4$
$\tau \to \mu\mu\bar{\mu}$	$< 2.1 \times 10^{-8}$	$\frac{1}{8}\left(\frac{v_{\rm w}}{v_2}\right)^4 \left\|V_{3\mu}^* V_{3\tau}\right\|^2 \mathrm{Br}_{\tau\to\mu\nu_\tau\bar{\nu}_\mu}$	$= 3.1 \times 10^{-9} \left\|\frac{g_{32}}{g_{33}}\right\|^2 \varepsilon_{20}^2 \left(\frac{2\,\rm TeV}{v_2}\right)^4$
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\rm w}}{v_2}\right)^4 \left\|V_{3e}^* V_{3\mu}\right\|^2$	$= 3.1 \times 10^{-15} \left\|\frac{g_{31}^* g_{32}}{g_{33}^2}\right\|^2 \tilde{\varepsilon}_{20}^2 \varepsilon_{20}^4 \left(\frac{2\,\rm TeV}{v_2}\right)^4$
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\rm w}}{v_2}\right)^4 \left\|V_{3\mu}^* V_{3\tau}\right\|^2 \mathrm{Br}_{\tau\to\mu\nu_\tau\bar{\nu}_\mu}$	$= 8.7 \times 10^{-11} \left\|\frac{g_{32}}{g_{33}}\right\|^2 \varepsilon_{20}^2 \left(\frac{2\,\rm TeV}{v_2}\right)^4$

Table 2.1: Experimental limits on the branching fractions for the LFV decays [111] vs. those predicted in our model. For the LFV decays of τ lepton the branching ratio $\mathrm{Br}(\tau \to \mu\nu_\tau\bar{\nu}_\mu) = 0.174$ is taken into account. In last column Eqs. (2.21) is used for the elements of mixing matrix V_R and the scale $v_2 = 2$ TeV is set from the compositeness limits. Both parameters ε and $\tilde{\varepsilon}$ are normalized to $1/20 \simeq m_\mu/m_\tau$; $\varepsilon_{20} = 20\varepsilon$, $\tilde{\varepsilon}_{20} = 20\tilde{\varepsilon}$. The latter choice is a clear overestimation and it would be more consistent to take $\tilde{\varepsilon} \simeq m_e/m_\mu \simeq 1/200$. However, all of the LFV limits are obeyed even with strongly overestimated $\tilde{\varepsilon}$.

Then each of these gauge factors would have triangle $SU(3)^3$ anomalies. For their cancellation, for each triplet in (2.50) one must additionally introduce *ad hoc* fermions of the opposite chiralities which are singlets of the SM and are triplets under the respective horizontal symmetry.

An interesting possibility is to introduce a mirror sector [65, 66, 67, 68, 69] as a mirror copy of the SM gauge symmetry $SU(3) \times SU(2) \times U(1)$, so that for every (LH or RH) fermion species of the SM, there exists its mirror twin with the opposite chirality in the identical representation of the mirror SM' gauge group $SU(3)' \times SU(2)' \times U(1)'$ (for a review see e.g. Ref. [70, 71]), and assume that the gauge horizontal symmetries (2.49) are common symmetries between ordinary and mirror particles as it was suggested in Ref. [72]. In other words, for ordinary quark and lepton species in (2.50), their mirror twins ('primed' quarks and leptons) should be respectively in representations:

$$\ell'_R \sim 3_\ell\,, \qquad e'_L \sim 3_e\,, \qquad Q'_R \sim 3_q\,, \qquad u'_L \sim 3_u\,, \qquad d'_L \sim 3_d \tag{2.51}$$

In this picture, the parity can be understood as a discrete symmetry of exchange between ordinary and mirror species, $\ell_L \leftrightarrow \ell'_R$ etc. with respective exchange of ordinary and mirror gauge bosons and Higgses ϕ and ϕ'. As for the horizontal gauge factors (2.49), they in fact all become vector-like, with their triangle anomalies reciprocally cancelled between the ordinary (2.50) and mirror (2.51) particle species of the opposite chiralities.

In particular, in our model "reduced" to leptons in which only $SU(3)_e$ is considered as a gauge symmetry, triangle $SU(3)_e^3$ anomaly is cancelled between the ordinary RH leptons $e_{R\alpha} \sim (1, -2, 3_e)$ and their LH mirror partners $e'_{L\alpha} \sim (1, -2', 3_e)$, where $-2'$ denotes $U(1)'$ hypercharge of mirror leptons.

Operators (2.9) must be complemented by similar couplings of flavons ξ_n with mirror leptons. Hence, we have

$$\sum_n \frac{g_{in}\xi_n^\alpha}{M} \left(\phi\, \overline{\ell_{Li}} e_{R\alpha} + \phi'\, \overline{\ell'_{Ri}} e'_{L\alpha}\right) \,+\, \text{h.c.} \tag{2.52}$$

Then, if mirror symmetry is exact, i.e. $\langle\phi'\rangle = \langle\phi\rangle = v_{\rm w}$, the ordinary and mirror leptons should have identical mass spectra. As for neutrinos, now besides the operator (2.22) generating the neutrino Majorana masses, we should have its mirror copy generating Majorana masses of mirror neutrinos, and also a mixed operator between ordinary ℓ and mirror ℓ' leptons [73, 74, 75, 76]:

$$\begin{aligned} &\frac{Y_\nu^{ij}}{\mathcal{M}} \left(\phi\phi\, \ell_{Li}^T C \ell_{Lj} + \phi'\phi'\, \ell_{Ri}'^T C \ell'_{Rj}\right) \\ &\qquad + \frac{\tilde{Y}_\nu^{ij}}{\mathcal{M}}\, \phi\phi'\, \overline{\ell_{Li}} \ell'_{Rj} \,+\, \text{h.c.}\,. \end{aligned} \tag{2.53}$$

The last operator mixes ordinary (active) and mirror (sterile) neutrinos, and also can play a key role in co-leptogenesis scenario which can generate baryon asymmetries in both ordinary and mirror sectors [77, 78]. Interestingly, if lepton numbers (or better B − L and B′ − L′) are conserved in each sector, then these operators are forbidden and all neutrinos remain massless. However, if the combination (B − L) + (B′ − L′) is conserved, then the last operator is allowed. In this case the neutrinos will be Dirac particles having masses $\sim v_{\rm w}^2/\mathcal{M}$, with their LH components living in ordinary world and the RH components living in mirror world.

However, introduction of the mirror fermions does not fully solve the anomaly problem: there remains a mixed triangle anomaly of hypercharge–flavor $U(1) \times SU(3)_e^2$. For its cancellation, new fermion species should be introduced in the proper representations of the SM and $SU(3)_e$. There are several ways of doing this. Let us consider one of the possibilities by introducing in our sector, in addition to the regular leptons (2.8), the new lepton species in representations

$$\mathcal{E}_{L\alpha} \sim (1,-2,3_e;X), \quad \mathcal{E}_{Ri} \sim (1,-2,1;X), \tag{2.54}$$

and, for mirror parity, analogous species in mirror sector:

$$\mathcal{E}'_{R\alpha} \sim (1,-2',3_e;X), \quad \mathcal{E}'_{Li} \sim (1,-2',1;X), \tag{2.55}$$

where $\alpha = 1,2,3$ is a gauge $SU(3)_e$ index and $i = 1,2,3$ is just for numbering three species. We assign to these fermions a new charge X of additional gauge symmetry $U(1)_X$ while ordinary leptons have no X-charges. This additional charge is introduced in order to forbid the mixing of new fermions (2.54) with ordinary leptons (2.8) due to the mass term $M\overline{\mathcal{E}_{L\alpha}}e_{R\alpha}$ and the Yukawa terms $\overline{\ell_{Li}}\mathcal{E}_{Rj}\phi$ which would ruin the flavor structure induced by the operator (2.9). It is easy to check that by introducing extra fermions (2.54) and (2.55) the mixed triangle anomalies including $U(1) \times SU(3)_e^2$, $U(1)_X \times SU(3)_e^2$, $U(1) \times U(1)_X^2$ and $U(1)_X \times U(1)^2$ are all cancelled.

The new fermions get masses from couplings with flavons ξ_n:

$$y_{in}\xi_n^\alpha \overline{\mathcal{E}_{Ri}}\mathcal{E}_{L\alpha} + y_{in}\xi_n^\alpha \overline{\mathcal{E}'_{Li}}\mathcal{E}'_{R\alpha} + \text{h.c.} \tag{2.56}$$

where y_{in} are order 1 Yukawa constants. Therefore, their mass spectrum should reflect the hierarchy $v_3 : v_2 : v_1 \sim 1 : \varepsilon : \varepsilon\tilde{\varepsilon}$. In particular, if v_2 is in the TeV range, then v_1 should be in the range of 100 GeV and thus the lightest of new leptons will have a mass of this order. In addition, if $U(1)_X$ symmetry is unbroken, then the lightest of these states should be stable (the heavier ones will decay into the lighter one via $SU(3)_e$ flavor boson mediated operators). Interestingly, the LEP direct experimental lower limit on the mass of new charged leptons is 102.6 GeV [79]. Such heavy leptons can be within the reach of new e^+e^- machines as ILC/CLIC or CEPC/FCC-ee. If $U(1)_X$ symmetry is spontaneously broken, then the mixing of e, μ, τ with new leptons can be allowed and thus the latter will be rendered unstable.

Let us turn to flavor gauge bosons of $SU(3)_e$ which now interact with both normal leptons and mirror leptons. Now their exchange should create mixed effective operators involving both ordinary and mirror leptons. In particular, the bosons $\mathcal{F}^\mu_{1,2,3}$ mediate the following LFV operators involving first two families of both sectors:

$$\frac{1}{v_2^2}\sum_{a=1}^{3} J_a^\mu J'_{a\mu} = \frac{1}{4v_2^2}\sum_{a=1}^{3} (\overline{\mathbf{e}_R}\,\lambda_a\gamma^\mu \mathbf{e}_R)(\overline{\mathbf{e}'_L}\,\lambda_a\gamma_\mu \mathbf{e}'_L) \tag{2.57}$$

Thus, this operator induces muonium - mirror muonium conversion $M(\bar{\mu}e) \to M'(\mu'\bar{e}')$ with $G_{MM'}/G_F = (v_\mathrm{w}/v_2)^2 = 7.6 \times 10^{-3}(2\,\mathrm{TeV}/v_2)^2$. Differently from the muonium-antimuonium conversion (2.44), here is no suppression by small mixing angles. The present limit on the muonium disappearance reads Br($M \to$ invisible) $< 5.7 \times 10^{-6}$ [80] which is clearly respected for $v_2 = 2$ TeV. However, this limit can be improved by several orders of magnitude as discussed in Ref. [80]. Analogously, this operator should induce positronium conversion into mirror positronium [81, 82], but for $v_2 = 2$ TeV the positronium disappearance rate is much below the present experimental limit Br(o-Ps$\to$invisible) $< 6 \times 10^{-4}$ [83].

As far as the presence of mirror sector is concerned, mirror matters is a viable candidate for light dark matter dominantly consisting of mirror helium and hydrogen atoms [84, 85, 86, 87]. The flavor gauge bosons interacting with both ordinary and mirror fermions appear as messengers between the two sectors and can give an interesting portal for mirror matter direct detection, complementary to the dark photon portal related to the photon-mirror photon kinetic mixing [88, 89]. However, they will also give rise to the mixing between the neutral ordinary and mirror mesons. Namely, the lighter flavor bosons induce mixings as $\pi^0 - \pi^{0\prime}$, $K^0 - K^{0\prime}$, etc. [65, 66, 67, 68, 69], with implications for the invisible decay channels of neutral mesons (for a recent discussion, see also Ref. [90]). In the supersymmetric version, the respective flavor gauginos complemented by R-parity breaking can induce the mixing between the ordinary and mirror neutral baryons. Interestingly, neutron-mirror neutron oscillation $n - n'$ related to physics at the scale of few TeV can be rather fast, in fact much faster than the neutron decay itself [91, 92, 93]. Recent summary of experimental bounds on the $n - n'$ oscillation time can be found in Ref. [94]). There are some anomalies in existing experiments on $n - n'$ oscillation search [95] which can be tested in planned experiments on the neutron disappearance and regeneration [96, 97].

Oscillation phenomena between ordinary and mirror neutral particles are effective if they are degenerate in mass, i.e. mirror parity is unbroken and the weak scales $\langle\phi\rangle = v_\mathrm{w}$ and $\langle\phi'\rangle = v'_\mathrm{w}$ are exactly equal in two sectors, $v'_\mathrm{w} = v_\mathrm{w}$. However, the cancellation of horizontal anomalies between two sectors does not require that mirror parity is unbroken, and in fact one can consider models where it is spontaneously broken, e.g. $v'_\mathrm{w} > v_\mathrm{w}$, with interesting implications for mirror dark matter properties and sterile mirror neutrinos [98] and axion physics [99]. In particular, in the context of the mirror twin Higgs mechanism for solving the little hierarchy problem, in supersymmetric [170, 101] or non-supersymmetric [102] versions, one expects v'_w in the TeV range.

The following remark is in order. The viability of mirror sector is subject of strong cosmological restrictions. Namely, the Big Bang nucleosynthesis (BBN) constraints require that at the BBN epoch its temperature should be smaller than the temperature of the ordinary sector, $T'/T < 0.6$ [84]. The constraints from the CMB and large scale structure are at least twice stronger, $T'/T <$ 0.2–0.3 [86, 87]. On the other hand, the interactions (2.57) induce the process $e\bar{e} \to e'\bar{e}'$ which in the early universe would bring two sectors into equilibrium. The freeze-out temperature T_d of this process can be easily estimated, just by rescaling by a factor $(2v_2/v_{\rm w})^{4/3}$ the neutrino decoupling temperature $T_\nu \simeq 2$ MeV. Thus, for respecting the cosmological bounds, the reheating temperature of the universe should not exceed $T_d \simeq (v_2/2\,\text{TeV})^{4/3} \times 130$ MeV. (Analogous problem of low reheating temperature is typical also for the TeV scale gravity models with large extra dimensions, as discussed in Ref. [103], and for twin Higgs models [170, 101, 102].) Alternatively, one has to assume the possibility of additional entropy production in ordinary sector below the temperatures T_d which in turn implies the necessity of mirror symmetry breaking. In asymmetric mirror model, with $v'_{\rm w} \gg v_{\rm w}$, T_d becomes significantly larger. In particular, for $v'_{\rm w}$ larger than few PeV, as e.g. in heavy axion (axidragon) model [99], one can have $T_d > 1$ TeV or so.

2.2 $SU(3)_\ell$

A bigger symmetry concerning lepton sector can be considered, that is $SU(3)_\ell \times SU(3)_e$. The LH and RH lepton fields are in the following representations:

$$\ell_{L\alpha} = \begin{pmatrix} \nu_\alpha \\ e_\alpha \end{pmatrix}_L \sim (2, -1, 3_\ell, 1), \qquad e_{R\gamma} \sim (1, -2, 1, 3_e) \tag{2.58}$$

where the multiplet content with respect to the EW $SU(2) \times U(1)$ and horizontal $SU(3)_\ell \times SU(3)_e$ is indicated. $\alpha = 1, 2, 3$ and $\gamma = 1, 2, 3$ are the indices of $SU(3)_\ell$ and $SU(3)_e$ respectively. This set of fermions is not anomaly free. Anomaly cancellation can be addressed following the discussion in section 2.1.4.

2.2.1 Lepton masses and mixing

Yukawa couplings of ϕ with fermions ℓ_{Li} and $e_{R\alpha}$ are forbidden by $SU(3)_\ell \times SU(3)_e$ symmetry. For breaking $SU(3)_\ell \times SU(3)_e$ three triplets $\eta_{i\alpha}$ of $SU(3)_\ell$ and three triplets $\xi_{i\gamma}$ of $SU(3)_e$, $i = 1, 2, 3$ are introduced. Then, as regards charged leptons, masses emerge from the gauge invariant dimension–6 operator

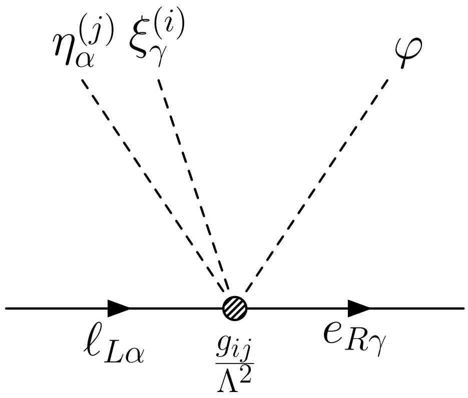

Figure 2.2: Effective operator (2.59) for charged lepton masses.

(see Fig. 2.2):

$$\frac{y_{ij}}{\mathcal{M}^2}\,\eta_{i\alpha}\overline{\xi}^\gamma_j\phi\,\overline{\ell_{L\alpha}}e_{R\gamma} + \text{h.c.} \tag{2.59}$$

where y_{ij} are order one constants, ϕ is the Higgs doublet and $\mathcal{M}$ is a cutoff scale.

As already stated, in an UV-complete theory such operators can be induced via seesaw-like mechanism by integrating out some heavy scalar or fermion states [39, 40].

As regards neutrinos, their Majorana masses are induced by the higher order operator (shown in Fig. 2.3):

$$\frac{h_{ij}}{\mathcal{M}_\nu^3}\,\overline{\eta}_i^\alpha\overline{\eta}_j^\beta\,\phi\phi\,\ell^T_{L\alpha}C\ell_\beta + \text{h.c.} \tag{2.60}$$

where $h_{ij} = h_{ji}$. The cutoff scale $\mathcal{M}_\nu$ of this operator is not necessarily the same as the scale $\mathcal{M}$ of operator (2.59).

Operator (2.59) has a global symmetry $U(3)_\ell \times U(3)_e$, $U(3)_e = SU(3)_e \times U(1)_e$, $U(3)_\ell = SU(3)_\ell \times U(1)_\ell$, and in order to generate non-zero masses of all three leptons e, μ, τ, that global symmetry must be fully broken. This means that all three $SU(3)_\ell$ flavons η_i as well as $SU(3)_e$ ξ_i should have non-zero VEVs with disoriented directions. Thus VEVs $\langle\eta_{i\alpha}\rangle$ should form a rank-3 matrix. Without losing generality, the flavon basis can be chosen so that the matrix $\langle\eta_{i\alpha}\rangle$ is diagonal, $\langle\eta_{i\alpha}\rangle = w_i\delta_{i\alpha}$, i.e. the flavon VEVs are orthogonal:

$$\langle\eta_1\rangle = \begin{pmatrix} w_1 \\ 0 \\ 0 \end{pmatrix}, \qquad \langle\eta_2\rangle = \begin{pmatrix} 0 \\ w_2 \\ 0 \end{pmatrix}, \qquad \langle\eta_3\rangle = \begin{pmatrix} 0 \\ 0 \\ w_3 \end{pmatrix} \tag{2.61}$$

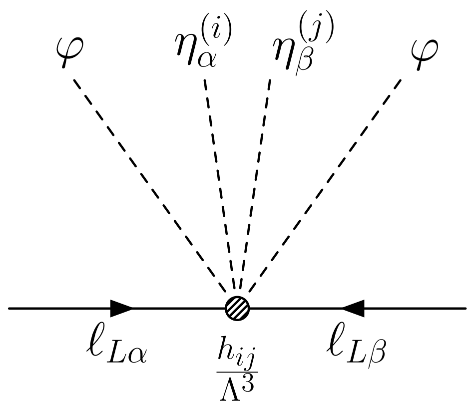

Figure 2.3: Effective operator (2.60) for neutrinos Majorana masses.

Analogously, for ξ–flavons:

$$\langle \xi_1 \rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix} , \qquad \langle \xi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix} , \qquad \langle \xi_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix} \tag{2.62}$$

as in (2.13). By substituting these VEVs in operator (2.59), the leptonic Yukawa matrices in the SM Lagrangian are obtained:

$$Y_e^{ij} = y_{ij} \frac{w_i v_j}{\mathcal{M}^2} \tag{2.63}$$

Operator (2.60) should give instead the observed neutrino mass pattern:

$$m_\nu^{ij} = h_{ij} w_i w_j v_{\rm w}^2 / \mathcal{M}_\nu^3 \tag{2.64}$$

and in particular the large neutrino mixing. This implies that $SU(3)_\ell$ breaking flavons η should have comparable VEVs, $w_3 \sim w_2 \sim w_1$. Then the big ratios of lepton masses are due to the VEVs of the triplets ξ_n: $v_3 \gg v_2 \gg v_1$ in $SU(3)_e$ symmetry breaking, i.e. $v_3 : v_2 : v_1 \sim m_\tau : m_\mu : m_e$. and small ratios between the VEVs breaking $SU(3)_\ell$ are negligible for obtaining the hierarchical mass spectrum of leptons. In fact, a change in the basis of the triplets η is simply connected with large mixings between neutrinos after spontaneous symmetry breaking, then ℓ_{Li} states can also be turned to the basis in which matrix Y_e^{ij} has a triangular form as in (2.15) with $g_{\alpha j}/M = y_{ij} \langle \eta_{i\alpha} \rangle / \mathcal{M}^2$.

2.2.2 $SU(3)_\ell$ gauge bosons

Gauge bosons $\mathcal{F}^\mu_{\ell a}$ of $SU(3)_\ell$ associated to the Gell-Mann matrices λ_a, $a = 1, 2, ...8$, interact as

$$g\mathcal{F}^\mu_{\ell a} J^\ell_{a\mu} \tag{2.65}$$

with the respective currents

$$J^\ell_{a\mu} = J^{(e)}_{a\mu} + J^{(\nu)}_{a\mu} = \frac{1}{2}\overline{\boldsymbol{e}_L}\gamma_\mu\lambda_a\boldsymbol{e}_L + \frac{1}{2}\overline{\boldsymbol{\nu}_L}\gamma_\mu\lambda_a\boldsymbol{\nu}_L \tag{2.66}$$

where g is the gauge coupling constant (an index ℓ is omitted here), $\boldsymbol{e}_L = (e_1, e_2, e_3)^T_L$ and $\boldsymbol{\nu}_L = (\nu_1, \nu_2, \nu_3)^T_L$ respectively denote the family triplets of the LH charged leptons and neutrinos.

At low energies these couplings induce four-fermion (current $\times$ current) interactions:

$$\mathcal{L}_{\text{eff}} = -\frac{g^2}{2} J^{\ell\mu}_a \left(M^2_\ell\right)^{-1}_{ab} J^\ell_{b\mu} \tag{2.67}$$

where $M^2_{\ell ab}$ is the squared mass matrix of gauge bosons $\mathcal{F}^\mu_{\ell a}$ which in the flavon VEV basis (2.61) is essentially diagonal apart of a non-diagonal 2×2 block related to $\mathcal{F}^\mu_{\ell 3}$ - $\mathcal{F}^\mu_{\ell 8}$ mixing. Namely, the masses of $\mathcal{F}^\mu_{\ell 1,2}$, $\mathcal{F}^\mu_{\ell 4,5}$ and $\mathcal{F}^\mu_{\ell 6,7}$ are

$$\begin{aligned} M^2_{\ell 1,2} &= \frac{g^2}{2}(w_2^2 + w_1^2) = \frac{g^2}{2} v^2_{\mathcal{F}}, \\ M^2_{\ell 4,5} &= \frac{g^2}{2}(w_3^2 + w_1^2), \quad M^2_{\ell 6,7} = \frac{g^2}{2}(w_3^2 + w_2^2) \end{aligned} \tag{2.68}$$

As for $\mathcal{F}^\mu_{\ell 3}$ and $\mathcal{F}^\mu_{\ell 8}$ they have a mass mixing and their mass matrix reads

$$M^2_{\ell 38} = \frac{g^2}{2}\begin{pmatrix} w_2^2 + w_1^2 & \frac{1}{\sqrt{3}}(w_1^2 - w_2^2) \\ \frac{1}{\sqrt{3}}(w_1^2 - w_2^2) & \frac{1}{3}(4w_3^2 + w_1^2 + w_2^2) \end{pmatrix}. \tag{2.69}$$

Notice that if $w_1 = w_2 = v_{\mathcal{F}}/\sqrt{2}$, this matrix becomes diagonal. In the following, in order to simplify the demonstration, this case is analyzed, although a similar analysis can be done also for a general case $w_1 \neq w_2$, along the lines of section 2.1. Then for the gauge boson masses we have $M^2_{\ell a} = (g^2/2)(x_a v_{\mathcal{F}})^2$, where

$$x^2_{1,2,3} = 1, \quad x^2_{4,5,6,7} = \frac{r+1}{2}, \quad x^2_8 = \frac{2r+1}{3} \tag{2.70}$$

and $r = 2w_3^2/v_{\mathcal{F}}^2$. Then operators (2.67) can be rewritten as

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}^{e\nu} + \mathcal{L}_{\rm eff}^{ee} + \mathcal{L}_{\rm eff}^{\nu\nu} \tag{2.71}$$

where

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{e\nu} &= -\frac{2G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\,\gamma^\mu \frac{\lambda_a}{x_a}\,\boldsymbol{e}_L\right)\left(\overline{\boldsymbol{\nu}_L}\,\gamma_\mu \frac{\lambda_a}{x_a}\,\boldsymbol{\nu}_L\right) \\
\mathcal{L}_{\rm eff}^{ee} &= -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\,\gamma_\mu \frac{\lambda_a}{x_a}\,\boldsymbol{e}_L\right)^2 \\
\mathcal{L}_{\rm eff}^{\nu\nu} &= -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{\nu}_L}\,\gamma_\mu \frac{\lambda_a}{x_a}\,\boldsymbol{\nu}_L\right)^2
\end{aligned} \tag{2.72}$$

with $4G_{\mathcal{F}}/\sqrt{2} = 1/v_{\mathcal{F}}^2$. Obviously, the factor $g^2/2$ in operators cancels out and the strength of these operators is determined solely by the VEVs (2.61).

2.2.3 $SU(3)_\ell$ gauge bosons mediated intaractions

The first term $\mathcal{L}_{\rm eff}^{e\nu}$ contains the operator

$$-\frac{4G_{\mathcal{F}}}{\sqrt{2}}(\overline{e_L}\gamma_\mu\mu_L)(\overline{\nu_\mu}\gamma^\mu\nu_e) \tag{2.73}$$

which contributes to the muon decay $\mu \to e\nu_\mu\bar{\nu}_e$ as

$$G_\mu = G_F + G_{\mathcal{F}} \tag{2.74}$$

that is, in this case the muon decay constant is not equal to the Fermi constant measured in beta decays anymore. The operator (3.30) is induced by exchange of gauge bosons $\mathcal{F}_{\ell 1}^\mu$ and $\mathcal{F}_{\ell 2}^\mu$, or more precisely by the combination $(\mathcal{F}_{\ell 1}^\mu \pm i\mathcal{F}_{\ell 2}^\mu)/\sqrt{2}$, as in second diagram of Fig. 2.4. As it will be discussed in details in the next chapter, since the relation $G_F = G_\mu$ is used for determining CKM elements, Eq. (2.74) is a possible solution to the discrepancy of the first row of the CKM matrix from unitarity. As it will be shown, for restoring the CKM unitarity it is needed that:

$$\delta_\mu = G_{\mathcal{F}}/G_F = (v_{\rm w}/v_{\mathcal{F}})^2 \approx 7 \times 10^{-4} \tag{2.75}$$

which corresponds to the flavor scale

$$v_{\mathcal{F}} = 6\text{–}7\,\text{TeV} \tag{2.76}$$

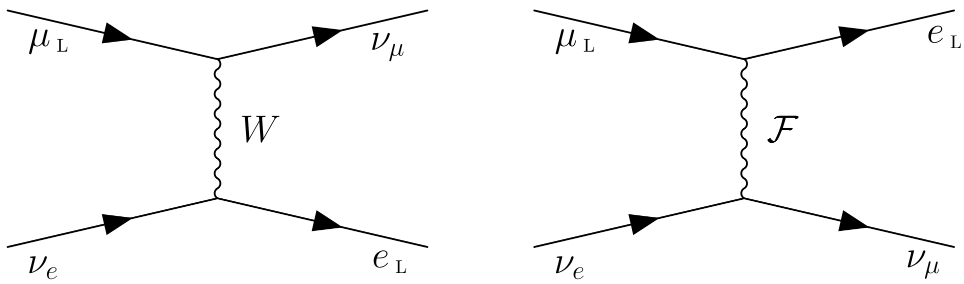

Figure 2.4: The SM contribution to the muon decay mediated by W-boson (left), and the BSM contribution mediated by the flavor-changing $\mathcal{F}$–boson (right).

The similar operators in $\mathcal{L}_{\rm eff}^{e\nu}$ mediated by the gauge bosons $\mathcal{F}^\mu_{\ell 4,5}$ and $\mathcal{F}^\mu_{\ell 6,7}$ contribute to the taon leptonic decays $\tau \to e\nu_\tau\bar{\nu}_e$ and $\tau \to \mu\nu_\tau\bar{\nu}_\mu$ whose rates are well consistent with the SM predictions [104]. Then, in the case $w_{1,2,3} \sim v_\mathcal{F}$ but $w_1 \neq w_2$, the branching ratio $\Gamma(\tau \to \mu\nu_\tau\bar{\nu}_\mu)/\Gamma(\tau \to e\nu_\tau\bar{\nu}_e)$ can have order $G_\mathcal{F}/G_F \sim \delta_\mu$ deviation from the SM prediction which can be experimentally testable. For a comparison, the present experimental value of this ratio is 0.9762(28) [111], which is 1.3σ larger than the SM predicted value 0.9726. In addition, the terms in $\mathcal{L}_{\rm eff}^{e\nu}$ with the diagonal generators λ_3 and λ_8 give rise to non-standard neutrino interactions with leptons. But respective coupling constants are of order $G_\mathcal{F} = \delta_\mu G_F$, and hence well below the experimental constraints.

The last term $\mathcal{L}_{\rm eff}^{\nu\nu}$ in (2.72) contains the non-standard interactions between neutrinos, but present experimental limits on the neutrino self-interactions are very weak. However, second term $\mathcal{L}_{\rm eff}^{ee}$ in (2.72) containing charged leptons in principle is testable for the scale $v_\mathcal{F}$ of few TeV.

Interestingly, if the flavor eigenstates e_1, e_2, e_3 are the mass eigenstates e, μ, τ, the terms (2.72) do not contain any LFV operators inducing processes like $\mu \to 3e$, $\tau \to 3\mu$ etc. However, the lepton flavor-conserving contact operators $-\frac{4\pi}{\Lambda_L^2}(\overline{e_L}\gamma_\mu e_L)^2$, $-\frac{2\pi}{\Lambda_L^2}(\overline{e_L}\gamma^\mu e_L)(\overline{\mu_L}\gamma_\mu\mu_L)$, etc. are restricted by the 'compositeness' limits $\Lambda_L^-(eeee) > 10.3$ TeV and $\Lambda_L^-(ee\mu\mu) > 9.5$ TeV. Comparing these operators with the corresponding terms in (2.72) and taking into account the relations (2.70), the 'compositeness' scales can be expressed in terms of the scale $v_\mathcal{F}$. Hence, we obtain the limit

$$v_\mathcal{F} > \left(\frac{r+1}{r+0.5}\right)^{1/2} \times 2.1 \text{ TeV}\,. \tag{2.77}$$

Here the r–dependent pre-factor approaches 1 when $r \gg 1$ and it becomes $\sqrt{2}$ in the opposite limit $r \ll 1$. Thus, the strongest limit emerges in the latter case, $v_{\mathcal{F}} > 3$ TeV or so, which is anyway fulfilled for the benchmark range $v_{\mathcal{F}} \simeq (6–7)$ TeV needed to solve the CKM unitarity problem.

The flavor eigenstates e_1, e_2, e_3 coincide with the mass eigenstates e, μ, τ, if the Yukawa matrix Y_e^{ij} in (2.63) is diagonal. This can be achieved by imposing some additional discrete symmetries between the flavons η_i and ξ_i of $SU(3)_\ell$ and $SU(3)_e$ sectors which would forbid the non-diagonal terms y_{ij} in operator (2.59). However, in general case the initial flavor basis of the LH leptons is related to the mass basis by the unitary transformation

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_L = U_L \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L = \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} \\ U_{2e} & U_{2\mu} & U_{2\tau} \\ U_{3e} & U_{3\mu} & U_{3\tau} \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L \tag{2.78}$$

Then, in the basis of mass eigenstates, the operators $\mathcal{L}_{\rm eff}^{ee}$ read as in (2.72) but with the substitution $\lambda_a/x_a \to U_L^\dagger(\lambda_a/x_a)U_L$. Interestingly, in the limit $r = 1$, i.e. when the VEVs $w_{1,2,3}$ are equal and so $x_a = 1$, all flavor bosons $\mathcal{F}_{\ell a}^\mu$ have equal masses, and the substitution $\lambda_a \to U_L^\dagger \lambda_a U_L$ is simply a basis redetermination of the Gell-Mann matrices. Therefore, no LFV effects will emerge in this case since the global $SO(8)_\ell$ symmetry acts as a custodial symmetry. Namely, by Fierz transformations, using also the Fierz identities for the Gell-Mann matrices, it is obtained the result:

$$-\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\gamma_\mu \lambda_a \boldsymbol{e}_L\right)^2 = -\frac{4}{3}\frac{G_{\mathcal{F}}}{\sqrt{2}} \left(\overline{\boldsymbol{e}_L}\gamma_\mu \boldsymbol{e}_L\right)^2 \tag{2.79}$$

Obviously, the latter expression is invariant under the unitary transformation (2.78).

In general case $r \neq 1$, the mixing (2.78) gives rise to the LFV operators as e.g. the one inducing $\mu \to 3e$ decay:

$$\begin{aligned} &-\frac{4G_{\mu eee}}{\sqrt{2}} \left(\overline{e_L}\gamma^\mu \mu_L\right)\left(\overline{e_L}\gamma^\mu e_L\right) + \text{h.c.}\,, \\ &\frac{4G_{\mu eee}}{\sqrt{2}} = \frac{C(r)}{2v_{\mathcal{F}}^2} \left[1 + \frac{1-r}{r}|U_{3e}|^2\right] U_{3e}^* U_{3\mu} \end{aligned} \tag{2.80}$$

where

$$C(r) = \frac{r(r-1)}{(r+1)(r+0.5)} \tag{2.81}$$

The function $C(r)$ is limited as $|C(r)| < 1$, reaching the maximal value at $r \gg 1$, and it vanishes at $r = 1$. Then, taking $|U_{3e}| \ll 1$, we obtain for the branching ratio of $\mu \to 3e$ decay

$$\frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)} = \frac{1}{2}\left|\frac{G_{\mu eee}}{G_F}\right|^2 = \frac{1}{8}\left(C(r)\delta_\mu|U^*_{3e}U_{3\mu}|\right)^2 \tag{2.82}$$

The experimental upper bound on this branching ratio is 10^{-12} [111]. Taking $\delta_\mu = (v_{\rm w}/v_{\mathcal{F}})^2 = 7 \times 10^{-4}$, the limit $\delta_\mu|CU^*_{3e}U_{3\mu}|/\sqrt{8} < 10^{-6}$ translates into

$$|C(r)U^*_{3e}U_{3\mu}| < 0.4 \times 10^{-2} \tag{2.83}$$

which is nicely satisfied if the lepton mixing angles in (2.78) are comparable with the CKM mixing angles in (**??**) or even larger. E.g. if the VEV ratio is in between $r = 0.5$–1.5, then $|C(r)| < 1/7$ so that $|U^*_{3e}U_{3\mu}| < (1/6)^2$ or so would suffice for properly suppressing the $\mu \to 3e$ decay rate. This means that in this case the matrix elements $|U_{3\mu}|$ and $|U_{3e}|$ can be almost as large as the Cabibbo angle $\sin\theta_C = V_{us}$. The experimental limits on other LFV effects as e.g. $\tau \to 3\mu$ are weaker, and following the lines of the previous section it can be shown that with $v_{\mathcal{F}} \sim 6$ TeV they are fulfilled even for whatever large mixings in (2.78). Once again, for $r = 1$ all LFV effects are vanishing owing to custodial symmetry, see Eq. (2.79).

2.2.4 $G_F \neq G_\mu$ vs. Standard Model precision tests

Here it is briefly discussed how the hypothesis $G_\mu \neq G_F$ could affect the SM precision tests. In the SM, at tree level, the weak gauge boson masses are $M_W = gv_{\rm w}/\sqrt{2} = ev_{\rm w}/\sqrt{2}\sin\theta_W$ and $M_Z = M_W/\cos\theta_W$ where θ_W is the weak angle. Radiative corrections, which depend also on the top quark and Higgs mass, are important for precision tests.

The world averages of experimentally measured masses of Z and W reported by PDG 2018 are [111]:

$$\begin{aligned} M_Z^{\rm exp} &= 91.1876(21)\,\text{GeV}, \\ M_W^{\rm exp} &= 80.379(12)\,\text{GeV}, \end{aligned} \tag{2.84}$$

while the SM global fit yields to the following values:

$$\begin{aligned} M_Z^{\rm SM} &= 91.1884(20)\ \text{GeV}, \\ M_W^{\rm SM} &= 80.358(4)\ \text{GeV}. \end{aligned} \tag{2.85}$$

Hence, the theoretical and experimental values of Z-mass are in perfect agreement while for W-boson the two values have about 1.6σ discrepancy:

$$M_W^{\rm exp} - M_W^{\rm SM} = (21 \pm 13)\ {\rm MeV} \tag{2.86}$$

In the SM the mass of W-boson, including radiative corrections, is determined as

$$M_W = \frac{A_0}{\hat{s}_Z (1-\Delta\hat{r}_W)^{1/2}} \tag{2.87}$$

where $A_0 = (\pi\alpha/\sqrt{2}G_F)^{1/2} = 37.28039(1)$ GeV taking $G_F = G_\mu$, the factor $1 - \Delta\hat{r}_W = 0.93084(8)$ includes the main radiative corrections and $\hat{s}_Z^2 = 1.0348(2)s_W^2$ is the corrected value of $\sin^2\theta_W(M_Z)$ by including the top and Higgs mass dependent corrections. The theoretical mass $M_W = 80.358(4)$ GeV (2.85) is then obtained by substituting in (2.87) the value $\hat{s}_Z^2 = 0.23122(3)$ obtained from the SM global fit [111]. In our scenario, however, $G_F \neq G_\mu$. Should we just set in A_0 instead of $G_F = G_\mu$ the "corrected" value $G_F = (1+\delta_\mu)^{-1}G_\mu$, then A_0 should be rescaled by a factor $(1+\delta_\mu)^{1/2}$, and correspondingly the "theoretical" value of M_W (2.87) too. In particular, for $\delta_\mu = 7\times 10^{-4}$ we would get $M_W = 80.386$ GeV, right in the ball-park of the experimental values (2.85). However, this is not the right thing to do.

In the global fit of SM M_Z is one of the input parameters with smallest experimental errors, along with the fine structure constant α and the "muon" Fermi constant G_μ. Essentially, this is the main reason of the good coincidence between $M_Z^{\rm exp}$ and $M_Z^{\rm SM}$. In fact, the SM implies the relation

$$M_Z = \frac{M_W}{\hat{c}_Z\hat{\rho}^{1/2}} = \frac{A_0}{\hat{s}_Z\hat{c}_Z(1-\Delta\hat{r}_W)^{1/2}\hat{\rho}^{1/2}} \tag{2.88}$$

where $\hat{\rho} = 1 + \rho_t + \delta\rho = 1.01013(5)$ includes the weak isospin breaking effects, dominantly from the quadratic m_t dependent corrections $\rho_t = 3G_F m_t^2/8\sqrt{2}\pi^2$. Therefore, taking the experimental value of Z-mass (2.84), Eq. (2.88) can be used for determination of $\hat{s}_Z^2$ parameter, $\hat{s}_Z^2 = 0.23123(3)$. This, in turn, from $M_W = M_Z\hat{\rho}^{1/2}\hat{c}_Z$ gives $M_W = 80.357(4)_{\rm SM}$ GeV, i.e. practically the same as the global fit result (2.85). This is because the determination of the parameter $\hat{s}_Z^2$ in the SM global fit is dominated by the results of Z-pole measurements.

However, in our scenario rescaling $A_0 \to A_0(1+\delta_\mu)^{1/2}$ changes the value of $\hat{s}_Z^2$. In particular, taking $\delta_\mu = (7.6\pm 1.6)\times 10^{-4}$, we get $\hat{s}_Z^2 = 0.23148(3)_{\rm SM}(5)_{\delta_\mu}$. Then, again from $M_W = M_Z\hat{\rho}^{1/2}\hat{c}_Z$, we get $M_W = 80.344(4)_{\rm SM}(3)_{\delta_\mu}$ GeV. Thus, unfortunately, while the effect is there, in reality it goes right to the opposite direction. So, our determination of M_W differs from $M_W^{\rm SM}$, $M_W^{\rm SM} - M_W^{\rm our} = (13 \pm$

3) MeV. Thus, with $M_W^{\rm SM}$ already being in tension with the experimental value (2.84), our result has more tension: $M_W^{\rm exp} - M_W^{\rm our} = (35 \pm 13)$ MeV (2.7σ).[2] If the tension will increase with future precision, this would mean that one has to admit at least some minimal step beyond the SM. The relation between W and Z masses can be improved by increasing of ρ-parameter via e.g. the VEV ~ 1 GeV of a scalar triplet of the electroweak $SU(2) \times U(1)$, or by diminishing Z mass by few MeV e.g. via its mixing with some extra gauge bosons like Z' at the TeV scale or perhaps also with the flavor gauge bosons considered in the previous section.

2.3 Quarks and Horizontal Symmetries $SU(3)_q \times SU(3)_u \times SU(3)_d$

Masses in the range of few TeV are allowed also for the gauge bosons related to horizontal symmetries in the quark sector. In order to show that this is indeed possible a brief description about quarks is given in this section. The maximal family symmetry of the quark sector is $U(3)_q \times U(3)_d \times U(3)_u$ with gauge factors $SU(3)_q \times SU(3)_d \times SU(3)_u$. Flavon triplets are introduced for breking the symmetry and masses emerge from effective dimension–6 operators:

$$\frac{y^d_{ij}}{\mathcal{M}^2}\,\eta^q_{i\alpha}\overline{\xi}^{d\gamma}_j\,\phi\,\overline{q_{L\alpha}}d_{R\gamma} + \frac{y^u_{ij}}{\mathcal{M}^2}\,\eta^q_{i\alpha}\overline{\xi}^{u\gamma}_j\,\tilde{\phi}\,\overline{q_{L\alpha}}u_{R\gamma} + \text{h.c.} \tag{2.89}$$

As shown for leptons, the quark mass hierarchy can be related with hierarchies in breaking of $SU(3)_q \times SU(3)_d \times SU(3)_u$ gauge symmetry, i.e. with the ratios $\varepsilon_d = v_{d2}/v_{d3}$ and $\tilde{\varepsilon}_d = v_{d1}/v_{d2}$ between the VEVs of $SU(3)_d$ triplet flavons, and the same for $SU(3)_u$ and $SU(3)_q$. In this way, the hierarchy of down quark masses will go parametrically as

$$m_b : m_s : m_d = 1 : \varepsilon_d\varepsilon_q : \varepsilon_d\tilde{\varepsilon}_d\varepsilon_q\tilde{\varepsilon}_q \tag{2.90}$$

$$m_t : m_c : m_u = 1 : \varepsilon_u\varepsilon_q : \varepsilon_u\tilde{\varepsilon}_u\varepsilon_q\tilde{\varepsilon}_q \tag{2.91}$$

It can be noticed that if in $SU(3)_u$ breaking there is stronger hierarchy between the VEVs of flavons ξ^u (parameter ϵ_u) than in $SU(3)_d$ breaking (parameter ϵ_d), then the larger splitting of the up-sector mass scales is generated.

It can be shown that the quark flavor violating processes mediated by gauge bosons of $SU(3)_d$ and $SU(3)_u$ will be suppressed due to custodial symmetry in

[2] Let us remark that the tension with the latest results of ATLAS $M_W^{\rm ATL} = 80.370(19)$ is much weaker (1.3σ), $M_W^{\rm ATL} - M_W^{\rm our} = (26 \pm 20)$ MeV.

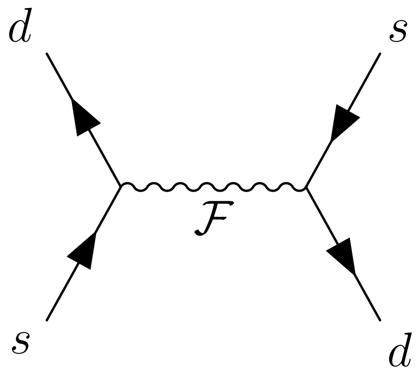

Figure 2.5: Tree level contribution to $K^0 \leftrightarrow \bar{K}^0$ mixing.

the same way as the LFV processes mediated by $SU(3)_e$ bosons. For example, some features of $SU(3)_d$ can be described. As for charged leptons, at low energies the flavor bosons induce four-fermion (current $\times$ current) interactions:

$$\mathcal{L}_{\text{eff}} = -\frac{g^2}{2} J_a^{d\mu} (M_d^2)^{-1}_{ab} J^d_{b\mu} \tag{2.92}$$

where g is the $SU(3)_d$ gauge coupling, $M^2_{d\,ab}$ is the mass matrix of gauge bosons $\mathcal{F}^\mu_{d\,a}$, associated with the currents

$$J^d_{a\mu} = \overline{\mathbf{d}_R}\frac{\lambda_a}{2}\gamma_\mu \mathbf{d}_R \tag{2.93}$$

where λ^a are the Gell-Mann matrices and $\mathbf{d}_R = (d_1, d_2, d_3)^T_R$ is the triplet of the RH down-type quarks. In the basis of the mass eigenstates (d, s, b) the four fermion operators in (2.92) can be written:

$$\begin{aligned}
&-\frac{1}{4v_{d2}^2}\left[\begin{pmatrix} \overline{d_R} & \overline{s_R} & \overline{b_R} \end{pmatrix}\gamma^\mu \begin{pmatrix} 1-|V_{3d}|^2 & -V_{3d}^*V_{3s} & -V_{3d}^*V_{3b} \\ -V_{3d}V_{3s}^* & 1-|V_{3s}|^2 & -V_{3s}^*V_{3b} \\ -V_{3d}V_{3b}^* & -V_{3s}V_{3b}^* & |V_{1b}|^2+|V_{2b}|^2 \end{pmatrix}\begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}\right]^2 \\
&-\frac{\epsilon_d^2}{4v_{d2}^2}\left[\begin{pmatrix} \overline{d_R} & \overline{s_R} & \overline{b_R} \end{pmatrix}\gamma^\mu \begin{pmatrix} 1-|V_{2d}|^2 & -V_{2d}^*V_{2s} & -V_{2d}^*V_{2b} \\ -V_{2d}V_{2s}^* & |V_{1s}|^2+|V_{3s}|^2 & -V_{2s}^*V_{2b} \\ -V_{2d}V_{2b}^* & -V_{2s}V_{2b}^* & 1-|V_{2b}|^2 \end{pmatrix}\begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}\right]^2 \\
&-\frac{\epsilon_d^2}{v_{d2}^2}(\overline{d_{R2}}\gamma_\mu d_{R2})(\overline{d_{R3}}\gamma_\mu d_{R3})
\end{aligned} \tag{2.94}$$

From (2.94) both flavour changing and flavour conserving new contributions arise. As regards flavour changing processes, operators in (2.94) contribute to neutral mesons mixing at tree level, as in Fig. 2.5. For example $K^0 - \bar{K}^0$ oscillation is induced by:

$$-\frac{1}{4v_{d2}^2}\left[(V_{3d}V_{3s}^*)^2 + \epsilon_d^2(V_{2d}V_{2s}^*)^2\right](\overline{s_R}\gamma^\mu d_R)^2 \tag{2.95}$$

Then it is clear that flavour changing phenomena are suppressed. In particular, taking into account that $|V_{2d}V_{2s}^*| \sim \tilde{\epsilon}_d$, $|V_{3d}V_{3s}^*| \sim \epsilon_d^2\tilde{\epsilon}_d$, K^0 mixing is suppressed by a factor $\sim \epsilon_d^2\tilde{\epsilon}_d^2 \ll 1$. Defining the scale of the effective operator as $\frac{1}{\Lambda^2}(\overline{s_R}\gamma^\mu d_R)^2$, the scales of the CP conserving and CP violating contribution can be computed to be $\Lambda = 1.4\cdot 10^3$ TeV and $\Lambda = 1.5\cdot 10^4$ TeV respectively. The new contribution can be constrained to be less than the SM contribution. By taking $\epsilon_d\tilde{\epsilon}_d \sim 10^{-2}$, the mass scale $v_{d2} \sim 7\,\text{TeV}$ is compatible with the contraint from the neutral kaons mass difference. As regards the imaginary part contributing to the CP violating parameter ϵ_K, with the same choice for the benchmark value of $\epsilon_d\tilde{\epsilon}_d \sim 10^{-2}$, $v_{d2} \sim 7\,\text{TeV}$ is still allowed if the phase of $V_{2d}V_{2s}^*$ is of order 0.1.

Limits on flavour conserving operators comes from quark compositeness bounds. From Ref. [105], the limit on the energy scale of the contact interaction operator:

$$\pm\frac{2\pi}{\Lambda_{RR}^{\pm 2}}(\overline{q_R}\gamma^\mu q_R)(\overline{q_R}\gamma_\mu q_R) \tag{2.96}$$

is $\Lambda_{RR}^- > 17.5$ TeV. Then $v_{d2} = \Lambda_{RR}^-/(2\sqrt{2\pi}) > 3.5$ TeV.

Analogously to $SU(3)_\ell$, also the flavor bosons of $SU(3)_q$ can give also anomalous contributions imitating the charged current $\times$ current operators of the SM. $SU(3)_q$ bosons should induce e.g. operator $(\overline{u_L}\gamma_\rho c_L)(\overline{s_L}\gamma^\rho d_L)$ which also interferes with the charged current operators in the SM.

Chapter 3

CKM unitarity

As already mentioned, the charged current weak interactions are described by the Lagrangian density:

$$\frac{g}{\sqrt{2}}\begin{pmatrix} u & c & t \end{pmatrix}_L V_L^{(u)\dagger}\gamma^\mu V_L^{(d)}\begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^\dagger + \text{h.c.} = \tag{3.1}$$

$$= \frac{g}{\sqrt{2}}\begin{pmatrix} u & c & t \end{pmatrix}_L \gamma^\mu V_{\text{CKM}}\begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^\dagger + \text{h.c.} \tag{3.2}$$

where $V_{\text{CKM}} = V_L^{(u)\dagger}V_L^{(d)}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{3.3}$$

In the context of the SM, V_{CKM} is unitary. Deviation from the CKM unitarity can be a signal of new physics beyond the Standard Model. The unitarity condition for the first row is:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \tag{3.4}$$

After the experimental precision and control of theoretical uncertainties in the determination of $|V_{us}|$ and $|V_{ud}|$, unitarity of the first row of the Cabibbo Kobayashi Maskawa matrix can be tested. In fact, since $|V_{ub}| \simeq 0.004$ is very small, its contribution is negligible and (3.4) reduces essentially to the check of the Cabibbo mixing: $|V_{us}| = \sin\theta_C$, $|V_{ud}| = \cos\theta_C$ and $|V_{us}/V_{ud}| = \tan\theta_C$. In essence, this is the universality test for the W-boson coupling $(g/\sqrt{2})W_\mu^+ J_L^\mu$+h.c. to the relevant part of the charged left-handed current:

$$J_L^\mu = V_{ud}\overline{u_L}\gamma^\mu d_L + V_{us}\overline{u_L}\gamma^\mu s_L + \overline{\nu_e}\gamma^\mu e_L + \overline{\nu_\mu}\gamma^\mu \mu_L \tag{3.5}$$

For energies smaller than W-boson mass this coupling gives rise to the effective current $\times$ current interactions:

$$-\frac{4G_F}{\sqrt{2}}\,\overline{u_L}\big(V_{ud}\gamma_\mu d_L + V_{us}\gamma_\mu s_L\big)\big(\overline{e_L}\gamma^\mu \nu_e + \overline{\mu_L}\gamma^\mu \nu_\mu\big) \tag{3.6}$$

which are responsible for leptonic decays of the neutron, pions, kaons etc., as well as to the interaction:

$$-\frac{4G_F}{\sqrt{2}}\,\big(\overline{e_L}\gamma_\mu \nu_e\big)\big(\overline{\nu_\mu}\gamma^\mu \mu_L\big) \tag{3.7}$$

responsible for the muon decay. All these couplings contain the Fermi constant $G_F/\sqrt{2} = g^2/8M_W^2$.

In Ref. [46] we found about 4σ deviation from unitarity of the first row of CKM matrix (3.4) after comparing indipendent determinations of V_{us} obtained from the results in Refs. [106], [107] and [108]. Two possible scenarios restoring unitarity were investigated.

One of the proposed solutions was about extending the unitarity condition by introducing a fourth state of down-type vectorlike $SU(2)$ isosinglet also involved in the mixing with the SM three families. This scenario is discussed in detail in Section 4.1.

The second scenario considered the existence of a new effective four-lepton interaction contributing to muon decay in interference with SM. In fact, the muon decay constant is used as a determination of G_F in order to determine the CKM elements. However in this case, the Fermi constant G_F is slightly smaller than the muon decay constant G_μ so that unitarity is recovered. As shown in section 2.2, this kind of effective operator can be induced by the flavor changing gauge bosons of the horizontal gauge symmetry between lepton families, which should be spontaneously broken at the scale of few TeV. Also the neutron lifetime problem, that is about 4σ discrepancy between the neutron lifetimes measured in beam and trap experiments, should be analyzed in the light of the these determinations of the CKM matrix elements.

The analysis in Ref. [46] focused on the gap generated by the determination of V_{ud} from super-allowed beta decays and the determination of V_{us} from kaon physics. Then other determinations exaggerated the tension among the determinations of V_{us} obtained from leptonic and semileptonic kaon decays themselves. In this chapter the present situation of the determinations of V_{us} and V_{ud} is shown.

Then two approaches to the problem can be considered. The incompatibility inside kaon physics may be attributed to some uncertainties which can disappear maybe soon with more precise determinations. Then this inner discrepancy can be considered almost neglected in the analysis, helping in the cure but focusing more on the average of determinations from kaons. The two just mentioned scenarios are on this line, they are analyzed in sections 3.32, 4.1, 4.2. Otherwise the discrepancy inside kaon physics can be considered seriously by looking for a solution addressing the whole situation. For example, as it will be shown in section 4.4, in principle a weak isodoublet can recover all the gaps.

3.1 The present situation of CKM unitarity

As already stated, the precision of recent determinations of $|V_{us}|$ and $|V_{ud}|$ allows to test the unitarity in the first row of the Cabibbo Kobayashi Maskawa matrix (3.4). Since $|V_{ub}| \simeq 0.004$ is very small, its contribution is negligible in the unitarity test. Precision experimental data on kaon decays, in combination with the lattice QCD calculations of the decay constants and form-factors, provide accurate information about $|V_{us}|$. Regarding $|V_{ud}|$, recent calculations of short-distance radiative corrections in the neutron decay allow a remarkable precision in its determination. Then the relation in Eq. (3.4) can be used to compare different values of V_{us} obtained from different determinations of the CKM elements assuming unitarity.

First, $|V_{us}|$ can be directly determined from semileptonic $K\ell 3$ decays ($K_L\mu 3$, $K_L e3$, $K^{\pm}e3$, etc.) [110]:

$$f_+(0)|V_{us}| = 0.21654 \pm 0.00041 \tag{3.8}$$

where $f_+(0)$ is the $K \to \pi\ell\nu$ vector form factor at zero momentum transfer. We call the value of $|V_{us}|$ resulting from (3.8) determination “A".

A second determination (which we call determination “B") can be obtained from the ratio of kaon and pion inclusive radiative decay rates $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$ [111]:

$$|V_{us}/V_{ud}| \times (f_{K^\pm}/f_{\pi^\pm}) = 0.27599 \pm 0.00038 \tag{3.9}$$

By assuming unitarity, the value of $|V_{us}|$ can be obtained. Both the values of the form factor $f_+(0)$ and the decay constant ratio f_K/f_π can be taken from lattice QCD results.

Another determination of V_{us} (labeled as A′) has been obtained recently in Ref. [109] directly from leptonic kaon decay:

$$V_{us} = 0.22567(42) \tag{3.10}$$

Last, the value of $|V_{ud}|$ is obtained from beta decay. The most precise determination comes from superallowed 0^+– 0^+ nuclear β-decays, which are pure Fermi transitions sensitive only to the vector coupling constant $G_V = G_F|V_{ud}|$ [112]:

$$|V_{ud}|^2 = \frac{K}{2G_F^2 \mathcal{F}t\,(1+\Delta_R^V)} = \frac{0.97147(20)}{1+\Delta_R^V} \tag{3.11}$$

where $K = 2\pi^3 \ln 2/m_e^5 = 8120.2776(9) \times 10^{-10}$ s/GeV^4, $\mathcal{F}t$ is a "corrected ft-value", that is the nucleus independent value obtained from the combination of the individual ft-values of different 0^+– 0^+ nuclear transitions with all nucleus-dependent corrections, (where f is the statistical rate function, while the half-life and branching ratio combine to yield the partial half-life t) and Δ_R^V accounts for short-distance (transition independent) radiative corrections. $\mathcal{F}t = 3072.07(72)$ s [113] is obtained by averaging the individual $\mathcal{F}t$-values of the fourteen best determined superallowed 0^+– 0^+ transitions and $G_F = G_\mu = 1.1663787(6) \times 10^{-5}$ GeV^{-2} determined from the muon decay [114]. The major uncertainty is related to the so called inner radiative correction Δ_R^V. Again, by imposing unitarity, a determination of $|V_{us}|$ (determination "C") can be obtained from (3.11).

The present scenario of different values of V_{us} obtained from independent determinations is shown in Table 4.2 and in Fig. 3.1. As regards determination A, FLAG 2019 4-flavor average is $f_+(0) = 0.9706(27)$ [107]. By including the result from 4–flavor lattice QCD $f_+(0) = 0.9696(19)$ quoted by Ref. [106] the average yields $f_+(0) = 0.9699(15)$, which from Eq. (3.8) gives:

$$A: \qquad |V_{us}| = 0.22326(55) \tag{3.12}$$

Determination B is obtained from Eq. (3.9) by employing the result the FLAG 2019 $N_f = 2+1+1$ average $f_{K^\pm}/f_{\pi^\pm} = 1.1932(19)$ [107] [107] which gives:

$$|V_{us}|/|V_{ud}| = 0.23130(49) \tag{3.13}$$

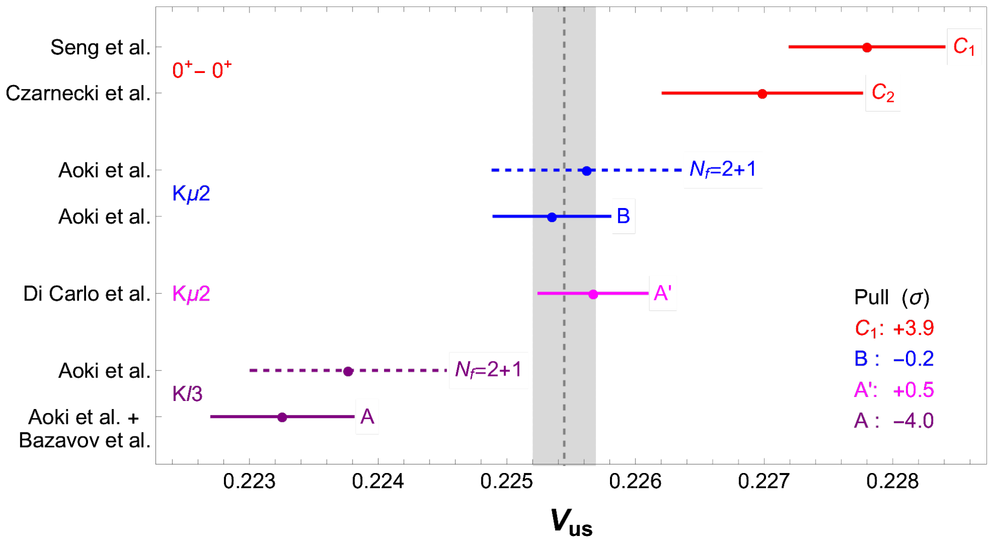

Figure 3.1: Values of V_{us} obtained from different methods, by assuming unitarity. Values from Bazavov et al [106], Aoki et al. [107], Di Carlo et al. [109], Czarnecki et al. [117], Seng et al. [108] are used. The grey region shows the 1σ interval of the average $A + B + A' + C_2$.

and with unitarity:

$$B \; : \qquad |V_{us}| = 0.22535(45) \tag{3.14}$$

which is represented by the blue continuous line in Fig. 3.1. The dashed lines refer to the values obtained from Eqs. (3.8) and (3.9) using FLAG 2019 $N_f = 2 + 1$ average $f_{K^\pm}/f_{\pi^\pm} = 1.1917(37)$ [107] and three flavor lattice QCD average of $f_+(0) = 0.9677(27)$ from Ref. [107] (which is the same as in Ref. [115]). Also the computation of V_{us} from leptonic kaon decay given in Ref. [109] is shown and labeled as A':

$$A' \; : \qquad |V_{us}| = 0.22567(42) \tag{3.15}$$

The upper red line (determination C_1) comes after the significant redetermination of inner radiative corrections with reduced hadronic uncertainties, $\Delta_R^V = 0.02467(22)$ (which comes after the result in Ref. [116]) reported by Ref. [108], which gives from Eq. (3.11):

$$|V_{ud}| = 0.97370(14) \tag{3.16}$$

then using unitarity:

$$C_1 \; : \qquad |V_{us}| = 0.22780(60) \tag{3.17}$$

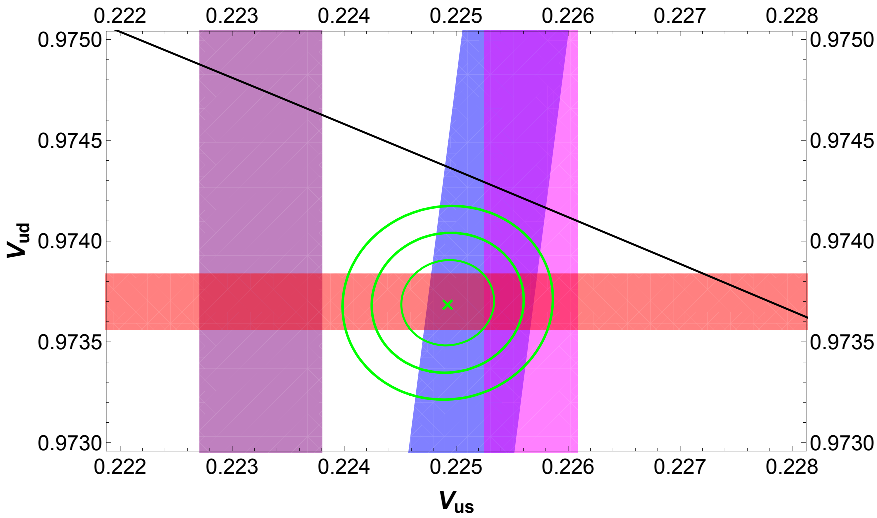

Figure 3.2: The red, purple, magenta and blue bands correspond to the values of $|V_{ud}|$ (Ref. [108]), $|V_{us}|$ (Refs. [106], [107]), $|V_{us}|$ (Ref. [109]) and $|V_{us}/V_{ud}|$ (Ref. [107]). The best fit point (green cross) and 1, 2 and 3σ contours are shown. The black line corresponds to the three family unitarity condition.

The second red line (determination C_2) refers to the determination of Ref. [117] $\Delta_R^V = 0.02426(32)$ which gives

$$|V_{ud}| = 0.97389(18) \tag{3.18}$$

from which

$$C_2 : \qquad |V_{us}| = 0.22699(77) \tag{3.19}$$

It is clear from Fig. 3.1 that these determinations are not compatible, which means that unitarity is not holding anymore.

Namely, there is a 5.6σ discrepancy between determinations A and C_1. Determinations A, B, A' are all from kaons physics, then, more conservatively, one can take their average $\overline{A + B + A'}$. The discrepancy of the latter with determination C_1 is 4.3σ.

However, since the values A, B, A' are not compatible, a very conservative treatment can be thought. One can take a conservative average between determinations A and A' (respectively from semileptonic and leptonic kaon decays, obtained without using unitarity) $|V_{us}| = 0.22478(69)$, where the errors are quadratically combined because of their poor compatibility. This average is well compatible with determination B $|V_{us}| = 0.22535(45)$ deduced from $|V_{us}/V_{ud}|$ in leptonic kaon decays, so that for the average $\overline{A + B + A'}$ it is obtained $|V_{us}| = 0.22518(38)$.

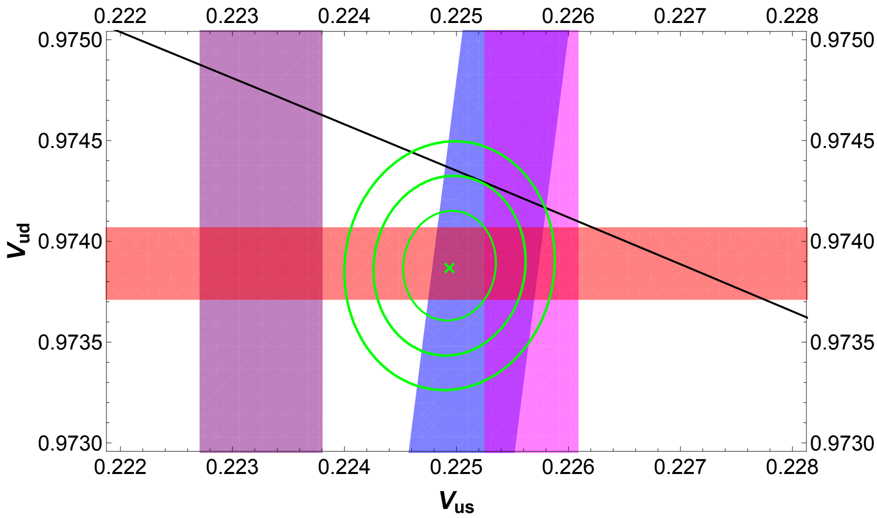

Figure 3.3: The red, purple, magenta and blue bands correspond to the values of $|V_{ud}|$ (Ref. [117]), $|V_{us}|$ (Refs. [106], [107]), $|V_{us}|$ (Ref. [109]) and $|V_{us}/V_{ud}|$ (Ref. [107]). The best fit point (green cross) and 1, 2 and 3σ contours are shown. The black line corresponds to the three family unitarity condition.

Regarding the determination of $|V_{ud}|$, as already stated, determination C_1 is obtained from $\Delta_R^V = 0.02467(22)$ of Ref. [108], while the value $\Delta_R^V = 0.02426(32)$ is deduced in Ref. [117]. For being more conservative, we can average these two results as $\Delta_R^V = 0.02454(32)$, without reducing the largest uncertainty. In doing so, from Eq. (3.11) it is obtained that $|V_{ud}| = 0.97376(10)_{\mathcal{F}t}(15)_{\Delta_R^V} = 0.97376(18)$ which in turn gives determination C as $|V_{us}| = 0.22756(78)$. Therefore, between this determination C and $A + B$ remains $2.8\,\sigma$ tension even with this very conservative treatment.

By performing a fit of the four determinations of V_{us} A, A', B, C_1 the minimum is obtained in:

$$|V_{us}| = 0.22545(25) \tag{3.20}$$

and consequently $|V_{ud}| = 0.974248(57)$, but the χ^2 in the minimum is $\chi^2_{\rm dof} = 10.5$, which is very high. In fact, unitarity is not verified at 3.9σ level (99.9916% C.L.), as can be seen by fitting the four determinations 3.12, 3.13, 3.15, 3.16, 3.18, without imposing unitarity, with V_{us} and V_{ud} as independent parameters as shown in Fig. 3.2. The best fit point is:

$$|V_{us}| = 0.22492(27) \qquad\qquad |V_{ud}| = 0.97369(14) \tag{3.21}$$

The χ^2 per degree of freedom is $\chi^2_{\rm dof} = 6.4$, which is still high since there is strong incompatibility among the determinations themselves besides the lack of unitarity.

The discrepancy is softened if the value obtained with the result in [117] is used in order to obtain determination C_2. By imposing unitarity, the minimum is obtained in:

$$|V_{us}| = 0.22520(25) \tag{3.22}$$

and consequently $|V_{ud}| = 0.974306(59)$, with χ^2 in the minimum $\chi^2_{\rm dof} = 6.4$. Unitarity is not verified at 2σ level (95.75% C.L.), as can be understood after the fit shown in Fig. 3.3 with V_{us} and V_{ud} as independent parameters, without imposing unitarity, which gives as best fit point (green cross in Fig. 3.3):

$$|V_{us}| = 0.22494(27) \qquad\qquad |V_{ud}| = 0.97388(18) \tag{3.23}$$

with $\chi^2_{\rm dof} = 6.4$.

As a brief digression, it is worth noticing that in this case it seems that the most important issue is with the vector coupling of semileptonic kaon decays.

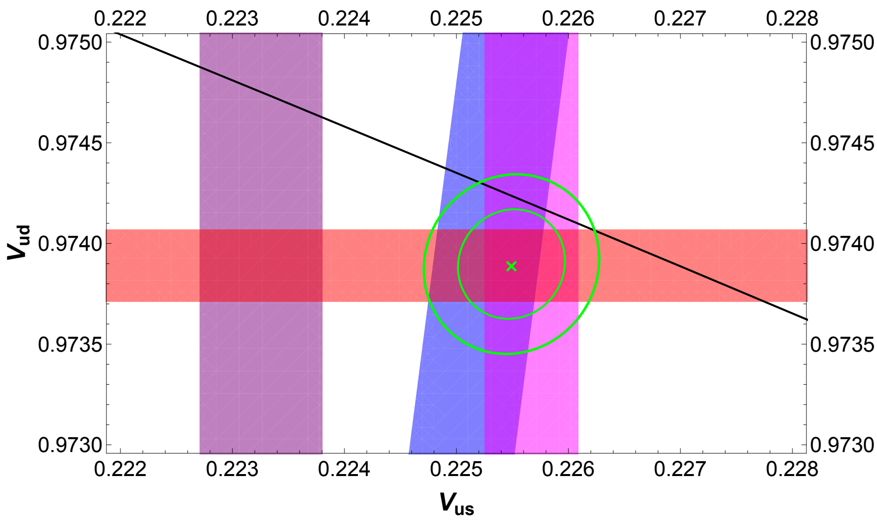

Figure 3.4: The red, purple, magenta and blue bands correspond to the values of $|V_{ud}|$ (Ref. [117]), $|V_{us}|$ (Refs. [106], [107]), $|V_{us}|$ (Ref. [109]) and $|V_{us}/V_{ud}|$ (Ref. [107]). The fit is performed without determination A (purple band). The best fit point (green cross) and 1, 2 and 3σ contours are shown. The black line corresponds to the three family unitarity condition.

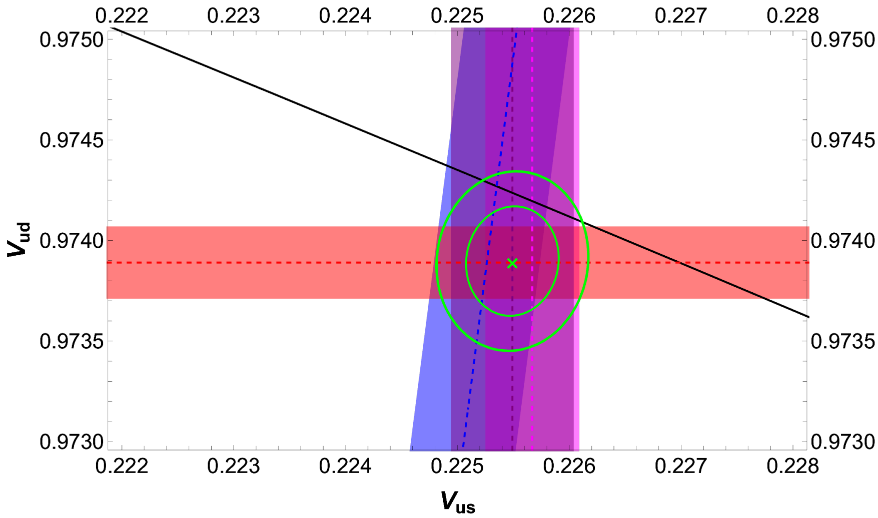

Figure 3.5: The red, magenta and blue bands correspond to the values of $|V_{ud}|$ (Ref. [117]), $|V_{us}|$ (Ref. [109]) and $|V_{us}/V_{ud}|$ (Ref. [107]). The purple band corresponds to the value of $|V_{us}|$ from Refs. [106], [107], shifted with $v_{us} = -0.00224$. The best fit point (green cross) and 1, 2 and 3σ contours are shown related to the two parameter fit of the four determinations after adding $v_{us} = -0.00224$. The black line corresponds to the three family unitarity condition.

In fact, by performing a fit of the three determinations A', B, C_2 with V_{us} as a parameter, by imposing unitarity, it is obtained the minimum $\chi^2_{\rm dof} = 1.7$ in:

$$V_{us} = 0.22572(29) \tag{3.24}$$

This agreement among axial couplings and unitarity is also evident in the two parameters fit shown in Fig. 3.4. Unitarity is satisfied at 1.2σ level (77.1% C.L.). Then the discrepancy could be fixed in this case by adding a vector current to the SM current involving V_{us}:

$$-\frac{G_F}{\sqrt{2}} \left(V_{us}\, \overline{u}\gamma^{\mu}(1-\gamma_5)s \, + v_{us}\, \overline{u}\gamma^{\mu} s\right) \cdot \left(\ell\gamma_{\mu}(1-\gamma_5)\nu_{\ell}\right) + \text{h.c.} \tag{3.25}$$

The two parameters fit of the determinations A', B, C_2 (all except the determination from the vector coupling) returns $\chi^2_{\rm dof} = 0.4$, with the best fit point in $V_{us} = 0.22549$, $V_{ud} = 0.973898$ (green cross in Fig. 3.4). In order to receive the same values also from the vector coupling, the needed value of v_{us} in (3.25) is $v_{us} = -0.00224$ as shown in Fig. 3.5. This additional vector coupling corresponds to the additional operator:

$$\frac{1}{\Lambda_{us}^2}\,(\overline{u}\gamma^\mu s)\,(\ell\gamma_\mu(1-\gamma_5)\nu_\ell) \tag{3.26}$$

where

$$\frac{1}{\Lambda_{us}^2} = -\frac{G_F}{\sqrt{2}}v_{us} \tag{3.27}$$

with

$$\Lambda_{us} = 7.3\,\mathrm{TeV} \tag{3.28}$$

corresponding to $v_{us} = -0.00224$.

3.2 $G_F \neq G_\mu$

3.2.1 New effective operator

Here one possible solution to the CKM unitarity problem is discussed: by making G_μ different from G_F, instead of moving the unitarity line in Fig. 3.5, the probability distribution is moved towards the unitarity line, which instead does not move.

In this scenario the inner incompatibility of the determinations of V_{us} from kaon physics cannot be address completely. For this reason, the same data set used in Ref. [46] can be indicative enough and the same analysis of Ref. [46] is reported here, in order to estimate the scale of the new operator. The determination $|V_{ud}|$ in (4.38) obtained from Ref. [108] is used, together with the result $f_+(0) = 0.9696(18)$ from new 4–flavor ($N_f = 2{+}1{+}1$) lattice QCD simulations [106] and the FLAG 2019 four-flavor average $f_{K^\pm}/f_{\pi^\pm} = 1.1932(19)$ [107]. Then the chosen dataset is:

$$\begin{aligned} |V_{us}| &= 0.22333(60) \\ |V_{us}/V_{ud}| &= 0.23130(50) \\ |V_{ud}| &= 0.97370(14) \end{aligned} \tag{3.29}$$

which reduces by imposing the CKM unitarity to independent $|V_{us}|$ which here can be called A, B, C respectively (numerical values are given in Table 4.2). There is a 5.3σ discrepancy between A and C, and 3.2σ between B and C. The

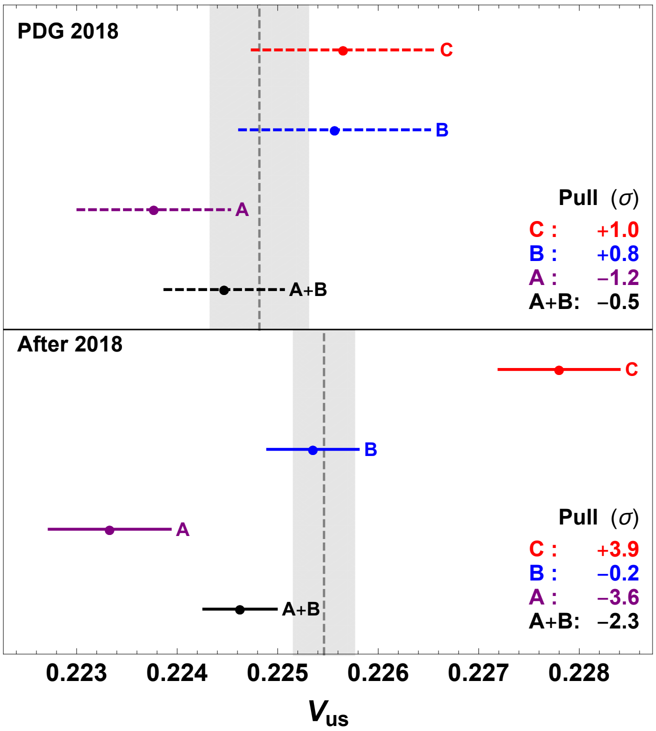

Figure 3.6: *Lower panel:* Three independent $|V_{us}|$ determinations A, B, C obtained from the dataset (3.29). The grey shaded band corresponds to the average A+B+C (with formal error not rescaled by a factor $\sqrt{\chi^2_{\rm dof}}$). Pulls of C, B, A and A+B are shown. *Upper panel:* For comparison, three independent $|V_{us}|$ determinations A, B, C obtained from the PDG 2018 data $|V_{us}| = 0.2238(8)$, $|V_{us}/V_{ud}| = 0.2315(10)$, $|V_{ud}| = 0.97420(21)$ obtained from FLAG 2017 averages of 3–flavor lattice QCD simulations $f_+(0) = 0.9677(27)$ and $f_{K^\pm}/f_{\pi^\pm} = 1.192(5)$ [115] and by taking $\Delta_R^V = 0.02361(38)$ as calculated in Ref. [116] respectively.

	CKM [PDG]	CKM [post 2018]	CKM+ b'	CKM+$\mathcal{F}$
C	0.2257(9)	0.22780(60)	0.22443(61)	0.22460(61)
B	0.2256(10)	0.22535(45)	0.22518(45)	0.22535(45)
A	0.2238(8)	0.22333(60)	0.22333(60)	0.22350(60)
$\overline{A+B}$	0.2245(6)	0.22463(36)	0.22452(36)	0.22469(36)
$\overline{A+B+C}$	0.2248(5)	0.22546(31)	0.22449(31)	0.22467(31)
	$\chi^2 = 3.4$	$\chi^2 = 27.7$ †	$\chi^2 = 6.1$	$\chi^2 = 6.1$
$\lvert V_{us}\rvert$	0.2248(7)	0.2255(12) †	0.2245(5)	0.2247(5)
$\lvert V_{ud}\rvert$	0.97440(16)	0.97424(27) †	0.97369(12)	0.97443(12)

Table 3.1: The second column shows independent $|V_{us}|$ determinations A, B, C obtained from dataset (3.29) by assuming 3-family CKM unitarity (**??**). For comparison, it is shown in the first column the dataset obtained from the PDG 2018 data $|V_{us}| = 0.2238(8)$, $|V_{us}/V_{ud}| = 0.2315(10)$, $|V_{ud}| = 0.97420(21)$ obtained from FLAG 2017 averages of 3–flavor lattice QCD simulations $f_+(0) = 0.9677(27)$ and $f_{K^\pm}/f_{\pi^\pm} = 1.192(5)$ [115] and by taking $\Delta_R^V = 0.02361(38)$ as calculated in Ref. [116] respectively. The last column shows the same dataset but assuming 3-family CKM with $G_\mu/G_F = 1 + \delta_\mu$ with $\delta_\mu = 7.6 \times 10^{-4}$. For comparison it is shown in the second coulumn the dataset obtained by extending unitarity to 4th quark b' with $|V_{ub'}| = 0.04$. The mark † in the first column indicates that for that large χ^2 the error-rescaling by $\sqrt{\chi^2_{\rm dof}} = 3.7$ does not make much sense since the data are incompatible. The last two rows show the conservative estimation of $|V_{us}|$ with error-bar rescaled by $\sqrt{\chi^2_{\rm dof}}$ and the corresponding value of $|V_{ud}|$.

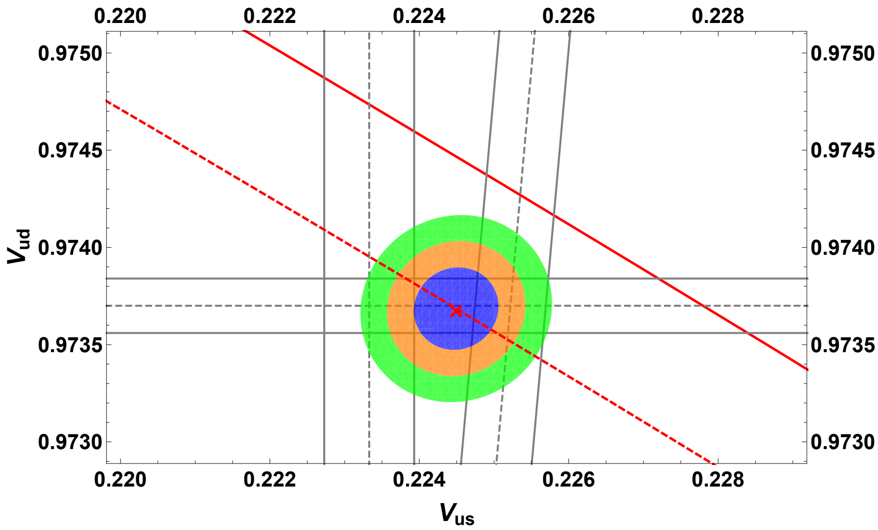

Figure 3.7: The horizontal, vertical and slightly bended bands correspond to $|V_{ud}|$, $|V_{us}|$ and $|V_{us}/V_{ud}|$ from (3.29). The best fit point (red cross) and 1, 2 and 3σ contours are shown. The red solid line corresponds to the three family unitarity condition (**??**), and the dashed red line corresponds to the "extended" unitarity (**??**) with $|V_{ub'}| = 0.04$.

tension between the determinations A and B from kaon physics is 2.7σ. More conservatively, one can take their average $\overline{A+B}$. The discrepancy of the latter with C is 4.5σ. Fitting these values, it is obtained $\overline{A+B+C} = 0.22546(31)$ but the fit is bad, $\chi^2_{\rm dof} = 13.9$. The two parameter fit of the three independent values (3.29) is shown in Fig. 3.7, the best fit point is ($|V_{us}| = 0.22449$, $|V_{ud}| = 0.97369$), with $\chi^2_{\rm min} = 6.1$. The red solid line corresponding to the three family unitarity condition is about 4.3σ away from this hill top ($\Delta\chi^2 = 21.6$) which means that the dataset (3.29) disfavors the CKM unitarity at 99.998% C.L.

Now it is considered the scenario in which the Fermi constant G_F in the effective interaction (3.6) which is responsible for leptonic decays of hadrons is different from the effective constant G_μ determined from the muon lifetime. In order to obtain this result, we assume that besides the SM interaction (3.7) mediated by charged $W-$boson, there is also a new operator

$$- \frac{4G_{\mathcal{F}}}{\sqrt{2}} (\overline{e_L}\gamma_\mu \mu_L)(\overline{\nu_\mu}\gamma^\mu \nu_e) \tag{3.30}$$

which, as already shown in section 2.2, can be mediated by a hypothetical lepton flavor changing neutral gauge boson. The respective diagrams, shown in Fig. 2.4, have positive interference for the muon decay. Namely, by Fierz transformation this new operator can be brought to the form (3.7), so that the sum of these two diagrams effectively gives the operator

$$-\frac{4G_\mu}{\sqrt{2}}\left(\overline{e_L}\gamma_\mu\nu_e\right)\left(\overline{\nu_\mu}\gamma^\mu\mu_L\right) \tag{3.31}$$

the same as (3.7) but with the coupling constant

$$G_\mu = G_F + G_{\mathcal{F}} = G_F(1+\delta_\mu), \qquad \frac{G_{\mathcal{F}}}{G_F} \equiv \delta_\mu > 0\,. \tag{3.32}$$

Constant $G_\mu = 1.1663787(6)\times 10^{-5}$ GeV^{-2} is determined with great precision from the muon decay [114] and it is used as a determination of G_F in order to determine the CKM elements. In this case instead the Fermi constant G_F would be slightly smaller than the muon decay constant G_μ.

Now Eqs. (3.11) and (3.8), instead of $|V_{ud}|$ and $|V_{us}|$, are determining respectively the values $|V_{ud}|\times G_F/G_\mu$ and $|V_{us}|\times G_F/G_\mu$. Instead the value of $|V_{us}/V_{ud}|$ determined from (3.9) remains unchanged since the Fermi constant cancels out. Thus, under our hypothesis, the dataset (3.29) in Table 4.2 should be modified to the following:

$$\begin{aligned} |V_{us}| &= 0.22326(55)\times(1+\delta_\mu) \\ |V_{us}/V_{ud}| &= 0.23130(50) \\ |V_{ud}| &= 0.97370(14)\times(1+\delta_\mu) \end{aligned} \tag{3.33}$$

Now, involving the extra parameter δ_μ but assuming the 3-family unitarity (3.4), the fit of the above dataset has acceptable quality, $\chi^2 = 6.1$, and the best fit point corresponds to $\delta_\mu = 0.00076$. This situation is shown in Fig. 3.8 in which the values of $|V_{ud}|$ and $|V_{us}|$ are determined by taking $\delta_\mu = 0.00076$. By this choice of the extra parameter the fit becomes perfectly compatible with the unitarity (3.4). The probability distribution is moved up so that its top now lies on the unitarity line.

By imposing the unitarity condition $|V_{ud}|^2+|V_{us}|^2 = 1-|V_{ub}|^2$, the list (3.33) can be transformed in δ_μ dependent determinations A, B, C of $|V_{us}|$. Fig. 3.9 shows these determinations for $\delta_\mu = 0.00076$. Taking into account that $G_F/\sqrt{2} = g^2/8M_W^2 = 1/4v_{\rm w}^2$, where $v_{\rm w} = 174$ GeV is the weak scale, and parameterizing similarly $G_{\mathcal{F}}/\sqrt{2} = 1/4v_{\mathcal{F}}^2$, we see that $\delta_\mu = G_{\mathcal{F}}/G_F = 0.00076$ corresponds to $v_{\mathcal{F}}/v_{\rm w} = 36.3$, or to the flavor symmetry breaking scale

$$v_{\mathcal{F}} = 6.3\text{TeV} \tag{3.34}$$

More widely, the 1σ interval of the parameter δ_μ consistent with unitarity at the 68% C.L. is $\delta_\mu = (7.6\pm 1.6)\times 10^{-4}$ which corresponds to the new scale in the interval $v_{\mathcal{F}} = 5.7$-7.1 TeV.

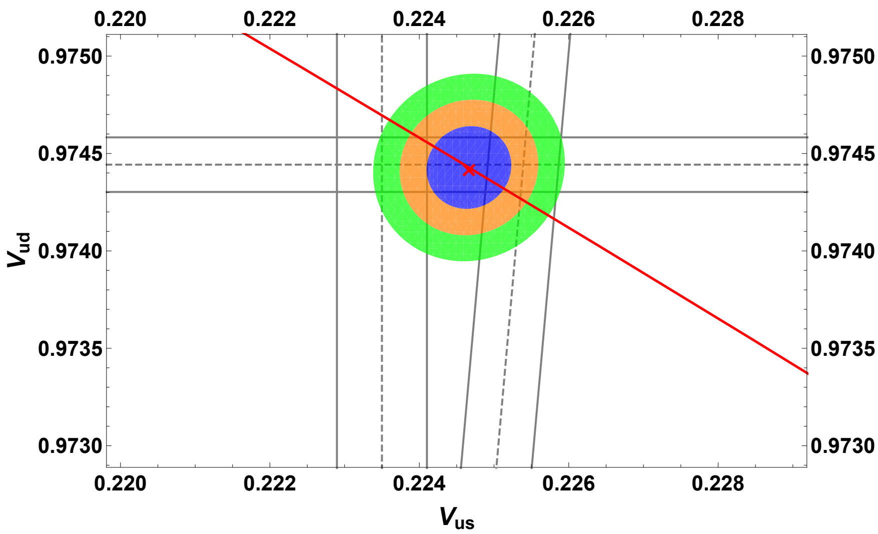

Figure 3.8: The same as on Fig. 3.7 but with the bands of $|V_{ud}|$, $|V_{us}|$ and $|V_{ud}/V_{us}|$ taken as in (3.33) with $1+\delta_\mu = 1.00076$. The red line corresponds to 3-family unitarity (3.4) as in Fig. 3.7.

3.2.2 Determination of CKM elements and neutron lifetime problem

The value $|V_{ud}|$ can be extracted also from free neutron decay by combining the results on the measurements of the neutron lifetime τ_n with those of the axial current coupling constant g_A. The master formula reads (see e.g. in a recent review [118]):

$$|V_{ud}|^2 = \frac{K/\ln 2}{G_F^2 \mathcal{F}_n \tau_n \, (1+3g_A^2)(1+\Delta_R^V)} = \frac{5024.46(30)\ \mathrm{s}}{\tau_n (1+3g_A^2)(1+\Delta_R^V)} \tag{3.35}$$

where $\mathcal{F}_n = f_n(1+\delta_R')$ is the neutron f-value $f_n = 1.6887(1)$ corrected by the long-distance QED correction $\delta_R' = 0.01402(2)$ [119]. This equation, taking the values $\tau_n = 880.2 \pm 1.0$ s, $g_A = 1.2724 \pm 0.0023$ adopted in PDG 2018 [111], and $\Delta_R^V = 0.02361(38)$ [116], would give the value

$$|V_{ud}| = 0.97577(55)_{\tau_n}(146)_{g_A}(18)_{\Delta_R^V} = 0.97577(157) \tag{3.36}$$

which has an order of magnitude larger error than $|V_{ud}| = 0.97420(10)_{\mathcal{F}t}(18)_{\Delta_R^V} = 0.97420(21)$ obtained from (3.11) and used in section 3.1, due to large uncertain-

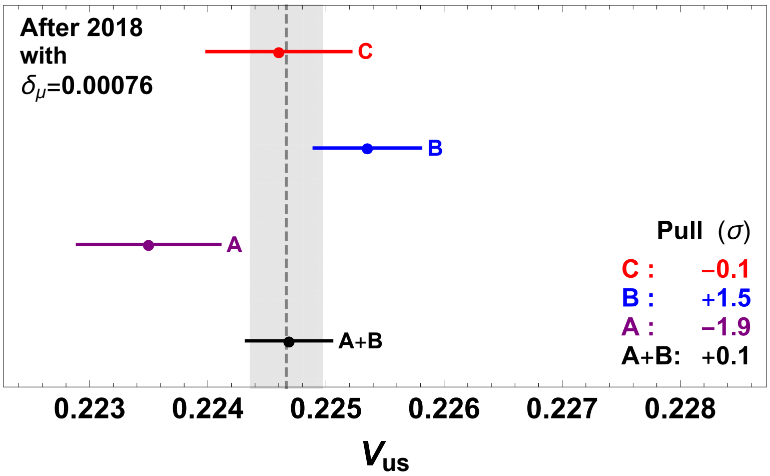

Figure 3.9: Determinations of $|V_{us}|$ obtained from (3.33).

ties in τ_n and g_A.

However, rather than for determination of $|V_{ud}|$, Eq. (3.35) can be used for a consistency check. Namely, by comparing it with Eq. (3.11) we get a relation between τ_n and g_A [120]:

$$\tau_n = \frac{2\mathcal{F}t}{\ln 2\, \mathcal{F}_n(1+3g_A^2)} = \frac{5172.0(1.1)\text{ s}}{1+3g_A^2} \tag{3.37}$$

In Fig. 3.10 this relation is shown by the red band. This formula is very accurate since the common factors in Eqs. (3.11) and (3.35) including the Fermi constant and radiative corrections Δ_R^V cancel out.

For the axial current coupling g_A, the PDG 2018 quotes the value $g_A = 1.2724 \pm 0.0023$. However, the results of the latest and most accurate experiments [121, 122, 123] which measured β-asymmetry parameter using different techniques (the cold neutrons in PERKEO II and PERKEO III experiments [121, 123] and ultra-cold neutrons in the UCNA experiment [122]), are in perfect agreement among each other, and their average determines the axial current coupling g_A with impressive (better than one per mille) precision:

$$g_A = 1.27625 \pm 0.00050\,. \tag{3.38}$$

Fig. 3.10 shows the results of Refs. [121, 122, 123] and their average (vertical grey band). For g_A in this range Eq. (3.37) gives the Standard Model prediction for the neutron lifetime

$$\tau_n^{\text{SM}} = 878.7 \pm 0.6\text{ s} \tag{3.39}$$

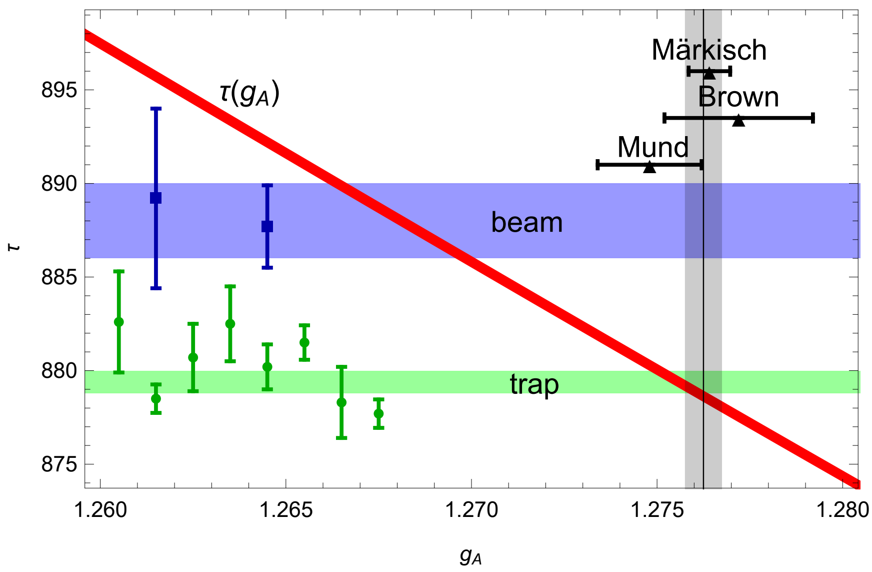

Figure 3.10: The red band shows the precision relation (3.37 between g_A and τ_n. Black triangles with horizontal error bars show values of g_A reported in Refs. [121, 122, 123] and vertical grey band corresponds to their average (3.38). Green circles show values of τ_n reported by trap experiments [125, 126, 127, 128, 129, 130, 131, 132] with respective error bars and horizontal green band shows their average (3.40). Blue squares and blue horizontal band show the the same for beam experiments [133, 134].

From the experimental side, the neutron lifetime is measured in two types of experiments. The trap experiments measure the disappearance rate of the ultra-cold neutrons (UCN) by counting the survived neutrons after storing them for different times in the UCN traps and determine the neutron decay width $\Gamma_n = \tau_n^{-1}$. The beam experiments are the appearance experiments, measuring the width of β-decay $n \to pe\bar{\nu}_e$, $\Gamma_\beta = \tau_\beta^{-1}$, by counting the produced protons in the monitored beam of cold neutrons. In the Standard Model the neutron decay should always produce a proton, and so both methods should measure the same value $\Gamma_n = \Gamma_\beta$.

However, there is a tension between the results obtained using these two methods, which was pointed out in Refs. [124]. Fig. 3.10 clearly demonstrates the discrepancy. Namely, by averaging the presently available results of eight trap experiments [125, 126, 127, 128, 129, 130, 131, 132] one obtains[1]:

$$\tau_{\rm trap} = 879.4 \pm 0.6 \text{ s}\,, \tag{3.40}$$

which is compatible with $\tau_n^{\rm SM}$ (3.39). In particular, this value of τ_n together with new g_A (3.38) and new value $\Delta_R^V = 0.02467(22)$ [108], determines $|V_{ud}|$ with the precision more than 3 times better than in (3.36):

$$|V_{ud}| = 0.97327(33)_{\tau_n}(32)_{g_A}(10)_{\Delta_R^V} = 0.97327(47). \tag{3.41}$$

This is compatible with $|V_{ud}| = 0.97370(10)_{\mathcal{F}t}(10)_{\Delta_R^V} = 0.97370(14)$ from superallowed $0^+ - 0^+$ decays used in (3.29) but has 3 times larger error. For making it competitive, the neutron lifetime should be measured with precision of 0.1 s and g_A with precision 3 times better than in (3.38), which can be realistic in future experiments.

On the other hand, the beam experiments [133, 134] yield the value

$$\tau_{\rm beam} = 888.0 \pm 2.0 \text{ s} \tag{3.42}$$

which is 4.4σ away from the SM prediction (3.39). Therefore, it is more likely that the true value of the neutron lifetime is the one measured by trap experiments (3.40) which is consistent with the SM prediction (3.37).

About 1 per cent deficit of produced protons in the beam experiments [133, 134] might be due to some unfixed systematic errors. Alternatively, barring the

[1] The PDG 2018 average $\tau_n = 880.2 \pm 1.0$ s includes the results of five trap experiments [125, 126, 127, 128, 129] and two beam experiments [133, 134]. The error is enlarged by a factor $\sqrt{\chi^2_{\rm dof}} \approx 2$, essentially for a loose compatibility between the data obtained from the trap and the beam experiments. This average does not include the results of three recent trap experiments [130, 131, 132] published in 2018.

possibility of uncontrolled systematics and considering the problem as real, a new physics must be invoked which could explain about one per cent deficit of protons produced in the beam experiments. One interesting possibility can be related to the neutron–mirror neutron ($n - n'$) oscillation [91, 92, 93], provided that ordinary and mirror neutrons have a tiny mass difference 300 neV or so [135]. Then in large magnetic fields (5 Tesla) used in beam experiments $n - n'$ conversion probability can be resonantly enhanced to about ~ 0.01, and corresponding fraction of neutrons converted in mirror neutrons will decay in an invisible (mirror) channel without producing ordinary protons.

Concluding this section, let us remark that the recent accurate calculations of the short-range radiative corrections Δ_R^V [108, 117] and respective redetermination of V_{ud} has no influence on the determination of τ_n (3.39) obtained from Eq. (3.37). In fact, the latter equation directly relates the neutron lifetime to the value $\mathcal{F}t$ accurately measured in superallowed $0^+ - 0^+$ nuclear transitions and to the value g_A obtained from accurate measurements of β-asymmetry. Notice that the relation (3.37) remains valid also in the presence of non-standard vector G_V or axial G_A coupling constants (which can be the case if some non-standard interactions mediated by new vector bosons also contribute to the neutron decay) since the value G_V (independently whether it is equal to $G_F|V_{ud}|$ or not) anyway cancels out [136]. Hence, only the ratio $g_A = G_A/G_V$ remains relevant which is accurately determined from the measurements of β-asymmetry. In particular, Eq. (3.37) remains valid in our model with $G_F \neq G_\mu$ discussed in previous section.

Chapter 4

Vector-like quarks

Quantity	Value	Quantity	Value
$\|V_{us}\|$	0.22545(25)	m_W[GeV]	80.379(12)
$\|V_{cd}\|$	0.218(4)	$m_c(m_c)$[GeV]	1.27(2)
$\|V_{cb}\|$	0.0422(8)	m_s(2GeV)[MeV]	93.12(69) [107]
$\|V_{ts}\|$	0.0394(23)	M_t[GeV]	172.9(4)
$\|V_{td}\|$	0.0081(5)	M_{K^0}[MeV]	497.611(13)
$\|V_{ub}\|$	0.00394(36)	τ_{K_L}[s]	$5.116(21) \cdot 10^{-8}$
λ	0.22453(44)	M_{K^+}[MeV]	493.677(16)
$\bar{\rho}$	0.122(18)	τ_{K^+}[s]	$1.2380(20) \cdot 10^{-8}$
A	0.836(15)	m_μ[MeV]	105.6583745(24)
$\bar{\eta}$	0.355(12)	τ_{D^+}[s]	$1.040(7) \cdot 10^{-12}$
$\alpha_s(M_Z)$	0.1187(16)	M_{D^+}[MeV]	1869.65(5)
$\alpha(M_Z)^{-1}$	127.955(10)	τ_{D^0}[s]	$4.101(15) \cdot 10^{-13}$
$\sin^2\theta_W(M_Z)$	0.23122(3)	M_{D^0}[MeV]	1864.83(5)
G_F[GeV^{-2}]	$1.1663787(6) \cdot 10^{-5}$		

Table 4.1: The central values are employed in the computations. All values are taken from Ref. [111], except $|V_{us}| = 0.22545(25)$ from Table 4.2 and $m_s(2\text{GeV}) = 93.12(69)$ MeV from Ref. [107].

In Ref. [46] we analyzed the determinations of the CKM elements of the first row and the problem of of the tensions among them leading to CKM unitarity violation was raised. One of the suggested solutions was the introduction of a down-type vector-like couple of $SU(2)$ singlets b'_L, b'_R. Vector-like fermions exist in several scenarios beyond the SM, as GUT theories or string theories, but also in the context of flavour physics as UV content of effective operators generating the fermion masses, e.g. vectorlike fermions emerge in the scenario of family symmetries, as also shown in this thesis. The existence of a fourth sequential family is excluded by the limits from electroweak precision data combined with the LHC data. However the existence of vector-like quarks, singlets or doublets, is still allowed. Since this isosinglet fourth species participates in the mixing, then the first row unitarity condition is modified to:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 + |V_{ub'}|^2 = 1 \tag{4.1}$$

In Ref. [46] we showed that $|V_{ub'}| = 0.04$ was needed in order to restore unitarity. Then in order to obtain this large mixing the extra vector-like quark should be no more than 6 TeV. In fact, the value of the mixing element $|V_{ub'}| = 0.04$ is rather large, being comparable to $|V_{cb}|$ and ten times larger than $|V_{ub}|$. Then the consequences of so large mixing need to be investigated. Namely, the mixing of ordinary quarks with extra quarks changes Z-boson couplings, inducing quark flavour changing (FC) couplings at tree level and affecting SM observables [137]. In this chapter possiblities and conditions are analyzed in which the needed large mixing of the first family with the 4th species, $|V_{ub'}| \approx 0.04$, can be realized.

Considering the dataset A, A', B, C_1 in Table 4.2, the best fit point is obtained for the value $|V_{ub'}| = 0.036$ in (4.1), with $\chi^2_{\rm dof} = 6.4$, mainly due to the discrepancy of A from B and A'.

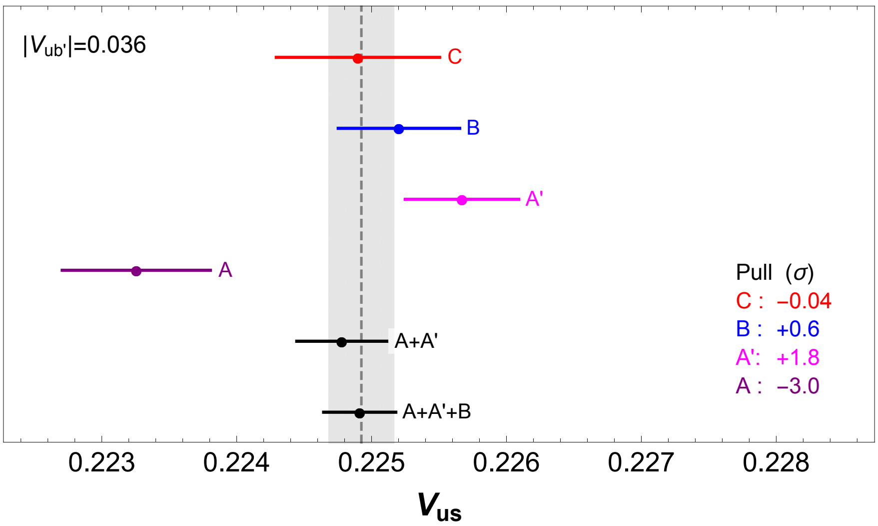

Figure 4.1: Determinations of $|V_{us}|$ obtained using Eq. (4.1) with $|V_{ub'}| = 0.036$, to be compared with Fig. 3.1.

In Fig. 4.1 it is shown how the present situation would look like if the relation (4.1) holds, with $|V_{ub'}| = 0.036$. The data are listed in Table 4.2. The best fit point is:

$$|V_{us}| = 0.22492(25) \tag{4.2}$$

which gives $|V_{ud}| = 0.973694(57)$. Determinations B and C are shifted with respect to the ones in Fig. 3.1, while A and A' remain unchanged. The compatibility increases but some tension between determination A and the values B and A' is still there. At 95% C.L. the needed additional mixing is $|V_{ub'}| = 0.036 \pm 0.009$.

However, the analysis carried out in Ref. [46] with the addition of the extra weak isosinglet was motivated by the gap between the determination of V_{us} extracted from the determination of V_{ud} by using unitarity and the average of the other determinations obtained from kaon physics, which seemed the most important issue. In the meantime other determinations exaggerated the tension among the determinations of V_{us} arising from leptonic and semileptonic kaon decays themselves, mostly after the determination of $|V_{us}|$ from $K\mu 2$ decays in Ref. [109]. Hence in this chapter scenarios which can address the whole situation are also considered. As it will be shown, in principle a weak isodoublet can recover all the gaps.

Then two approaches to the problem are considered. The incompatibility inside kaon physics may be attributed to some uncertainties which can disappear maybe soon with more precise determinations, focusing instead on the average of determinations from kaons. The scenarios considering new isosinglet quarks analyzed in sections 4.1, 4.2 are on this line. Otherwise the discrepancy inside kaon physics can be considered seriously by looking for a solution addressing the whole situation, as described in sections 4.4, 4.5, considering additional weak isodoublet species.

4.1 Extra down-type quark

First, the case of the addition of a fourth down-type vector-like isosinglet couple of quarks (d_{4L}, d_{4R}) is examined. New Yukawa terms and Dirac mass terms should be added to the Lagrangian density apart from the standard Yukawa terms:

$$\sum_{i=1}^{4}\sum_{j=1}^{3} h'_{d\,j}\varphi\overline{q_{Lj}}d'_{iR} + \sum_{i=1}^{4} m_i\overline{d_{4L}}d'_{iR} + \text{h.c.} \tag{4.3}$$

Determination	$\lvert V_{us}\rvert$ value	$\lvert V_{ub'}\rvert = 0.036$
A+B+A′+C_2	0.22545(25)	0.22492(25)
A+B+A′+C_1	0.22520(25)	
A	0.22326(55)	0.22326(55)
A′	0.22567(42)	0.22567(42)
B	0.22535(45)	0.22520(45)
C_1	0.22780(60)	0.22490(61)
C_2	0.22699(77)	
A+A′	0.22478(33)	0.22478(33)
A+A′+B	0.22498(27)	0.22491(27)
A $2+1$	0.22377(75)	
B $2+1$	0.22562(73)	

Table 4.2: Values of V_{us} obtained from different methods, by assuming unitarity. Determination A is obtained using Eq. (3.8) after including the result for $f_+(0)$ in Ref. [106] in FLAG 2019 average [107]. Determination A′ is taken from Ref. [109]. Determination B is obtained inserting the result in Ref. [107] in Eq. (3.9), making use of CKM unitarity. Determination C from the result in Ref. [108] after using unitarity. Determinations A and B in the last two rows are obtained using 3-flavours averages of lattice QCD simulations from Ref. [107]. After 2018 data are not compatible, then the fitting gives back high χ^2, more precisely $\chi^2_{\text{dof}} = 10.5$. In the last column the values of V_{us} from different determinations assuming the extended unitarity (4.1) are shown.

Since the four species of right-handed singlets d'_{iR} have identical quantum numbers, a unitary transformation can be applied on the four components d'_{iR} so that $m_j = 0$ for $j = 1, 2, 3$ and d_{4R} is identified with the combination making the Dirac mass term with the left handed singlet d_{4L}. Thus the new Yukawa couplings can be written as:

$$\sum_{j=1}^{3} h_{d\,j}\varphi\overline{q_{Lj}}d_{4R} + M_4\overline{d_{4L}}d_{4R} + h.c. \tag{4.4}$$

Then the down-type quarks mass matrix looks like:

$$\overline{d_{Li}}\mathbf{m}^{(d)}_{ij}d_{Rj} + h.c. = \tag{4.5}$$

$$=(\overline{q_{1L}}, \overline{q_{2L}}, \overline{q_{3L}}, \overline{d_{4L}}) \left(\begin{array}{ccc|c} & & & h_{d1}v_w \\ & \mathbf{m}^{(d)}_{3\times 3} & & h_{d2}v_w \\ & & & h_{d3}v_w \\ \hline 0 & 0 & 0 & M_4 \end{array}\right) \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}_R + h.c. \tag{4.6}$$

where $v_w = 174$ GeV is the SM Higgs vacuum expectation value (VEV) (for a convenience, we use this normalization of the Higgs VEV instead of the "standard" normalization $\langle\phi\rangle = v/\sqrt{2}$, i.e. $v = \sqrt{2}v_{\mathrm{w}}$) and $\mathbf{m}^{(d)}_{3\times 3}$ is a 3×3 mass matrix. The mass matrix $\mathbf{m}^{(d)}$ can be diagonalized with positive eigenvalues by a biunitary transformation:

$$V_L^{(d)\dagger}\mathbf{m}^{(d)}V_R^{(d)} = \mathbf{m}^{(d)}_{\mathrm{diag}} = \mathrm{diag}(y_1^d v_w, y_2^d v_w, y_3^d v_w, M_d) \tag{4.7}$$

where $V^{(d)}_{L,R}$ are two unitary 4×4 matrices. $\mathbf{m}^{(d)}_{\mathrm{diag}}$ is the diagonal matrix of mass eigenvalues $m_{d,s,b} = y^d_{1,2,3}v_w$ and $M_d \approx M_4$. Since no new weak interaction has been added in the right-handed sector, the only observable mixing remains inside the left-handed sector:

$$V_L^{(d)} = \begin{pmatrix} V_{1d} & V_{1s} & V_{1b} & V_{1b'} \\ V_{2d} & V_{2s} & V_{2b} & V_{2b'} \\ V_{3d} & V_{3s} & V_{3b} & V_{3b'} \\ V_{4d} & V_{4s} & V_{4b} & V_{4b'} \end{pmatrix}_L \tag{4.8}$$

Flavour eigenstates in terms of mass eigenstates are:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}_L = V_L^{(d)} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_L \tag{4.9}$$

while the three down-type quarks involved in charged weak interactions expressed in terms of mass eigenstates are:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} V_{1d} & V_{1s} & V_{1b} & V_{1b'} \\ V_{2d} & V_{2s} & V_{2b} & V_{2b'} \\ V_{3d} & V_{3s} & V_{3b} & V_{3b'} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_L = \tilde{V}_L^{(d)} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_L \tag{4.10}$$

where $\tilde{V}_L^{(d)}$ is the 3×4 submatrix of $V_L^{(d)}$ obtained by cutting the last row. Although it still holds that $\tilde{V}_L^{(d)} \tilde{V}_L^{(d)\dagger} = 1_{3\times3}$, $\tilde{V}_L^{(d)}$ is not unitary anymore.

The Lagrangian for the charged current interaction is:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} \end{pmatrix} \gamma^\mu \tilde{V}_{\rm CKM} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_L W_\mu^+ + \text{h.c.} \tag{4.11}$$

where

$$\tilde{V}_{\rm CKM} = V_L^{(u)\dagger} \tilde{V}_L^{(d)} = \left(\begin{array}{c|c} & V_{ub'} \\ V_{\rm CKM} & V_{cb'} \\ & V_{tb'} \end{array} \right) \tag{4.12}$$

is a 3×4 matrix. Again $\tilde{V}_{\rm CKM} \tilde{V}_{\rm CKM}^\dagger = 1_{3\times3}$, but $\tilde{V}_{\rm CKM}$ is not unitary. $V_L^{(u)}$ is the unitary 3×3 matrix diagonalizing the up-type quark mass matrix from the left:

$$V_L^{(u)} = \begin{pmatrix} V_{1u} & V_{1c} & V_{1t} \\ V_{2u} & V_{2c} & V_{2t} \\ V_{3u} & V_{3c} & V_{3t} \end{pmatrix}_L \tag{4.13}$$

The elements of the fourth column of $\tilde{V}_{\rm CKM}$ determine the strength of the violation of CKM unitarity.

The weak neutral current lagrangian for down quarks reads:

$$\begin{aligned} \mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W} & \left[-\frac{1}{2} \begin{pmatrix} \overline{d_L} & \overline{s_L} & \overline{b_L} & \overline{b'_L} \end{pmatrix} \gamma^\mu V_{\rm nc}^{(d)} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix} \right. \\ & \left. + \frac{1}{3} \sin^2\theta_W \left(\overline{\mathbf{d}_L} \gamma^\mu \mathbf{d}_L + \overline{\mathbf{d}_R} \gamma^\mu \mathbf{d}_R \right) \right] Z_\mu \end{aligned} \tag{4.14}$$

$$V_{\rm nc}^{(d)} = V_L^{(d)\dagger} \text{diag}(1,1,1,0) V_L^{(d)} \tag{4.15}$$

where $\mathbf{d}$ is the column vector of d, s, b. As comes out from Eqs. (4.14) and (4.15), the non-unitarity of $\tilde{V}_L^{(d)}$ is at the origin of non-diagonal couplings with Z boson, which means flavor changing neutral currents (FCNC) at tree level. The strength of the coupling of flavor changing interactions is determined by the elements of the matrix:

$$V_{\rm nc}^{(d)} = V_L^{(d)\dagger}\mathrm{diag}(1,1,1,0)V_L^{(d)} =$$
$$= \begin{pmatrix} 1-|V_{4d}|^2 & -V_{4d}^*V_{4s} & -V_{4d}^*V_{4b} & -V_{4d}^*V_{4b'} \\ -V_{4s}^*V_{4d} & 1-|V_{4s}|^2 & -V_{4s}^*V_{4b} & -V_{4s}^*V_{4b'} \\ -V_{4b}^*V_{4d} & -V_{4b}^*V_{4s} & 1-|V_{4b}|^2 & -V_{4b}^*V_{4b'} \\ -V_{4b'}^*V_{4d} & -V_{4b'}^*V_{4s} & -V_{4b'}^*V_{4b} & |V_{1b'}|^2+|V_{2b'}|^2+|V_{3b'}|^2 \end{pmatrix}_L \tag{4.16}$$

If the second and third families are not mixed with the fourth, that is $V_{L4s} = V_{L4b} = 0$, then there is not any FCNC at tree level between the first three families.

$V_L^{(d)}$ can be parameterized by 6 angles and 10 phases:

$$V_L^{(d)} = \begin{pmatrix} V_{1d} & V_{1s} & V_{1b} & V_{1b'} \\ V_{2d} & V_{2s} & V_{2b} & V_{2b'} \\ V_{3d} & V_{3s} & V_{3b} & V_{3b'} \\ V_{4d} & V_{4s} & V_{4b} & V_{4b'} \end{pmatrix}_L \simeq$$
$$\simeq \mathbf{D}_{\phi L}^d \begin{pmatrix} & & & 0 \\ & V_{3\times3}^{(d)} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{L3}^d & -\tilde{s}_{L3}^d \\ 0 & 0 & \tilde{s}_{L3}^{d*} & c_{L3}^d \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{L2}^d & 0 & -\tilde{s}_{L2}^d \\ 0 & 0 & 1 & 0 \\ 0 & \tilde{s}_{L2}^{d*} & 0 & c_{L2}^d \end{pmatrix} \cdot \begin{pmatrix} c_{L1}^d & 0 & 0 & -\tilde{s}_{L1}^d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \tilde{s}_{L1}^{d*} & 0 & 0 & c_{L1}^d \end{pmatrix} =$$
$$= \mathbf{D}_{\phi L}^d \begin{pmatrix} & & & 0 \\ & V_{3\times3}^{(d)} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} c_{L1}^d & 0 & 0 & -\tilde{s}_{L1}^d \\ -\tilde{s}_{L2}^d\tilde{s}_{L1}^{d*} & c_{L2}^d & 0 & -\tilde{s}_{L2}^d c_{L1}^d \\ -\tilde{s}_{L3}^d\tilde{s}_{L1}^{d*}c_{L2}^d & -\tilde{s}_{L2}^{d*}\tilde{s}_{L3}^d & c_{L3}^d & -\tilde{s}_{L3}^d c_{L2}^d c_{L1}^d \\ \tilde{s}_{L1}^{d*}c_{L2}^d c_{L3}^d & \tilde{s}_{L2}^{d*}c_{L3}^d & \tilde{s}_{L3}^{d*} & c_{L1}^d c_{L2}^d c_{L3}^d \end{pmatrix} \tag{4.17}$$

where $\mathbf{D}_{\phi L}^d = \mathrm{diag}(e^{i\phi_{L1}^d}, e^{i\phi_{L2}^d}, e^{i\phi_{L3}^d}, e^{i\phi_{L4}^d})$. $V_{3\times3}^{(d)}$ contains 3 angles and 3 phases. $\tilde{s}_{Li}^d, c_{Li}^d$ are complex sines and cosines of angles in the 14, 24, 34 family planes parameterizing the mixing of the first three families with the fourth family:

$$\tilde{s}_{Li}^d = \sin\theta_{Li4}^d e^{-i\delta_{Li}^d} = s_{Li}^d e^{-i\delta_{Li}^d} \tag{4.18}$$

Equation (4.323) is true at order $O(|h_{d3}|(y_3^d)^2\frac{v_w^3}{M_d^3} + s_i^{d2}s_j^d)$, with

$$s_{Li}^d \simeq \frac{|h_{di}|v_w}{M_d} + O(|h_{di}|(y_i^d)^2\frac{v_w^3}{M_d^3}) \tag{4.19}$$

$$\tan\delta_{Li}^d \simeq -\frac{\mathrm{Im}(h_{di})}{\mathrm{Re}(h_{di})} \tag{4.20}$$

In natural models with mixing of down quarks of the same order of V_{CKM}, that is small angles and no big cancellation between mixing of up and down quarks, $V^{(d)}_{3\times 3}$ is practically equal to the 3×3 submatrix of $V^{(d)}_L$ in (4.315), with the main corrections regarding the elements:

$$\begin{aligned} V_{L3d} &\simeq V_{3\times 3\,3d} - \tilde{s}^d_{L1}\tilde{s}^d_{L3} \\ V_{L2d} &\simeq V_{3\times 3\,2d} - \tilde{s}^d_{L1}\tilde{s}^d_{L2} \\ V_{L3s} &\simeq V_{3\times 3\,3s} - \tilde{s}^d_{L2}\tilde{s}^d_{L3} \end{aligned} \tag{4.21}$$

and so $V^{(d)}_{3\times 3}$ basically diagonalizes the 3×3 submatrix $\mathbf{m}^{(d)}_{3\times 3}$. In fact, it will be shown in this work that s^d_{L2} and s^d_{L3} should be at least a factor λ less than $s^d_{L1} \approx |V_{ub'}| \approx 0.04 \sim \lambda^2$, then

$$[V^{(d)}_L]_{i\beta} \simeq [V^{(d)}_{3\times 3}]_{i\beta} \tag{4.22}$$

with corrections less than $O(\lambda^4)$ for all the elements, maybe only affecting $V^{(d)}_{3d}$ in (4.21). As regards the last column:

$$\begin{aligned} V_{L1b'} &\simeq -\tilde{s}^d_{L1} V_{L1d} \\ V_{L2b'} &\simeq -\tilde{s}^d_{L1} V_{3\times 3\,2d} - \tilde{s}^d_{L2} V_{L2s} - \tilde{s}^d_{L3} V_{L2b} \\ V_{L3b'} &\simeq -\tilde{s}^d_{L3} V_{L3b} \end{aligned} \tag{4.23}$$

with corrections at most of $O(\lambda^5)$.

As regards charged currents, $\tilde{V}_{\text{CKM}}$ in (4.12) can be described by 6 moduli and 9 phases, 6 of which can be absorbed into the quark fields. For the submatrix V_{CKM} in (4.12), it holds that:

$$[V_{\text{CKM}}]_{\alpha\beta} = \sum_{i=1}^{3} V^{(u)*}_{L\,i\alpha} V^{(d)}_{L\,i\beta} \simeq \sum_{i=1}^{3} V^{(u)*}_{L\,i\alpha} V^{(d)}_{3\times 3\,i\beta} \tag{4.24}$$

Then, after rephasing quark fields, V_{CKM} can be in the usual parameterization with 3 angles and one phase. Also another phase can be absorbed, we can choose to absorb the phase of $\tilde{s}^d_{L2}$. The elements of the fourth column of $\tilde{V}_{\text{CKM}}$ in (4.12) can be parameterized as:

$$\begin{aligned} V_{ub'} &\simeq -\tilde{s}^d_{L1} V^*_{L1u} V_{L1d} \simeq -\tilde{s}^d_{L1} \\ V_{cb'} &\simeq -s^d_{L2} V^*_{L2c} V_{L2s} - \tilde{s}^d_{L1}(V^*_{L1c} V_{L1d} + V^*_{L2c} V_{L2d}) \simeq -s^d_{L2} - \tilde{s}^d_{L1} V_{cd} \\ V_{tb'} &\simeq -\tilde{s}^d_{L3} V^*_{L3t} V_{L3b} \simeq -\tilde{s}^d_{L3} \end{aligned} \tag{4.25}$$

with corrections at most of order $O(\lambda^4)$.

Finally the couplings of neutral currents can be related to the elements of the new fourth column of $\tilde{V}_{\rm CKM}$. Remembering from (4.323) that:

$$V_{L4d} \approx \tilde{s}^{d*}_{L1}\,, \qquad V_{L4s} \approx \tilde{s}^{d*}_{L2}\,, \qquad V_{L4b} = \tilde{s}^{d*}_{L3} \tag{4.26}$$

and that the elements of $V_L^{(d)}$ and $V_L^{(u)}$ can be naturally chosen with small mixings, that is $V_{\rm CKM}$ is not obtained after large cancellations in their product, then, comparing (4.26) with (4.25):

$$|V^*_{L4d}V_{L4s}| \approx s^d_{L1}s^d_{L2} \approx |V_{ub'}||V_{cb'} + \tilde{s}^d_1 V_{cd}| \tag{4.27}$$

$$|V^*_{L4d}V_{L4b}| \approx s^d_{L1}s^d_{L3} \approx |V_{ub'}V^*_{tb'}| \tag{4.28}$$

$$|V^*_{L4s}V_{L4b}| \approx s^d_{L2}s^d_{L3} \approx |V^*_{tb'}(V_{cb'} + \tilde{s}^d_1 V_{cd})| \tag{4.29}$$

4.1.1 K decays

$K^+ \to \pi^+\nu\bar{\nu}$

In the Standard Model the decay $K^+ \to \pi^+\nu\bar{\nu}$ is one of the golden modes, since it is dominated by one-loop contributions, while long-distance contributions are negligible. The effective Lagrangian in the SM relevant for $K^+ \to \pi^+\nu\bar{\nu}$ is given by [138]:

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}}\frac{\alpha(M_Z)}{2\pi\sin^2\theta_{\rm W}}\sum_{\ell=e,\mu,\tau}[V^*_{cs}V_{cd}X^\ell(x_c) + V^*_{ts}V_{td}X(x_t)](\bar{s}\gamma^\mu d)(\overline{\nu_\ell}\gamma_\mu(1-\gamma_5)\nu_\ell) \tag{4.30}$$

where $X(x_t)$ is the Inami-Lim function with QCD and electroweak corrections for the top quark short-distance contribution and $X^\ell(x_c)$ accounts for the charm contribution ($x_i = m_i^2/M_W^2$), the index ℓ denotes the lepton flavor. In order to compute the short distance SM contribution to the amplitude of this process, the effective Lagrangian is also written as [140]:

$$\begin{aligned}\mathcal{L}_{\rm eff} &= -\frac{G_F}{\sqrt{2}}\frac{\alpha(M_Z)}{2\pi\sin^2\theta_{\rm W}}\left(V^*_{ts}V_{td}X(x_t) + {\rm Re}(V^*_{cs}V_{cd})\lambda^4 P_c(X)\right)\sum_{e,\mu,\tau}(\bar{s}\gamma^\mu d)(\overline{\nu_\ell}\gamma_\mu(1-\gamma_5)\nu_\ell) = \\ &= -\frac{G_F}{\sqrt{2}}\mathcal{F}^+\sum_{e,\mu,\tau}(\bar{s}\gamma^\mu d)(\overline{\nu_\ell}\gamma_\mu(1-\gamma_5)\nu_\ell)\end{aligned} \tag{4.31}$$

where [141]:

$$P_c(X) = \frac{1}{\lambda^4}\left(\frac{2}{3}X^e(x_c) + \frac{1}{3}X^\tau(x_c)\right) \tag{4.32}$$

and we defined $\mathcal{F}^+$ for later convenience. From [141] using the central values of $m_c(m_c)$ and $\alpha_s(M_Z)$ in Table 4.1:

$$P_c(X) = 0.3645 \pm 0.0091 \tag{4.33}$$

$$\lambda^4 P_c(X) = (0.92 \pm 0.02)\cdot 10^{-3} \tag{4.34}$$

The real and imaginary part of the elements of V_{CKM} can be expressed by using the Wolfenstein parameterization:

$$\begin{aligned} \mathrm{Re}(V^*_{cs}V_{cd}) &= -\lambda(1-\frac{\lambda^2}{2}) \\ \mathrm{Re}(V^*_{ts}V_{td}) &= -A^2\lambda^5(1-\rho) \\ \mathrm{Im}(V^*_{ts}V_{td}) &= A^2\lambda^5\eta \end{aligned} \tag{4.35}$$

where $\bar\rho = \rho(1-\frac{\lambda^2}{2})$, $\bar\eta = \eta(1-\frac{\lambda^2}{2})$. $X(x_t)$ was computed in [142], we use the central value $X(x_t) = 1.469$.

The new contribution arising from the non-unitarity of the mixing matrix after adding a vector-like b' is described by the effective Lagrangian:

$$\begin{aligned} \mathcal{L}_{\mathrm{new}} = &-\frac{g^2}{M_Z^2\cos^2\theta_W}\left(\frac{1}{2}V^*_{L4s}V_{L4d}\right)(\overline{s_L}\gamma^\mu d_L)\frac{1}{2}\sum_{e,\mu,\tau}(\overline{\nu_{\ell\mathrm{L}}}\gamma_\mu\nu_{\ell\mathrm{L}}) = \\ &-\frac{G_F}{2\sqrt{2}}V^*_{L4s}V_{L4d}(\overline{s}\gamma^\mu d)\sum_{e,\mu,\tau}(\overline{\nu_\ell}\gamma_\mu(1-\gamma_5)\nu_\ell) \end{aligned} \tag{4.36}$$

A limit on the new mixing elements can be set by requiring for the amplitude squared of the new contribution to be less than the SM one:

$$|V^*_{L4s}V_{L4d}|^2 < |2\mathcal{F}^+|^2 \tag{4.37}$$

$$|V^*_{L4s}V_{L4d}| \lesssim 8\cdot 10^{-6} \tag{4.38}$$

where:

$$|\mathcal{F}^+|^2 = \left(\frac{\alpha(M_Z)}{2\pi\sin^2\theta_W}\right)^2\left[\left(\mathrm{Re}(V^*_{ts}V_{td})X(x_t) + \mathrm{Re}(V^*_{cs}V_{cd})\lambda^4P_c(X)\right)^2 + (\mathrm{Im}(V^*_{ts}V_{td})X(x_t))^2\right] \tag{4.39}$$

Since $|V_{L4d}| \approx |V_{ub'}|$, setting $|V_{ub'}| = 0.04$, the constraint implies that $|V_{L4s}| \approx s^d_{L2} \lesssim 0.0002$. Then the mixing angle of the fourth species of quarks (which is the heaviest) with the second family should be two orders of magnitude less than the mixing with the first family (the lightest). Regarding charged currents, if for example the mixing in charged currents is almost coming by down-type quarks, that is $V_{\rm CKM} \simeq V^{(d)}_{3\times 3}$, then from (4.25) $|V_{cb'}| \simeq \tilde{s}^d_{L1} V^*_{L2c} V_{L2d}$, about 4 times less than $|V_{ub'}|$.

A different estimation may in principle come from the comparison with the experimental value of the branching ratio of this decay. The experimental branching ratio of this decay is $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm exp} = (1.7 \pm 1.1) \cdot 10^{-10}$ [143], compatible with the predicted SM contribution is $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm SM} = (8.5 \pm 0.5) \cdot 10^{-11}$ [111]. Following [140], the expression of the SM contribution to the branching ratio is:

$$
\begin{aligned}
&\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm SM} = \left(\frac{4 G_F\, \alpha(M_Z)}{\sqrt{2}\, 2\pi \sin^2\theta_{\rm W}} \right)^2 \cdot \\
&\cdot \left(\left(\mathrm{Re}(V^*_{ts} V_{td}) X(x_t) + \mathrm{Re}(V^*_{cs} V_{cd}) \lambda^4 (P_c(X) + \delta P_{c,u}) \right)^2 + \left(\mathrm{Im}(V^*_{ts} V_{td}) X(x_t) \right)^2 \right) \cdot \\
&\cdot \frac{\tau(K^+) M^5_{K^+} f^{K^+\pi^+}_+ (0)^2 \mathcal{I}^+_\nu (1 + \Delta_{\rm EM})}{512\pi^3} \qquad (4.40)
\end{aligned}
$$

where $\mathcal{I}^+_\nu$ is the phase space integral [140] and $\Delta_{\rm EM} = -0.003$ [140] are the long-distance QED corrections, $\delta P_{c,u} = 0.04 \pm 0.02$ is the contribution of dimension-eight operators and long-distance contributions computed in [144].

The new contribution to the decay can be constrained to be less than the experimental error:

$$
\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm new} \simeq \mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm SM} \frac{|V^*_{L4s} V_{L4d}|^2}{4|\mathcal{F}^+|^2} < 1.1 \cdot 10^{-10} \qquad (4.41)
$$

$$
|V^*_{L4s} V_{L4d}| \lesssim 10^{-5} \qquad (4.42)
$$

If the assumption $|V_{ub'}| = 0.04$ is made, it is obtained that $|V_{L4s}| \approx s^d_{L2} \lesssim 0.0002$. Alternatively, the limits obtained from the NA62 experiment combining 2016 and 2017 data can be used, $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) < 1.85 \cdot 10^{-10}$ at 90% C.L., or

$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) < 2.44 \cdot 10^{-10}$ at 95% C.L. [145]:

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{new}} \simeq \mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} \frac{|V^*_{L4s} V_{L4d}|^2}{16|\mathcal{F}^+|^2} < 1.85 \cdot 10^{-10} \tag{4.43}$$

$$|V^*_{L4s} V_{L4d}| \lesssim 1.2 \cdot 10^{-5} \tag{4.44}$$

$$|V_{L4s}| \approx s^d_{L2} \lesssim 0.0003 \tag{4.45}$$

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{new}} \simeq \mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} \frac{|V^*_{L4s} V_{L4d}|^2}{16|\mathcal{F}^+|^2} < 2.44 \cdot 10^{-10} \tag{4.46}$$

$$|V^*_{L4s} V_{L4d}| \lesssim 1.4 \cdot 10^{-5} \tag{4.47}$$

$$|V_{L4s}| \approx s^d_{L2} \lesssim 3.5 \cdot 10^{-4} \tag{4.48}$$

The limits on $|V_{L4s}|$ are set under the assumption $|V_{L4d}| \approx |V_{ub'}| = 0.04$.

$K_{\mathbf{L}} \to \pi^0 \nu\bar{\nu}$

The second golden mode is the decay $K_{\mathrm{L}} \to \pi^0 \nu\bar{\nu}$. In the Standard Model the effective Lagrangian related to $K_{\mathrm{L}} \to \pi^0 \nu\bar{\nu}$ is [138]:

$$\mathcal{L}_{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}} \frac{\alpha(M_Z)}{2\pi \sin^2\theta_{\mathrm{W}}} V^*_{ts} V_{td} X(x_t) (\bar{s}\gamma^\mu d) \sum_{\ell=e,\mu,\tau} (\overline{\nu_\ell} \gamma_\mu (1-\gamma_5)\nu_\ell) + h.c. = \tag{4.49}$$

$$= -\frac{G_F}{\sqrt{2}} \left(\mathcal{F}^0 (\bar{s}\gamma^\mu d) \sum_{\ell=e,\mu,\tau} (\overline{\nu_\ell} \gamma_\mu (1-\gamma_5)\nu_\ell) + \mathcal{F}^{0*} (\bar{d}\gamma^\mu s) \sum_{\ell=e,\mu,\tau} (\overline{\nu_\ell} \gamma_\mu (1-\gamma_5)\nu_\ell) \right) \tag{4.50}$$

where

$$\mathcal{F}^0 = \frac{\alpha(M_Z)}{2\pi \sin^2\theta_{\mathrm{W}}} V^*_{ts} V_{td} X(x_t) \tag{4.51}$$

Following [139], taking into account that:

$$K_{\mathrm{L}} = \frac{1}{\sqrt{2}} [(1+\bar{\epsilon}) K^0 - (1-\bar{\epsilon}) \bar{K}^0] \tag{4.52}$$

with

$$P|K^0\rangle = -|K^0\rangle \qquad C|K^0\rangle = -|\bar{K}^0\rangle \qquad CP|K^0\rangle = |\bar{K}^0\rangle \tag{4.53}$$

then:

$$\langle \pi^0 | (\bar{d}s)_{\mathrm{V-A}} | \bar{K}^0 \rangle = \langle \pi^0 | (\bar{s}d)_{\mathrm{V-A}} | K^0 \rangle \tag{4.54}$$

so:

$$\langle \pi^0 | \mathcal{L}_{\rm eff} | K^0 \rangle = -\frac{4G_F}{\sqrt{2}} \frac{2{\rm Im}\mathcal{F}^0}{\sqrt{2}} \langle \pi^0 | (\overline{s_L} \gamma^\mu d_L) | K^0 \rangle \sum_{\ell=e,\mu,\tau} (\overline{\nu_{\ell \rm L}} \gamma_\mu \nu_{\ell \rm L}) \tag{4.55}$$

The new contribution arising from the non-unitarity of the mixing matrix after adding a vector-like b' is described by the effective Lagrangian:

$$\begin{aligned} \mathcal{L}_{\rm new} = & -\frac{2G_F}{\sqrt{2}} V_{L4s} V_{L4d}^* (\overline{d_L} \gamma^\mu s_L) \sum_{\ell=e,\mu,\tau} (\overline{\nu_{\ell \rm L}} \gamma_\mu \nu_{\ell \rm L}) + \text{h.c.} = \\ & -\frac{G_F}{2\sqrt{2}} V_{L4s} V_{L4d}^* (\bar{d} \gamma^\mu s) \sum_{\ell=e,\mu,\tau} (\overline{\nu_\ell} \gamma_\mu (1-\gamma_5) \nu_\ell) + \text{h.c.} \end{aligned} \tag{4.56}$$

Then

$$|{\rm Im}(V_{L4s}^* V_{L4d})| < |2{\rm Im}\mathcal{F}^0| \tag{4.57}$$
$$|{\rm Im}(V_{L4s}^* V_{L4d})| \lesssim 2.3 \cdot 10^{-6} \tag{4.58}$$

However the experimental limit on this decay is ${\rm Br}(K_{\rm L} \to \pi^0 \nu\bar{\nu}) < 3 \cdot 10^{-9}$ at 90% CL while the predicted SM contribution is ${\rm Br}(K_{\rm L} \to \pi^0 \nu\bar{\nu}) = (3.0 \pm 0.2) \cdot 10^{-11}$ [111]. Then since:

$${\rm Br}(K_{\rm L} \to \pi^0 \nu\bar{\nu})_{\rm new} = {\rm Br}(K_{\rm L} \to \pi^0 \nu\bar{\nu})_{\rm SM} \frac{({\rm Im}(V_{L4s}^* V_{L4d}))^2}{(2{\rm Im}\mathcal{F}^0)^2} \tag{4.59}$$

by imposing the experimental limit on the new contribution to the branching ratio of the decay it is obtained:

$$|{\rm Im}(V_{L4s}^* V_{L4d})| < 2.3 \cdot 10^{-5} \tag{4.60}$$

In the given parameterization then:

$$|V_{L4s}^* V_{L4d}| |\sin(\delta_{L1}^d - \delta_{L2}^d)| < 2.3 \cdot 10^{-5} \tag{4.61}$$
$$|V_{L4s}| \approx s_{L2}^d < \frac{0.0006}{|\sin(\delta_{L1}^d - \delta_{L2}^d)|} \tag{4.62}$$

where the second relation follows with $|V_{L4d}| \approx s_{L1}^d \approx |V_{ub'}| = 0.04$.

$K_{\mathbf{L}} \to \pi^0 e^+ e^-$

The decay $K_\mathrm{L} \to \pi^0 e^+ e^-$ contains a direct CP violating contribution, indirect CP violating contribution, interference between them, and also a very small CP conserving contribution) [111]. The direct CP-violating amplitude is short distance dominated. The effective Lagrangian for $K_\mathrm{L} \to \pi^0 e^+ e^-$ can be written as [138]:

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ts}^* V_{td} \sum_{i=1}^{6,7V} \{(\tilde{z}_i(\mu) + y_i(\mu)) Q_i(\mu) + \bar{s}\gamma_\mu (1-\gamma_5) d \left[y_V (\bar{e}\gamma^\mu e) + y_A (\bar{e}\gamma^\mu \gamma^5 e) \right] \} + h.c. \tag{4.63}$$

where $Q_{1,2}$ are $\Delta S = 1$ current-current operators and $Q_{3,\ldots,6}$ the QCD penguin operators. The SM directly CP violating contribution to this decay basically comes from [146]:

$$\mathcal{L}_{\text{DCPV}} = \frac{G_F \alpha(M_Z)}{\sqrt{2}} V_{ts}^* V_{td} (\bar{s}\gamma_\mu d) \left(\bar{y}_V (\bar{e}\gamma^\mu e) + \bar{y}_A (\bar{e}\gamma^\mu \gamma^5 e) \right) + h.c. \tag{4.64}$$

where $y_{V,A} = \alpha(M_Z) \bar{y}_{V,A}$. The direct CP-violating branching ratio in the SM is given by [140]:

$$\mathrm{Br}(K_\mathrm{L} \to \pi^0 e^+ e^-)_{\text{DCPV}} = \left(\frac{G_F \alpha(M_Z)}{\sqrt{2}} \right)^2 \left(\frac{2 \mathrm{Im} V_{ts}^* V_{td}}{\sqrt{2}} \right)^2 \frac{1}{2} \frac{M_{K^0}^5 \tau(K_L) f_+(0)^2 \mathcal{I}_e}{192\pi^3} (\bar{y}_A^2 + \bar{y}_V^2) \tag{4.65}$$

where $\mathcal{I}_e$ is the phase space integral.

The new effective Lagrangian contributing to this decay is:

$$\mathcal{L}_{\text{new}} = -\frac{2G_F}{\sqrt{2}} V_{L4s} V_{L4d}^* \overline{d} \gamma^\mu s \left[(-\frac{1}{4} + \sin^2\theta_\mathrm{W}) \bar{e}\gamma_\mu e + \frac{1}{4} \bar{e}\gamma_\mu \gamma_5 e \right] + \text{h.c.} \tag{4.66}$$

Then

$$\begin{aligned} &\mathrm{Br}(K_\mathrm{L} \to \pi^0 e^+ e^-)_{\text{new}} = \\ &= 4G_F^2 \left[\mathrm{Im}(V_{L4s} V_{L4d}^*) \right]^2 \left[(-\frac{1}{4} + \sin^2\theta_\mathrm{W})^2 + \frac{1}{16} \right] \frac{M_{K^0}^5 \tau(K_L) f_+(0)^2 \mathcal{I}_e}{384\pi^3} \end{aligned} \tag{4.67}$$

The experimental limit on this decay is $\mathrm{Br}(K_\mathrm{L} \to \pi^0 e^+ e^-) < 2.8 \cdot 10^{-10}$ at 90% CL while the SM prediction for the direct CP violating decay is $\mathrm{Br}(K_\mathrm{L} \to \pi^0 e^+ e^-) =$

$0.45 \cdot 10^{-11}$ [111]. By imposing the experimental limit on the new contribution to the branching ratio of the decay we obtain:

$$|\mathrm{Im}(V_{L4s}V^*_{L4d})| < 1.7 \cdot 10^{-5} \tag{4.68}$$

where the central values of $\mathcal{I}_e = 0.16043 \pm 0.00031$ [140], $f_+(0) = 0.9699(15)$ have been used. In the given parameterization then:

$$|V_{L4s}V^*_{L4d}||\sin(\delta^d_{L1} - \delta^d_{L2})| < 1.7 \cdot 10^{-5} \tag{4.69}$$

$$|V_{L4s}| \lesssim \frac{0.0004}{|\sin(\delta^d_{L1} - \delta^d_{L2})|} \tag{4.70}$$

where the second relation follows with $|V_{L4d}| \approx |V_{ub'}| = 0.04$. A more stringent constraint can be imposed by constraining the new contribution to be at most the same as the SM contribution to the direct CP violating decay:

$$\mathrm{Br}(K_\mathrm{L} \to \pi^0 e^+ e^-)_\mathrm{new} < 4.5 \cdot 10^{-12} \tag{4.71}$$

$$|V_{L4s}V^*_{L4d}||\sin(\delta^d_{L1} - \delta^d_{L2})| < 2 \cdot 10^{-6} \tag{4.72}$$

$$|V_{L4s}| \lesssim \frac{5 \cdot 10^{-5}}{|\sin(\delta^d_{L1} - \delta^d_{L2})|} \tag{4.73}$$

Again the second relation follows with $|V_{L4d}| = 0.04$.

$K_\mathbf{L} \to \mu^+\mu^-$

The existence of the additional vector-like singlet leads to the following additional contribution to the effective Lagrangian for the decay $K_\mathrm{L} \to \mu^+\mu^-$:

$$\mathcal{L}_\mathrm{new} = -\frac{4G_F}{\sqrt{2}} V_{L4s}V^*_{L4d}(\overline{s_L}\gamma_\mu d_L)[g_L(\overline{\mu_L}\gamma^\mu \mu_L) + g_R(\overline{\mu_R}\gamma^\mu \mu_R)] + \mathrm{h.c.} = \tag{4.74}$$

$$= \frac{G_F}{\sqrt{2}} V_{L4s}V^*_{L4d}(\bar{s}\gamma_\mu\gamma_5 d)[g_V(\overline{\mu}\gamma^\mu\mu) - g_A(\overline{\mu}\gamma^\mu\gamma^5\mu)] + \mathrm{h.c.} \tag{4.75}$$

where

$$g_L = -\frac{1}{2} + \sin^2\theta_W \qquad g_R = \sin^2\theta_W \tag{4.76}$$

$$g_A = g_L - g_R = -\frac{1}{2} \qquad g_V = g_L + g_R = 2\sin^2\theta_w - \frac{1}{2} \tag{4.77}$$

By using the phase convention:

$$C|K^0\rangle = -|\bar{K}^0\rangle \qquad P|K^0\rangle = -|K^0\rangle \qquad CP|K^0\rangle = |\bar{K}^0\rangle \tag{4.78}$$

and

$$K_L \approx K_2 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \qquad\qquad CP|K_2\rangle = -|K_2\rangle \tag{4.79}$$

then:

$$\langle 0|(\bar{d}s)_{\text{V-A}}|\bar{K}^0\rangle = -\langle 0|(\bar{s}d)_{\text{V-A}}|K^0\rangle \tag{4.80}$$

then the amplitude of this process can be written:

$$-i\mathcal{M} = \frac{G_F}{\sqrt{2}}\frac{2\text{Re}(V_{L4s}V^*_{L4d})}{\sqrt{2}} f_K m_\mu 2(g_R - g_L)[\bar{u}(p^{(\mu)})\gamma^5 v(p^{(\bar{\mu})})] \tag{4.81}$$

Then

$$|\mathcal{M}|^2 = G_F^2 f_K^2 m_\mu^2 [\text{Re}(V_{L4s}V^*_{L4d})]^2 (2g_A)^2 2M^2_{K^0} \tag{4.82}$$

which gives for the rate:

$$\Gamma_{\text{new}} = \frac{1}{8\pi} G_F^2 f_K^2 m_\mu^2 M_{K^0}\left(1 - \frac{4m_\mu^2}{M^2_{K^0}}\right)^{\frac{1}{2}} (\text{Re}(V_{L4s}V^*_{L4d}))^2 \tag{4.83}$$

The experimental branching ratio $\text{Br}(K_\text{L} \to \mu^+\mu^-)_{\text{exp}} = (6.84 \pm 0.11) \cdot 10^{-9}$ but it is dominated by the imaginary part of the long-distance contribution $\text{Br}_{\text{abs}}(K_L \to \mu^+\mu^-) = (6.64 \pm 0.07) \cdot 10^{-9}$ [111]. The SM prediction for the short distance contribution is $\text{Br}(K_\text{L} \to \mu^+\mu^-)_{\text{SD}} \approx 10^{-9}$ (from [111], [146], $\text{Br}(K_\text{L} \to \mu^+\mu^-)_{\text{SD}} \approx 2.7 \cdot 10^{-4}|V_{cb}|^4(1.2 - \bar{\rho})^2$). In order to evaluate the branching ratio of the decay $K_\text{L} \to \mu^+\mu^-$ it is useful to consider the decay $K^+ \to \mu^+\nu_\mu$:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V^*_{us}(\bar{s}\gamma_\alpha\gamma_5 u)(\bar{\nu}_\mu\gamma^\alpha(1-\gamma_5)\mu) \tag{4.84}$$

$$\Gamma = \frac{1}{8\pi} G_F^2 |V_{us}|^2 f_K^2 m_\mu^2 M_{K^+}\left(1 - \frac{m_\mu^2}{M^2_{K^+}}\right)^2 \tag{4.85}$$

which gives:

$$\frac{\text{Br}(K_\text{L} \to \mu^+\mu^-)_{\text{new}}}{\text{Br}(K^+ \to \mu^+\nu_\mu)} = \frac{\tau(K_L)M_{K^0}\sqrt{1 - 4\frac{m_\mu^2}{M^2_{K^0}}}}{\tau(K^+)M_{K^+}\left(1 - \frac{m_\mu^2}{M^2_{K^+}}\right)^2}\frac{[\text{Re}(V_{L4s}V^*_{L4d})]^2}{|V_{us}|^2} \tag{4.86}$$

(with $\langle 0|(\bar{s}d)_{\mathrm{V-A}}|K^0\rangle = \langle 0|(\bar{s}u)_{\mathrm{V-A}}|K^+\rangle$). The new contribution can be constrained to be at most the same as the SM short distance contribution

$$\mathrm{Br}(K_{\mathrm{L}} \to \mu^+\mu^-)_{\mathrm{new}} < 10^{-9} \tag{4.87}$$

Using $\mathrm{Br}(K^+ \to \mu^+\nu_\mu) = 0.6356$ [111], $|V_{us}| = 0.22567(42)$ [109], it is obtained that:

$$|\mathrm{Re}(V_{L4s}V^*_{L4d})| = |V_{L4s}V^*_{L4d}||\cos\big(\delta^d_{L2} - \delta^d_{L1}\big)| < 4.4 \cdot 10^{-6} \tag{4.88}$$
$$|V_{L4s}| \lesssim 10^{-4}/|\cos\big(\delta^d_{L2} - \delta^d_{L1}\big)| \tag{4.89}$$

where the last inequality is obtained using $|V_{L4d}| \approx |V_{ub'}| = 0.04$.

A more stringent condition can be found by constraining the new contribution to be less than the experimental error:

$$\mathrm{Br}(K_{\mathrm{L}} \to \mu^+\mu^-)_{\mathrm{new}} < 1.1 \cdot 10^{-10} \tag{4.90}$$
$$|V_{L4s}V^*_{L4d}||\cos\big(\delta^d_{L2} - \delta^d_{L1}\big)| < 1.5 \cdot 10^{-6} \tag{4.91}$$
$$|V_{L4s}| \lesssim 4 \cdot 10^{-5}/|\cos\big(\delta^d_{L2} - \delta^d_{L1}\big)| \tag{4.92}$$

In the last expression $|V_{L4d}| = 0.04$ has been used.

$K_{\mathbf{S}} \to \mu^+\mu^-$

The LHCb collaboration found an upper limit on the decay $K_{\mathrm{S}} \to \mu^+\mu^-$ with data collected with the LHCb experiment during 2016, 2017 and 2018: $\mathrm{Br}(K_{\mathrm{S}} \to \mu^+\mu^-) < 2.4 \cdot 10^{-10}$ at 95% CL [147]. A constraint on the imaginary part of the mixing elements follow:

$$\mathrm{Br}(K_{\mathrm{S}} \to \mu^+\mu^-)_{\mathrm{new}} < 2.4 \cdot 10^{-10} \tag{4.93}$$
$$|\mathrm{Im}(V^*_{L4d}V_{L4s})| = |V_{L4s}V^*_{L4d}||\sin\big(\delta^d_{L2} - \delta^d_{L1}\big)| < 2.2 \cdot 10^{-6} \tag{4.94}$$
$$|V_{L4s}| < 5.4 \cdot 10^{-5}/|\sin\big(\delta^d_{L2} - \delta^d_{L1}\big)| \tag{4.95}$$

In the last expression $|V_{L4d}| = 0.04$ has been used.

4.1.2 Neutral mesons systems

K^0-$\bar{K}^0$ mixing

In the SM the short-distance contribution to the transition $\bar{K}^0(\bar{d}s) \leftrightarrow K^0(d\bar{s})$ arises from weak box diagrams. The effective lagrangian describing this weak contribution is given by:

$$\mathcal{L} = -\frac{G_F^2 m_W^2}{4\pi^2} \left[\lambda_c^2 S_0(x_c) + \lambda_t^2 S_0(x_t) + 2\lambda_c\lambda_t S_0(x_c, x_t)\right] (\overline{s_L}\gamma^\mu d_L)(\overline{s_L}\gamma^\mu d_L) + \text{h.c.} \tag{4.96}$$

where $\lambda_a = V_{as}^* V_{ad}$ and $x_a = \frac{m_a^2}{m_W^2}$, $S_0(x_a)$ are the Inami-Lim functions [149]:

$$\begin{aligned} S_0(x_c) &\simeq x_c \\ S_0(x_t) &= \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1-x_t)^3} \\ S_0(x_c, x_t) &\simeq x_c \left[\ln\frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln x_t}{4(1-x_t)^2}\right] \end{aligned} \tag{4.97}$$

keeping only linear terms in x_c [138]. When QCD corrections are included, the effective interaction becomes [138]:

$$\begin{aligned} \mathcal{L} = -\frac{G_F^2 m_W^2}{4\pi^2} &\left(\eta_1\lambda_c^2 S_0(x_c) + \eta_2\lambda_t^2 S_0(x_t) + 2\eta_3\lambda_c\lambda_t S_0(x_c, x_t)\right) (\overline{s_L}\gamma^\mu d_L)(\overline{s_L}\gamma^\mu d_L)\cdot \\ \cdot\, [\alpha_s^{(3)}(\mu)]^{-2/9} &\left[1 + \frac{\alpha_s^{(3)}(\mu)}{4\pi} J_{(3)}\right] + h.c. \end{aligned} \tag{4.98}$$

where:

$$\eta_1 = 1.87 \pm 0.76 \tag{4.99}$$

$$\eta_2 = 0.574 \tag{4.100}$$

$$\eta_3 = 0.496 \pm 0.047 \tag{4.101}$$

from [150], [138], [151] respectively. μ is a renormalization scale, $J_{(3)}$ a renormalization scheme dependent quantity. The weak short-distance contribution to the mass difference $\Delta m_K = M_{K_L} - M_{K_S}$ and the CP-violating parameter ϵ_K respectively originate from the off-diagonal term M_{12} of the mass matrix of neutral kaons, which therefore should be computed. M_{12} corresponds to the transition $\bar{K}^0 \to K^0$ [138]:

$$2m_K M_{12}^* = -\langle \bar{K}^0|\mathcal{L}|K^0\rangle \tag{4.102}$$

with

$$\langle \bar{K}^0|(\overline{s_L}\gamma^\mu d_L)^2|K^0\rangle = \frac{2}{3} B_K(\mu) f_K^2 m_K^2 \tag{4.103}$$

$$B_K = B_K(\mu)[\alpha_s^{(3)}(\mu)]^{-2/9}\left[1 + \frac{\alpha_s^{(3)}(\mu)}{4\pi} J_{(3)}\right] \tag{4.104}$$

($\frac{2}{3}f_K^2 M_K^2$ is computed after the (VIA) vacuum insertion approximation [152]), that is

$$M_{12} = \frac{G_F^2 m_W^2}{12\pi^2}(\eta_1 \lambda_c^{*2} S_0(x_c) + \eta_2 \lambda_t^{*2} S_0(x_t) + 2\eta_3 \lambda_c^* \lambda_t^* S_0(x_c, x_t)) f_K^2 m_K B_K \tag{4.105}$$

with $B_K = 0.7625(97)$ (three flavours FLAG 2019 average [107]). The mass difference between mass eigenstates is [138]:

$$\Delta m_K \approx 2|M_{12}| + (\Delta m)_{LD} \tag{4.106}$$

where $(\Delta m)_{LD}$ corresponds to the long-distance contributions. The CP violation is parameterized by ϵ_K, which can be written as:

$$\epsilon_K \approx \frac{\mathrm{Im} M_{12}}{\sqrt{2}\Delta m_K} e^{-i\frac{\pi}{4}} \tag{4.107}$$

The imaginary part of M_{12} is dominated by short distance physics.

The short distance contribution to Δm_K is dominated by the charm contribution, while for the imaginary part the top quark plays a central role. Then, using the central values [111] $m_c = 1.27 \pm 0.02$, $|V_{cd}| = 0.218$ [111], $f_K = 155.7(0.7)$ MeV (three flavours FLAG 2019 average) [107] the short distance contribution to Δm_K is

$$|M_{12}| \approx \frac{G_F^2}{12\pi^2} \eta_1 |\lambda_c^{*2}| m_c^2 f_K^2 m_K B_K \tag{4.108}$$

$$\Delta m_K \approx 2|M_{12}| \simeq 3 \cdot 10^{-15}\,\mathrm{GeV} \tag{4.109}$$

which almost saturates the measured value [111]:

$$\Delta M_K = (3.484 \pm 0.006) \cdot 10^{-15}\,\mathrm{GeV} \tag{4.110}$$

The imaginary part of M_{12} is dominated by:

$$\mathrm{Im} M_{12} = \frac{G_F^2 m_W^2}{6\pi^2} f_K^2 m_K B_K \left[\eta_2 \mathrm{Re}(V_{ts} V_{td}^*) S_0(x_t) + \eta_3 \mathrm{Re}(V_{cs} V_{cd}^*) S_0(x_c, x_t)\right] \mathrm{Im}(V_{ts} V_{td}^*) \tag{4.111}$$

from which $|\epsilon_K| \approx 2.83 \cdot 10^{-3}$. The SM prediction for the parameter ϵ_K has been computed in Ref. [153]:

$$|\epsilon_K^{\mathrm{SM}}| = (2.16 \pm 0.18) \cdot 10^{-3} \tag{4.112}$$

which is compatible (0.4σ) with the value of ϵ_K obtained by fitting the data of the decay $K \to \pi\pi$ [111]:

$$|\epsilon_K| = (2.228 \pm 0.011) \cdot 10^{-3} \tag{4.113}$$

The new contribution translates into the effective lagrangian:

$$\mathcal{L}_{\rm new} = -\frac{G_F}{\sqrt{2}}(V^*_{L4d}V_{L4s})^2(\overline{d_L}\gamma^\mu s_L)^2 + \text{h.c.} \tag{4.114}$$

In order to set a constraint for the mixing with b', it can be imposed that the new contribution to the mass difference should be less than the short-distance SM contribution. An estimate can be carried out comparing the coefficients of the effective operators in Eqs. (4.96) and (4.114):

$$\frac{G_F}{\sqrt{2}}\left|(V^*_{L4d}V_{L4s})^2\right| < \frac{G_F^2 m_c^2}{4\pi^2}|\lambda_c^2| \tag{4.115}$$

$$|V^*_{L4d}V_{L4s}| < 2 \cdot 10^{-4} \tag{4.116}$$

$$|V_{L4s}| < 0.0045 \tag{4.117}$$

where in the last equation $|V_{L4d}| = 0.04$ has been used. Regarding the contribution to the CP-violating parameter ϵ_K, the imaginary part of operators (4.96) and (4.114), can be compared:

$$\frac{G_F}{\sqrt{2}}2|\text{Im}(V^*_{L4d}V_{L4s}) \cdot \text{Re}(V^*_{L4d}V_{L4s})| < \frac{G_F^2 m_W^2}{4\pi^2}2\left|(\text{Re}(V^*_{ts}V_{td})\, S_0(x_t) + \text{Re}(V^*_{cs}V_{cd})\, S_0(x_c, x_t))\, \text{Im}(V^*_{ts}V_{td})\right| \tag{4.118}$$

which means:

$$|V^*_{L4d}V_{L4s}|^2|\sin\big(\delta^d_{L1} - \delta^d_{L2}\big)\cos\big(\delta^d_{L1} - \delta^d_{L2}\big)| < 5.1 \cdot 10^{-10} \tag{4.119}$$

$$|V^*_{L4d}V_{L4s}|\sqrt{|\sin[2(\delta^d_{L1} - \delta^d_{L2})]|} < 3 \cdot 10^{-5} \tag{4.120}$$

$$|V_{L4s}| < 0.0008/\sqrt{\sin[2(\delta^d_{L1} - \delta^d_{L2})]} \tag{4.121}$$

Alternatively, one can consider the theoretical estimation with its uncertainty and require to stay within $1.96\Delta\epsilon_K^{\rm SM}$, $\Delta\epsilon_K^{\rm SM} = 0.18 \cdot 10^{-3}$, from the experimental central value after adding the new contribution (positively interfering):

$$\frac{1}{3\sqrt{2}\Delta M_K} B_K f_K^2 M_{K^0} \frac{G_F}{\sqrt{2}} |[\mathrm{Im}(V^*_{L4d} V_{L4s})]^2| < \epsilon_K - (\epsilon_K^{\mathrm{SM}} - 1.96 \Delta\epsilon_K^{\mathrm{SM}}) \tag{4.122}$$

$$2|V^*_{L4d} V_{L4s}|^2 |\sin\big(\delta^d_{L1} - \delta^d_{L2}\big) \cos\big(\delta^d_{L1} - \delta^d_{L2}\big)| < 8.2 \cdot 10^{-11} \tag{4.123}$$

$$|V^*_{L4d} V_{L4s}| \sqrt{|\sin[2(\delta^d_{L1} - \delta^d_{L2})]|} < 9.1 \cdot 10^{-6} \tag{4.124}$$

$$|V_{L4s}| < 0.00023/\sqrt{\sin[2(\delta^d_{L1} - \delta^d_{L2})]} \tag{4.125}$$

$B^0_{d,s}$-$\bar{B}^0_{d,s}$ mixing

The effective lagrangian for $B^0_{d,s}$-$\bar{B}^0_{d,s}$ mixing is (for B^0_d as an example) [138]:

$$\mathcal{L} = -\frac{G_F^2}{4\pi^2} m_W^2 (V^*_{tb} V_{td})^2 \eta_B S(x_t) [\alpha_s^{(5)}(\mu_b)]^{-6/23} \left[1 + \frac{\alpha_s^{(5)}(\mu_b)}{4\pi} J_5\right] (\overline{b_L} \gamma_\mu d_L)^2 + h.c. \tag{4.126}$$

where the QCD factor $\eta_B = 0.551$ [138]. Then, analogously to the kaons system:

$$\langle \bar{B}^0_{d,s} | (\overline{b_L} \gamma^\mu q_L)^2 | B^0_{d,s} \rangle = \frac{2}{3} B_{B_{d,s}}(\mu_b) f^2_{B_{d,s}} m^2_{B_{d,s}} \tag{4.127}$$

and

$$\Delta M_{d,s} = 2|M_{12}^{(d,s)}| \tag{4.128}$$

$$2 m_{B_{d,s}} M^*_{12} = \langle \bar{B}^0_{d,s} | -\mathcal{L} | B^0_{d,s} \rangle \tag{4.129}$$

Putting together (4.127), (4.128), (4.129), in the SM:

$$\Delta M_{d,s} = \frac{G_F^2 m_W^2}{6\pi^2} f^2_{B_{d,s}} m_{B_{d,s}} B_{B_{d,s}} |(V_{tb} V^*_{td/s})^2| \eta_B S(x_t) \tag{4.130}$$

where $f_{B_d}\sqrt{B_{B_d}} = 225(9)$ MeV, $f_{B_s}\sqrt{B_{B_s}} = 274(8)$ MeV [107]. The experimental results are [111]:

$$\Delta M_{d\,\mathrm{exp}} = (3.334 \pm 0.013) \cdot 10^{-13}\,\mathrm{GeV} \tag{4.131}$$

$$\Delta M_{s\,\mathrm{exp}} = (1.1688 \pm 0.0014) \cdot 10^{-11}\,\mathrm{GeV} \tag{4.132}$$

The long range interactions are negligible for the neutral B meson systems. Therefore $B^0_{d,s}$-$\bar{B}^0_{d,s}$ oscillations contribute to the determination of the CKM elements $|V_{td}|$ and $|V_{ts}|$. The largest uncertainty comes from the hadronic matrix elements $f_{B_d}\sqrt{B_{B_d}}$ and $f_{B_s}\sqrt{B_{B_s}}$which carry a relative error of 8% and 6% respectively. The new piece of Lagrangian generating B-mesons mixing is:

$$\mathcal{L}^{(d)}_{\text{new}} = -\frac{G_F}{\sqrt{2}}(V^*_{L4d}V_{L4b})^2(\overline{d_L}\gamma^\mu b_L)^2 + \text{h.c.} \tag{4.133}$$

$$\mathcal{L}^{(s)}_{\text{new}} = -\frac{G_F}{\sqrt{2}}(V^*_{4s}V_{4b})^2(\overline{s_L}\gamma^\mu b_L)^2 + \text{h.c.} \tag{4.134}$$

It can be imposed on the new operator to give a contribution less important than the uncertainty generated by the hadronic matrix elements:

$$\frac{G_F}{\sqrt{2}}|V^*_{L4d}V_{L4b}|^2 < \frac{G_F^2}{4\pi^2}m_W^2|V_{td}|^2 S(x_t)\cdot\Delta_{B_d} \tag{4.135}$$

$$\frac{G_F}{\sqrt{2}}|V^*_{L4s}V_{L4b}|^2 < \frac{G_F^2}{4\pi^2}m_W^2|V_{ts}|^2 S(x_t)\cdot\Delta_{B_s} \tag{4.136}$$

where $\Delta_{B_d} = 0.08$ and $\Delta_{B_s} = 0.06$, which gives (with $m_t(m_t) = 163$ GeV):

$$|V^*_{L4d}V_{L4b}| \approx |V_{ub'}V^*_{tb'}| < 1.8\cdot 10^{-4} \tag{4.137}$$

$$|V_{L4b}| \approx s_3^d \approx |V_{tb'}| < 0.0045 \tag{4.138}$$

$$|V^*_{L4s}V_{L4b}| \approx s_2^d s_3^d < 7.6\cdot 10^{-4} \tag{4.139}$$

or, at 90% C.L. $1.64\Delta_{B_d}$ and $1.64\Delta_{B_s}$ giving:

$$|V^*_{L4d}V_{L4b}| \approx |V_{ub'}V^*_{tb'}| < 2.3\cdot 10^{-4} \tag{4.140}$$

$$|V_{L4b}| \approx s_3^d \approx |V_{tb'}| < 0.006 \tag{4.141}$$

$$|V^*_{L4s}V_{L4b}| \approx s_2^d s_3^d < 0.00096 \tag{4.142}$$

or using as comparison 1.96 times the above considered uncertainties:

$$|V^*_{L4d}V_{L4b}| \approx |V_{ub'}V^*_{tb'}| < 2.5\cdot 10^{-4} \tag{4.143}$$

$$|V_{L4b}| \approx s_3^d \approx |V_{tb'}| < 0.0063 \tag{4.144}$$

$$|V^*_{L4s}V_{L4b}| \approx s_2^d s_3^d < 0.001 \tag{4.145}$$

where $|V_{L4d}| = 0.04$ has been used in the second equation. If, as in the kaon case, in order to have an estimate we compare the coefficient of the effective operators in (4.126) and (4.133):

$$\frac{G_F}{\sqrt{2}}|V^*_{L4d}V_{L4b}|^2 < \frac{G_F^2}{4\pi^2}m_W^2|V_{td}|^2 S(x_t) \tag{4.146}$$

$$\frac{G_F}{\sqrt{2}}|V^*_{L4s}V_{L4b}|^2 < \frac{G_F^2}{4\pi^2}m_W^2|V_{ts}|^2 S(x_t) \tag{4.147}$$

we have:

$$|V^*_{L4d}V_{L4b}| \approx |V_{ub'}V^*_{tb'}| < 6.4 \cdot 10^{-4} \quad (4.148)$$
$$|V_{L4b}| \approx s^d_3 \approx |V_{tb'}| < 0.02 \quad (4.149)$$
$$(4.150)$$

D^0-$\bar{D}^0$ mixing

The effective Lagrangian for D^0-$\bar{D}^0$ mixing due to the s-quark running in the box diagram is:

$$\mathcal{L} \approx -\frac{G_F^2 m_W^2}{4\pi^2} V^*_{us} V_{cs} S_0(x_s)(\overline{u_L}\gamma^\mu c_L)^2 + h.c. \quad (4.151)$$

where $x_s = \frac{m_s^2}{m_W^2}$, $S_0(x_s)$ is the Inami-Lim function. The matrix element can be defined as:

$$\langle \bar{D}^0 |(\overline{u_L}\gamma^\mu c_L)^2| D^0 \rangle = \frac{2}{3} B_D f_D^2 m_D^2 \quad (4.152)$$

where B_D is the correction to the VIA approximation and should be of order unity. Similarly to the case of neutral kaons mixing, the following relations hold:

$$2m_D M^*_{D12} = -\langle \bar{D}^0 |\mathcal{L}| D^0 \rangle \quad (4.153)$$
$$\Delta m_D \approx 2|M_{D12}| + (\Delta m_D)_{LD} \quad (4.154)$$

where $(\Delta m_D)_{LD}$ corresponds to the long-distance contributions. The experimental value of the mass difference in the D^0 mesons system is

$$\Delta m_{D\,\mathrm{exp}} = (6.25 \pm 2.6) \cdot 10^{-15}\mathrm{GeV} \quad (4.155)$$

However the mass difference in the D^0 system is likely to be dominated by long-distance contributions. The SM short-distance contribution due to the s-quark running in the box diagram is of order

$$\Delta m_{D\,\mathrm{SM}} \approx 2|M_{12}| \approx \frac{G_F^2 m_W^2}{6\pi^2} |(V_{us}V^*_{cs})^2| S_0(x_s) f_D^2 m_D B_D \sim O(10^{-16}\mathrm{GeV}) \quad (4.156)$$

where $x_s = \frac{m_s^2}{m_W^2}$, $S_0(x_s)$ is the Inami-Lim function, and $f_D = 212.0 \pm 0.7$ MeV [107].

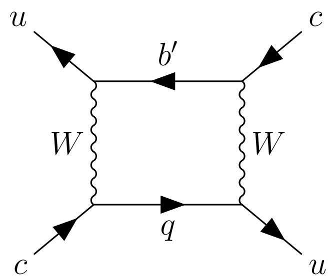

Figure 4.2: New contribution to $D^0 - \bar{D^0}$ mixing, $q = d, s, b, b'$.

The new down-quark contributes to the box diagram originating D mesons mixing, as shown in Fig. (4.2):

$$\begin{aligned}\mathcal{L}_{\text{new}} = -\frac{G_F^2 m_W^2}{4\pi^2} \big((V_{ub'}V_{cb'}^*)^2 S_0(x_{b'}) + 2(V_{ub'}V_{cb'}^*)(V_{cs}^* V_{us}) S_0(x_s, x_{b'}) + \\ +2(V_{ub'}V_{cb'}^*)(V_{cb}^* V_{ub}) S_0(x_b, x_{b'}) \big) \cdot (\overline{u_L}\gamma^\mu c_L)(\overline{u_L}\gamma^\mu c_L) + \text{h.c.}\end{aligned} \tag{4.157}$$

It is worth underlying that in this case the CKM matrix is concerned since this contribution involves charged currents. Then the limit can be set directly for the elements of V_{CKM}.

Since for the presumably lower estimate $|V_{ub'}V_{cb'}^*| = 0.00002$ (with $M_d = 1530$ GeV) the contribution from b' is $\Delta m_{D\,\text{new}} \simeq 5 \cdot 10^{-17}$ GeV, while the mixed one from s, b' is $\Delta m_{D\,\text{new}} \simeq 2 \cdot 10^{-19}$ GeV and also from b, b' $\Delta m_{D\,\text{new}} \simeq 2 \cdot 10^{-19}$ GeV, then the contribution to Δm_D coming from (4.157) is about:

$$\Delta m_{D\,\text{new}} \simeq \frac{G_F^2 m_W^2}{6\pi^2} (V_{ub'}V_{cb'}^*)^2 S_0(x_{b'}) f_D^2 m_D \tag{4.158}$$

In order to make the new contribution less than the SM short-distance contribution, it should be that:

$$\left|(V_{ub'}V_{cb'}^*)^2 S_0(x_{b'})\right| < \left|(V_{us}V_{cs}^*)^2 S_0(x_s)\right| \tag{4.159}$$

Choosing $M_d = 1530$ GeV:

$$|V_{ub'}V_{cb'}^*| < 3 \cdot 10^{-5} \tag{4.160}$$

$$|V_{cb'}| < 7 \cdot 10^{-4} \tag{4.161}$$

where the second relation is obtained with $|V_{ub'}| = 0.04$. In a more conservative way, by using $\Delta m_{D\,\text{new}}$ from (4.158) with $M_d = 1$ TeV, $\Delta m_{D\,\text{new}} < \Delta m_{D\,\text{exp}}$ gives:

$$|V_{ub'}V_{cb'}^*| < 2 \cdot 10^{-4} \tag{4.162}$$

$$|V_{cb'}| < 0.006 \tag{4.163}$$

4.1.3 B decays

Rare semileptonic B decays

Results on rare B-decays can constrain new mixings, due to the new contributions to FCNC processes $b \to s(d)$ originated at tree level by mixing with the extra singlet.

The new effective lagrangian contributing to $b \to s(d)\ell^+\ell^-$ is:

$$\mathcal{L}_{\rm new} = -\frac{4G_F}{\sqrt{2}} V_{L4d} V^*_{L4b} (\overline{b_L}\gamma^\mu d_L) \left[(-\frac{1}{2} + \sin^2\theta_W)(\overline{\ell_L}\gamma_\mu \ell_L) + \sin^2\theta_W (\overline{\ell_R}\gamma_\mu \ell_R) \right] \tag{4.164}$$

$$\mathcal{L}_{\rm new} = -\frac{4G_F}{\sqrt{2}} V^*_{L4b} V_{L4s} (\overline{b_L}\gamma^\mu s_L) \left[(-\frac{1}{2} + \sin^2\theta_W)(\overline{\ell_L}\gamma_\mu \ell_L) + \sin^2\theta_W (\overline{\ell_R}\gamma_\mu \ell_R) \right] \tag{4.165}$$

accounting for both inclusive decays $B \to X_{d,s}\ell^+\ell^-$, and exclusive decays such as $B \to \pi\ell^+\ell^-$ and $B \to K\ell^+\ell^-$.

As regards exclusive decays, experimental branching fractions are below SM predictions [111]. Recently also other observables were exctracted, in order to consider quantities less affected by hadronic uncertainties [111]. The decays $B^0 \to K^{0*}\ell^+\ell^-$ and $B^+ \to K^+\ell^+\ell^-$ are the best studied ([111] and references therein). Tensions with SM were found related to the quantity P_5' and to lepton universality test ([111] and references therein).

In order to avoid hadronic form factors, inclusive decays are studied. As regards $B \to X_s\ell^+\ell^-$, branching fractions can be analyzed both from a theoretical and experimental point of view in low and high dilepton invariant mass regions, that is q^2 ($1\text{GeV}^2 < q^2 < 6\text{GeV}^2$) and q^2 ($q^2 > 14.2\text{GeV}^2$) regions (the intermediate region is dominated by charmonia resonances). Experimental results were obtained by both Belle [154] and BaBar [155]. In Ref. [156] the SM expectations in these q^2-regions were computed and compared with experimental results, showing an agreement at the 95% C.L. between SM and experiments.

The total branching fraction averaged between electrons and muons is: $(5.8 \pm 1.3) \cdot 10^{-6}$ [111], [154], [155]. Then a rough estimate for the constraint on the mixings can be found out by limiting the new contribution to be less than the experimental error on the total inclusive branching fraction:

$$\text{Br}(B \to X_s\ell^+\ell^-)_{\rm new} \simeq \text{Br}(B \to X_c\ell^+\nu_\ell) \frac{|V^*_{L4b}V_{L4s}|^2[(-\frac{1}{2} + \sin^2\theta_W)^2 + \sin^4\theta_W]}{|V_{cb}|^2} < \sigma_{\rm exp} \tag{4.166}$$

with $\sigma_{\text{exp}} = 1.3 \cdot 10^{-6}$ resulting in the limit $|V^*_{L4b}V_{L4s}| < 4.2 \cdot 10^{-4}$. Or we can make an estimate by imposing on the new contribution to the branching ratio to be less than $1.96\sigma_{\text{exp}}$ resulting in the limit:

$$|V^*_{L4b}V_{L4s}| \approx s^d_{L3}s^d_{L2} < 5.8 \cdot 10^{-4} \tag{4.167}$$

where we take advantage of the experimental branching ratio $\text{Br}(B \to X_c\ell^+\nu_\ell) = 0.1065 \pm 0.0016$ [111].

As regards $b \to d\ell^+\ell^-$ transitions, branching ratios were measured for the decays $\text{Br}(B^+ \to \pi^+\mu^+\mu^-) = (1.76 \pm 0.23) \cdot 10^{-8}$ [157], $\text{Br}(B^0 \to \pi^+\pi^-\mu^+\mu^-) = (2.1 \pm 0.5) \cdot 10^{-8}$ [158]. Limits at 90% C.L. on exclusive decays include $\text{Br}(B^0 \to \pi^0\ell^+\ell^-) < 5.3 \cdot 10^{-8}$, $\text{Br}(B^0 \to \pi^0e^+e^-) < 8.4 \cdot 10^{-8}$, $\text{Br}(B^0 \to \pi^0\mu^+\mu^-) < 6.9 \cdot 10^{-8}$ and the lepton flavor and isospin averaged branching fraction upper limit $\text{Br}(B \to \pi\ell^+\ell^-) < 5.9 \cdot 10^{-8}$ [159], and $\text{Br}(B^+ \to \pi^+\ell^+\ell^-) < 4.9 \cdot 10^{-8}$, $\text{Br}(B^+ \to \pi^+e^+e^-) < 8.0 \cdot 10^{-8}$ [160]. Constraints on the mixing elements $|V_{L4d}V^*_{L4b}|$ can be estimated using the lepton-flavor combined limits:

$$\begin{aligned}
&\text{Br}(B^0 \to \pi^0\ell^+\ell^-)_{\text{new}} \simeq \qquad (4.168)\\
&\simeq \text{Br}(B^0 \to \pi^-\ell^+\nu_\ell)\frac{1}{2}\frac{|V_{L4d}V^*_{L4b}|^2[(-\frac{1}{2} + \sin^2\theta_W)^2 + \sin^4\theta_W]}{|V_{ub}|^2} < 5.3 \cdot 10^{-8}\\
&\text{Br}(B^\pm \to \pi^\pm\ell^+\ell^-)_{\text{new}} \simeq \qquad (4.169)\\
&\simeq \text{Br}(B^\pm \to \pi^0\ell^+\nu_\ell)2\frac{|V_{L4d}V^*_{L4b}|^2[(-\frac{1}{2} + \sin^2\theta_W)^2 + \sin^4\theta_W]}{|V_{ub}|^2} < 4.9 \cdot 10^{-8}
\end{aligned}$$

The factors 2 take into account the isospin symmetry relation. For convenience we took advantage of the results in [111] $\text{Br}(B^0 \to \pi^-\ell^+\nu_\ell) = (1.50 \pm 0.06) \cdot 10^{-4}$ $\text{Br}(B^\pm \to \pi^0\ell^+\nu_\ell) = (7.80 \pm 0.27) \cdot 10^{-5}$, $\ell = e$ or μ. The effective Lagrangian originating this decays is:

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}V^*_{ub}(\overline{b_L}\gamma^\mu u_L)(\overline{\nu_\ell}\gamma_\mu\ell_L) \tag{4.170}$$

From (4.169):

$$|V_{L4d}V^*_{L4b}| \approx s^d_{L1}s^d_{L3} \approx |V_{ub'}V^*_{tb'}| < 2 \cdot 10^{-4} \tag{4.171}$$
$$|V_{L4b}| \approx |V_{tb'}| < 0.005 \tag{4.172}$$

where the last result is obtained using $|V_{L4d}| = 0.04$.

$B^0_{d,s} \to \mu^+\mu^-$

The main effective weak Lagrangian for $B^0_{d,s} \to \mu^+\mu^-$ is [161]:

$$\mathcal{L}_{\rm SM} = \frac{G_F^2 m_W^2}{\pi^2} V_{tb}^* V_{tq} C_A(\mu_b)(\bar{b}\gamma^\mu\gamma^5 q)(\bar{\ell}\gamma_\mu\gamma_5\ell) \tag{4.173}$$

with $q = d, s$ and $C_A(\mu_b) = 0.4681$ the same as in [161] using the values in Table 4.1. The new contribution is:

$$\mathcal{L}_{\rm new} = \frac{G_F}{\sqrt{2}}\frac{1}{2} V_{L4b}^* V_{L4q}(\bar{b}\gamma^\mu\gamma^5 q)(\bar{\ell}\gamma_\mu\gamma_5\ell) \tag{4.174}$$

for $q = d, s$. The branching ratio for the decay $B^0_s \to \mu^+\mu^-$ was measured, $\mathrm{Br}(B^0_s \to \mu^+\mu^-)_{\rm exp} = (3.0 \pm 0.4)\cdot 10^{-9}$ [111], the 95% C.L. limit for the decay $B^0_d \to \mu^+\mu^-$ is $\mathrm{Br}(B^0_d \to \mu^+\mu^-)_{\rm exp} < 2.1\cdot 10^{-10}$ [162]. The branching ratios predicted by the SM are respectively: $\mathrm{Br}(B^0_s \to \mu^+\mu^-)_{\rm SM} = (3.35 \pm 0.06)\cdot 10^{-9}$, $\mathrm{Br}(B^0_d \to \mu^+\mu^-)_{\rm SM} = (0.89 \pm 0.02)\cdot 10^{-10}$, obtained from [161] using $f_{B_s} = 230.3(1.3)$ MeV, $f_{B_d} = 190.0(1.3)$ MeV (from [107]), and for the lifetimes $\tau_{B^0_{sH}} = 1.619$ ps, $\tau_{B^0_d} = 1.519$ ps from [111]. In order to have a first estimate of the limit on the mixings, the new contribution to the effective operator can be constrained to be less than the SM contribution:

$$|V_{L4b}^* V_{L4s}| < \frac{2\sqrt{2}G_F m_W^2}{\pi^2} C_A(\mu_b)|V_{tb}^* V_{ts}| \tag{4.175}$$

$$|V_{L4b}^* V_{L4s}| < 4\cdot 10^{-4} \tag{4.176}$$

$$|V_{L4b}^* V_{L4d}| \approx |V_{ub'}^* V_{tb'}| < \frac{2\sqrt{2}G_F m_W^2}{\pi^2} C_A(\mu_b)|V_{tb}^* V_{td}| \tag{4.177}$$

$$|V_{L4b}^* V_{L4d}| < 8\cdot 10^{-5} \tag{4.178}$$

$$|V_{L4b}| < 0.002 \tag{4.179}$$

where the last relation is obtained with $|V_{L4d}| = 0.04$. Alternatively, the 95% C.L. experimental limit on the B^0_d decay can be considered:

$$|V_{L4b}^* V_{L4d}| < \frac{2\sqrt{2}G_F m_W^2}{\pi^2} C_A(\mu_b)|V_{tb}^* V_{td}|\sqrt{\frac{2.1\cdot 10^{-10}}{\mathrm{Br}(B^0_d \to \mu^+\mu^-)_{\rm SM}}} \tag{4.180}$$

$$|V_{L4b}^* V_{L4d}| < 1.3\cdot 10^{-4} \tag{4.181}$$

$$|V_{L4b}| < 0.003 \tag{4.182}$$

Regarding B^0_s decaying into two muons, it can be required that the combination of the central value of the SM prediction and the new contribution stay within

$1.96\sigma_{\text{exp}}$, $\sigma_{\text{exp}} = 0.4 \cdot 10^{-9}$, from the experimental determination. Considering negative interference:

$$|V_{L4b}^* V_{L4s}]| < \frac{2\sqrt{2} G_F m_W^2}{\pi^2} C_A(\mu_b) |V_{tb}^* V_{ts}| \left(1 - \sqrt{\frac{3.0 \cdot 10^{-9} - 1.96\sigma_{\text{exp}}}{\text{Br}(B_s^0 \to \mu^+\mu^-)_{\text{SM}}}} \right) \tag{4.183}$$

$$|V_{L4b}^* V_{L4s}| < 7.4 \cdot 10^{-5} \tag{4.184}$$

4.1.4 Z-boson physics

Quantity	Experimental value	SM prediction
Γ_Z	2.4952 ± 0.0023 GeV	2.4942 ± 0.0008 GeV
$\Gamma(\text{had})$	1.17444 ± 0.0020 GeV	1.7411 ± 0.0008 GeV
R_b	0.21629 ± 0.00066	0.21582 ± 0.00002
R_c	0.1721 ± 0.0030	0.17221 ± 0.00003
σ_{had}	41.541 ± 0.037 nb	41.481 ± 0.008 nb
$\Gamma(Z \to b\bar{b})$		375.76 ∓ 0.16 MeV
$\Gamma(Z \to u\bar{u})$		299.91 ± 0.18 MeV
$\Gamma(Z \to b\bar{b})/\Gamma_Z$	0.1512 ± 0.0005	
$\Gamma(Z \to c\bar{c})/\Gamma_Z$	0.1203 ± 0.0021	
$\Gamma(Z \to (u\bar{u} + c\bar{c})/2)/\Gamma_Z$	0.116 ± 0.006	
$\Gamma(Z \to \text{had})/\Gamma_Z$	0.69911 ± 0.00056	
$Q_W(p)$	0.0719 ± 0.0045	0.0711 ± 0.0002
$Q_W(Cs)$	-72.58 ± 0.43	-73.23 ± 0.01
$Q_W(T\ell)$	-116.4 ± 3.6	-116.87 ± 0.02
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1030 ± 0.0002
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0735 ± 0.0001
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1031 ± 0.0002
A_b	0.923 ± 0.020	0.9347
A_c	0.670 ± 0.027	0.6677 ± 0.0001
A_s	0.895 ± 0.091	0.9356
$g_{AV}^{eu} + 2g_{AV}^{ed}$	0.4914 ± 0.0031	0.4950
$2g_{AV}^{eu} - g_{AV}^{ed}$	-0.7148 ± 0.0068	-0.7194

Table 4.3: Values of interest from Particle Data Group [111].

The presence of an additional quark affects the diagonal couplings of Z-boson with quarks, changing the prediction of many observables related to the Z boson physics e.g. the partial decay widths $\Gamma(Z \to q\bar{q})$ with $q = u, d, s, c, b$, the variables $R_c = \Gamma(c\bar{c})/\Gamma(Z \to \text{had})$, $R_b = \Gamma(b\bar{b})/\Gamma(Z \to \text{had})$, σ_{had}, Z-pole asymmetries. Both off-diagonal couplings and diagonal couplings also modify the prediction of the Z total width Γ_Z and the partial width into hadrons $\Gamma(Z \to \text{had})$.

The variation of the diagonal couplings of weak neutral-current interaction also modifies interactions of quarks with leptons, then low energy ($Q^2 << M_Z^2$) electroweak precision observables are affected.

Experimental values and SM predictions for Z pole quantities are given in averages and global fit results of Particle Data Group [111], as reported in Table 4.3.

$Z \to b\bar{b}$

As regards the diagonal coupling of b-quark, the predicted partial decay width for Z-boson decaying into $b\bar{b}$ is [111] $\Gamma(Z \to b\bar{b})_{\text{SM}} \simeq 375.76 \mp 0.16$ MeV, which can be compared to the experimental value $\Gamma(Z \to b\bar{b})_{\text{exp}} \simeq 377.3 \mp 1.3$ MeV where the values from [111] $\text{Br}(Z \to b\bar{b})_{\text{exp}} \simeq 0.1512 \pm 0.0005$ MeV and $\Gamma(Z)_{\text{exp}} = 2.4952 \pm 0.0023$ GeV have been used.

The SM decay rate $\Gamma(Z \to b\bar{b})$ at tree level is given by:

$$\Gamma(Z \to b\bar{b})_{0,\text{SM}} = \frac{G_F M_Z^3}{\sqrt{2}\pi}\left[\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)^2 + \left(\frac{1}{3}\sin^2\theta_W\right)^2\right] \tag{4.185}$$

By inserting a fourth down-quark, the decay rate changes in:

$$\Gamma(Z \to b\bar{b})_{0,\text{new}} = \frac{G_F M_Z^3}{\sqrt{2}\pi}\left[\left(-\frac{1}{2}(1-|V_{L4b}|^2)+\frac{1}{3}\sin^2\theta_W\right)^2 + \left(\frac{1}{3}\sin^2\theta_W\right)^2\right] \tag{4.186}$$

so the prediction for the decay rate is lowered. Then a limit can be set on $|V_{L4b}|$ by constraining the difference between the SM contribution (4.185) and the new contribution (4.186) to be less than the experimental error:

$$|\Gamma(Z \to b\bar{b})_{\text{new}} - \Gamma(Z \to b\bar{b})_{\text{SM}}| = \frac{G_F M_Z^3}{\sqrt{2}\pi}\left|-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right| |V_{L4b}|^2 < 0.0013\,\text{GeV} \tag{4.187}$$

and it is obtained that:

$$|V_{L4b}| = s_{L3}^d \approx |V_{tb'}| < 0.04 \tag{4.188}$$

or at 95% C.L.

$$|V_{L4b}| < 0.055 \tag{4.189}$$

$\Gamma(Z \to \mathbf{hadr})$, $\Gamma(Z)$

The SM predictions for the Z decay rate and partial decay rate into hadrons are $\Gamma(Z)_{\rm SM} = 2.4942 \pm 0.0008$ GeV, $\Gamma(Z \to {\rm hadr})_{\rm SM} = 1.7411 \pm 0.0008$ GeV, to be compared with the experimental results $\Gamma(Z)_{\rm exp} = 2.4952 \pm 0.0023$ GeV, $\Gamma(Z \to {\rm hadr})_{\rm exp} = 1.7444 \pm 0.0020$ GeV, showing a 0.4σ pull of the predicted width with respect to the experimental value and agreement as regards the partial decay rate into hadrons [111].

In this model the deviation of the Z decay rate from the SM prediction is (see also Ref. [137]):

$$
\begin{aligned}
&|\Gamma(Z)_{\rm new} - \Gamma(Z)_{\rm SM}| = |\Gamma(Z \to {\rm had})_{\rm new} - \Gamma(Z \to {\rm had})_{\rm SM}| = \\
&= \frac{G_F M_Z^3}{\sqrt{2}\pi} \left| \sum_{i,j=d,s,b} \left| -\frac{1}{2} \sum_{k=1}^{3} V_{Lki}^{(d)*} V_{Lkj}^{(d)} + \frac{1}{3}\sin^2\theta_W \delta_{ij} \right|^2 - 3\left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right)^2 \right| = \\
&= \frac{G_F M_Z^3}{\sqrt{2}\pi} \left| \sum_{q=d,s,b} \left(-\frac{1}{2}(1 - |V_{L4q}^{(d)}|^2) + \frac{1}{3}\sin^2\theta_W\right)^2 \right. \\
&\left. + \frac{1}{2}\left(|V_{L4d}^* V_{L4s}|^2 + |V_{L4d}^* V_{L4b}|^2 + |V_{L4s}^* V_{L4b}|^2\right) - 3\left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right)^2 \right|
\end{aligned}
\tag{4.190}
$$

We can impose that this deviation is less than the experimental error:

$$
|\Gamma(Z \to {\rm had})_{\rm new} - \Gamma(Z \to {\rm had})_{\rm SM}| < 0.0020\,{\rm GeV} \tag{4.191}
$$

The constraints in Table 4.4 imply that non-diagonal terms in (4.190) do not give any noticeable contribution. Then:

$$
\frac{G_F M_Z^3}{\sqrt{2}\pi}\left(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W\right)\left(|V_{L4d}|^2 + |V_{L4s}|^2 + |V_{L4b}|^2\right) < 0.0020\,{\rm GeV} \tag{4.192}
$$

that is

$$
|V_{L4d}|^2 + |V_{L4s}|^2 + |V_{L4b}|^2 < 0.0024 \tag{4.193}
$$

or

$$
|V_{L4d}|^2 + |V_{L4s}|^2 + |V_{L4b}|^2 < 0.0047 \tag{4.194}
$$

to remain within 1.96 times experimental error. For $|V_{L4d}| = 0.04$ the constraint means $|V_{L4s}|^2 + |V_{L4b}|^2 < 0.0008$ (or $|V_{L4s}|^2 + |V_{L4b}|^2 < 0.003$ within $1.96\sigma_{\rm exp}$), which is satisfied for example if both $|V_{L4s}|, |V_{L4b}| < 0.02$. If $V_{L4s} = V_{L4b} = 0$ this constraint implies $|V_{L4d}| < 0.05$ or $|V_{L4d}| < 0.07$ at 95% C.L..

Z-pole asymmetries

Z-pole asymmetries from $e^+e^- \to ff$ processes around Z resonance can also be studied in the light of the presence of an extra isosinglet. In particular, left-right asymmetries A_{LR}, forward-backward asymmetries A_{FB}, and left-right forward-backward asymmetries A_{LRFB} are measured [111], [163]:

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \tag{4.195}$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \tag{4.196}$$

$$A_{LRFB} = \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R} \tag{4.197}$$

where L, R are the incident electron helicities.

The diagonal Lagrangian can be written as:

$$\frac{g}{\cos\theta_W} Z^\mu \left(g_L^q \overline{q_L}\gamma_\mu q_L + g_R^q \overline{q_R}\gamma_\mu q_R\right) \tag{4.198}$$

where $q = d, s, b$, but also $q = u, c, t$ under the hypothesis of the existence of en extra up-type sinlglet. Then:

$$g_L^q = t_3^q(1 - |V_{L4q}|^2) - Q_q \sin^2\theta_W \tag{4.199}$$

$$g_R^q = -Q_q \sin^2\theta_W \tag{4.200}$$

SM couplings are obtained when $|V_{L4q}| = 0$.

Cross sections for Z-boson exchange are usually written in terms of the asymmetry parameters A_f, $f = e, \mu, \tau, b, c, s, q$, containing final-state couplings:

$$A_f = \frac{\bar{g}_L^{f2} - \bar{g}_R^{f2}}{\bar{g}_L^{f2} + \bar{g}_R^{f2}} = \frac{2\bar{g}_V^f \bar{g}_A^f}{\bar{g}_V^{f2} + \bar{g}_A^{f2}} = \frac{1 - 4|Q_f|\bar{s}_f^2}{1 - 4|Q_f|\bar{s}_f^2 + 8(|Q_f|\bar{s}_f^2)^2} \tag{4.201}$$

where in the SM at tree level $g_L^f = t_{3L}^f - Q_f \sin^2\theta_W$, $g_R^f = -Q_f \sin^2\theta_W$. where $g_V^f = g_L^f + g_R^f$, $g_A^f = g_L^f - g_R^f$. The bar in the couplings in (4.201) indicates that EW radiative corrections must be taken into account, so effective couplings are defined and for convenience the effective angles $\bar{s}_f^2$ are used [111], [164] [165], [166]. The parameters A_f can also be exctracted from the measured asymmetries. For example, they are related as $A_{LR} = A_e P_e$ where P_e is the initial electron polarization, $A_{FB}^{(0,f)} = \frac{3}{4}A_e A_f$ for $P_e = 0$, $A_{LRFB} = \frac{3}{4}A_f$ for $P_e = 1$.

As regards b-quark final state, the measured value $A_{FB}^{(0,b)} = 0.0992 \pm 0.0016$ exhibits a -2.3σ pull with respect the SM prediction $A_{FB}^{(0,b)} = 0.1030 \pm 0.0002$. On the other hand, the value $A_b = 0.923 \pm 0.020$ is obtained from $A_{LRFB}(b)$ at SLD, basically in agreement with the SM prediction $A_b = 0.9347$ (-0.6σ pull). However it can also be inferred from $A_{FB}^{(0,b)}$ using $A_e = 0.1501 \pm 0.0016$ (as suggested by PDG [111]) obtaining $A_b = 0.881 \pm 0.017$. The average between the two can be considered: $A_b = 0.899 \pm 0.013$, which is 2.8σ below the SM prediction.

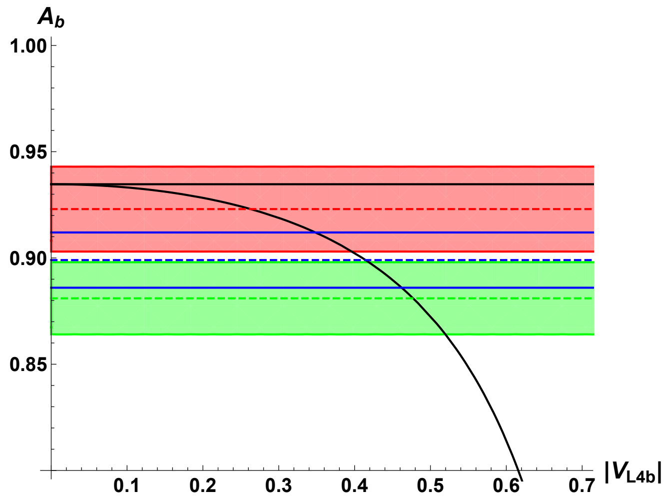

Figure 4.3: Determination of A_b obtained from Eq. (4.202) as a function of V_{L4b} (black curve). The 1σ intervals of the experimental values $A_b = 0.923 \pm 0.020$ (red band) and $A_b = 0.881 \pm 0.017$ obtained from $A_{FB}^{(0,b)}$ (green band) are shown. The blue lines correspond to the average of the two $A_b = 0.899 \pm 0.013$. The black straight line is the SM prediction $A_b = 0.9347$.

The presence of an additional isosinglet changes the couplings as in (4.199), and consequently the predictions for the asymmetries are also changed:

$$A_q = \frac{(1-|V_{L4q}|^2)^2 - 4|Q_q|\bar{s}_q^2(1-|V_{L4q}|^2)}{(1-|V_{L4q}|^2)^2 - 4|Q_q|\bar{s}_q^2(1-|V_{L4q}|^2) + 8Q_q^2\bar{s}_q^4} \tag{4.202}$$

Regarding the asymmetry for b-quark, as shown in Fig. 4.3, the presence of the mixing element V_{L4b} can heal the discrepancy, whatever is the chosen value for A_b. In fact, for $V_{L4b} < 0.52$ still gives a predicted value within one errorbar if A_b is inferred from $A_{FB}^{(0,b)}$ using $A_e = 0.1501 \pm 0.0016$.

4.1.5 Low energy electroweak observables

As a consequence of the insertion of the isosinglet quark, also the standard interactions of quarks with leptons are modified.

The low-energy four fermion interactions with Z-boson exchange related to parity violating e-hadron neutral current processes are contained in the Lagrangian [111]:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \sum_q \left[g^{eq}_{AV} \bar{e}\gamma_\mu\gamma^5 e \bar{q}\gamma^\mu q + g^{eq}_{VA} \bar{e}\gamma_\mu e \bar{q}\gamma^\mu\gamma^5 q \right] \tag{4.203}$$

where in the SM:

$$g^{eu}_{AV} = -\frac{1}{2} + \frac{4}{3}\sin^2\theta_W \qquad g^{ed}_{AV} = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \tag{4.204}$$

$$g^{eu}_{VA} = -\frac{1}{2} + 2\sin^2\theta_W \qquad g^{ed}_{VA} = \frac{1}{2} - 2\sin^2\theta_W \tag{4.205}$$

Weak isosinglets change the couplings into:

$$g^{eu}_{AV} = -\frac{1}{2}(1-|V_{L4u}|^2) + \frac{4}{3}\sin^2\theta_W \quad g^{ed}_{AV} = \frac{1}{2}(1-|V_{L4d}|^2) - \frac{2}{3}\sin^2\theta_W \tag{4.206}$$

$$g^{eu}_{VA} = (-\frac{1}{2} + 2\sin^2\theta_W)(1-|V_{L4u}|^2) \quad g^{ed}_{VA} = -(-\frac{1}{2} + 2\sin^2\theta_W)(1-|V_{L4d}|^2) \tag{4.207}$$

The weak charge of the proton and of nuclei are measurable quantities which can be expressed in terms of the couplings in (4.206). The weak charge of the proton is proportional to $2g^{eu}_{AV} + g^{ed}_{AV}$ and it is an observable quantity, measured by Qweak at Jefferson Lab, from which it is obtained [111], [167]:

$$2g^{eu}_{AV} + g^{ed}_{AV} = 0.0356 \pm 0.0023 \tag{4.208}$$

corresponding to the weak charge $Q_W(p) = 0.0719 \pm 0.0045$, in agreement with the SM expectation $Q_W(p) = 0.0711 \pm 0.0002$ [111]. After considering the mixing with an extra singlet, the expected weak charge changes. The additional contribution to $2g^{eu}_{AV} + g^{ed}_{AV}$ is $|V_{L4u}|^2 - \frac{1}{2}|V_{L4d}|^2$. With $V_{L4u} = 0$, $|V_{L4d}| < 0.068$ gives a contribution to the weak charge of the proton less than the experimental error, or $|V_{L4d}| < 0.095$ contribute for less than $1.96\sigma_{\text{exp}}$.

Also atomic parity violation is measured, and it is linked to the nuclear weak charge $Q^{Z,N}_W$ [111]:

$$Q^{Z,N}_W = -2[Z(g^{ep}_{AV} + 0.00005) + N(g^{en}_{AV} + 0.00006)](1 - \frac{\alpha}{2\pi}) \tag{4.209}$$

where Z and N are the numbers of protons and neutrons in the nucleus, $g_{AV}^{ep} = 2g_{AV}^{eu} + g_{AV}^{ed}$, $g_{AV}^{en} = g_{AV}^{eu} + 2g_{AV}^{ed}$. The most precise measurement of atomic parity violation is in Cesium, $Q_W(Cs) = -72.58 \pm 0.43$ [168], while the SM prediction is $Q_W(Cs) = -73.23 \pm 0.01$, so with 1.5σ discrepancy. The contribution of the new quark to the weak charge of Cesium is:

$$\Delta Q_W^{55,78}(Cs) = Q_W^{55,78}(Cs)_{\text{new}} - Q_W^{55,78}(Cs)_{\text{SM}} = -2[94|V_{L4u}|^2 - 105.5|V_{L4d}|^2](1 - \frac{\alpha}{2\pi}) \tag{4.210}$$

In this first scenario $V_{L4u} = 0$ and the fourth down-type quark is helping in filling the gap with the experimental result. With $|V_{L4d}| = 0.04$, $\Delta Q_W^{55,78}(Cs) = 0.34$, comparable to the experimental error and the gap. In order to completely fill the gap it should be that $|V_{L4d}| = 0.056$. Also the discrepancy in the weak charge of cesium would be less than $1\sigma_{\text{exp}}$ with $|V_{L4d}| < 0.072$ or within $1.96\sigma_{\text{exp}}$ with $|V_{L4d}| < 0.084$.

4.1.6 Discussion

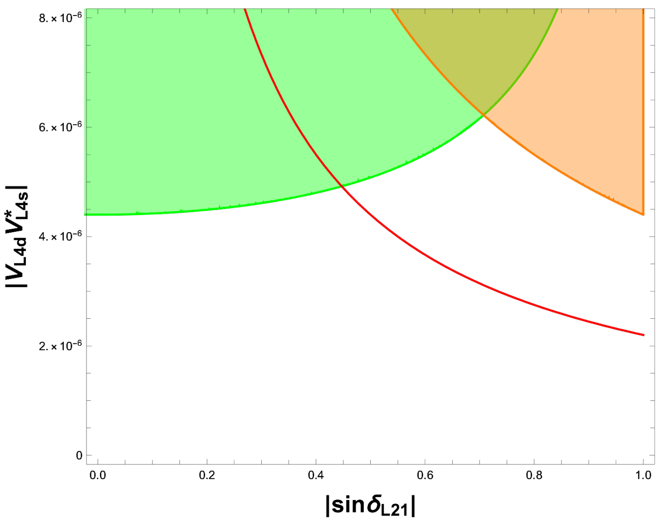

Figure 4.4: Upper limit on the product $|V_{L4s}^* V_{L4d}|$ as a function of the relative phase δ_{L21}^d. The limit is set by the constraints from $K_L \to \mu^+\mu^-$, $K_S \to \mu^+\mu^-$. The green area is excluded by setting $\text{Br}(K_{\text{L}} \to \mu^+\mu^-) < 10^{-9}$, as described in the text. The orange area is excluded by the constraint from the result $\text{Br}(K_{\text{S}} \to \mu^+\mu^-) < 1.0 \cdot 10^{-9}$ at 95% CL [148] adopted by PDG [111], the red line represents the boundary if the result $\text{Br}(K_{\text{S}} \to \mu^+\mu^-) < 2.4 \cdot 10^{-10}$ at 95% CL [147] is used.

Process	Constraint	$\lvert V_{L4d}\rvert \approx \lvert V_{ub'}\rvert = 0.04$	
$K^+ \to \pi^+\nu\bar{\nu}$	$\lvert V^*_{L4s}V_{L4d}\rvert < 1.4\cdot 10^{-5}$	$\lvert V_{L4s}\rvert < 3.5\cdot 10^{-4}$	$\lvert V_{cb'}\rvert \sim 0.01$
$K_S \to \mu^+\mu^-$	$\lvert V^*_{L4s}V_{L4d}\rvert\lvert\sin\delta^d_{L21}\rvert < 2.2\cdot 10^{-6}$	$\lvert V_{L4s}\rvert < \frac{0.000054}{\lvert\sin\delta^d_{L21}\rvert}$	
$K_L \to \mu^+\mu^-$	$\lvert V^*_{L4s}V_{L4d}\rvert\lvert\cos\delta^d_{L21}\rvert < 4.4\cdot 10^{-6}$	$\lvert V_{L4s}\rvert < \frac{0.0001}{\lvert\cos\delta^d_{L21}\rvert}$	
$K^0 - \bar{K}^0$	$\lvert V_{L4s}V^*_{L4d}\rvert\sqrt{\lvert\sin(2\delta_{L12})\rvert} < 9.1\cdot 10^{-6}$	$\lvert V_{L4s}\rvert < \frac{0.00023}{\sqrt{\lvert\sin(2\delta^d_{L12})\rvert}}$	
$B^\pm \to \pi^\pm\ell^+\ell^-$	$\lvert V^*_{L4b}V_{L4d}\rvert < 2\cdot 10^{-4}$	$\lvert V_{L4b}\rvert < 0.005$	$\lvert V_{tb'}\rvert < 0.005$
$B^0 \to \mu^+\mu^-$	$\lvert V^*_{L4b}V_{L4d}\rvert < 1.3\cdot 10^{-4}$	$\lvert V_{L4b}\rvert < 0.003$	$\lvert V_{tb'}\rvert < 0.003$
$B \to X_s\ell^+\ell^-$	$\lvert V^*_{L4b}V_{L4s}\rvert < 5.8\cdot 10^{-4}$		
$B^0_s \to \mu^+\mu^-$	$\lvert V^*_{L4b}V_{L4s}\rvert < 7.4\cdot 10^{-5}$		
$D^0 - \bar{D}^0$ th	$\lvert V_{ub'}V^*_{cb'}\rvert < 3\cdot 10^{-5}$		$\lvert V_{cb'}\rvert \lesssim 0.0007$
$D^0 - \bar{D}^0$ exp	$\lvert V_{ub'}V^*_{cb'}\rvert < 2\cdot 10^{-4}$		$\lvert V_{cb'}\rvert \lesssim 0.006$
$Z \to b\bar{b}$	$\lvert V_{L4b}\rvert < 0.055$		$\lvert V_{tb'}\rvert < 0.055$
Z decay	$\sum_{q=d,s,b}\lvert V_{L4q}\rvert^2 < 0.0047$	$\lvert V_{L4s,b}\rvert^2 < 0.003$	
EW obs.	$\lvert V_{L4d}\rvert < 0.084$		$\lvert V_{ub'}\rvert < 0.084$

Table 4.4: Limits on the mixing of the first three families with a fourth down-type vectorial family. Here $(\delta^d_{Li} - \delta^d_{Lj}) = \delta^d_{Lij}$.

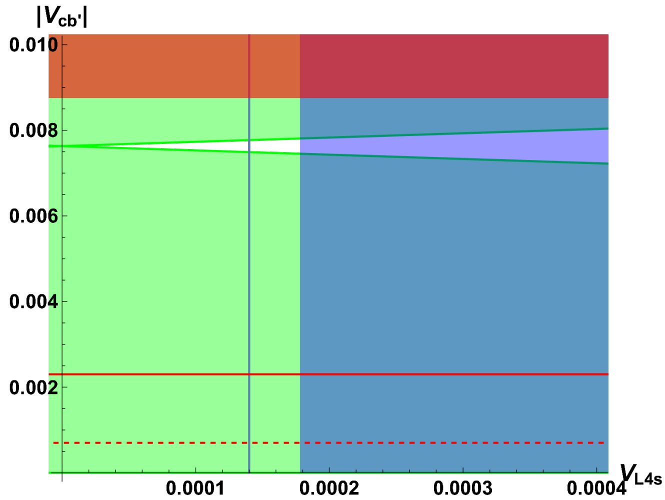

Figure 4.5: Constraints on $|V_{cb'}|$ and $|V_{L4s}|$. $|V_{L4d}| = 0.035$ is set as a benchmark value. The red area is excluded by the constraints on $|V_{cb'}|$ from D^0 systems by taking a conservative limit, that is the new contribution to D^0 mass difference stay within $1.96\sigma_{\rm exp}$ of the experimental value $\Delta m_{D\,\rm exp} = (6.25 \pm 2.6) \cdot 10^{-15}$GeV. The dashed red line shows where the boundary would be if the SM short distance expectation is taken as the limit. The continuous red line stands where the boundary is shifted if the mass of the extra quark is taken as $M_d = 6$ TeV. The blue area is excluded by K decays, using the result $\mathrm{Br}(K_S \to \mu^+\mu^-) < 1.0 \cdot 10^{-9}$ at 95% CL [148]. If the result $\mathrm{Br}(K_S \to \mu^+\mu^-) < 2.4 \cdot 10^{-10}$ at 95% CL [147] is used instead, then the boundary is shifted to the blue line. The green region is excluded by the relation in Eq. (4.215), for any value of the phase δ^d_{L1}.

It should be discussed in which range of couplings and masses the large mixing can be obtained. The LHC limit on extra b' mass $M > 1530$ GeV [111] implies that $|V_{ub'}| \simeq 0.04$ can be obtained if $|h_{d1}| > 0.35$, much larger than the Yukawa constant of the bottom quark y^d_3. In turn, by taking $|V_{ub'}| > 0.03$ in $M_d = |h_{d1}|v_w/|V_{ub'}|$, and assuming (for the perturbativity) $|h_{d1}| < 1$, an upper limit on the extra quark mass is obtained, $M_d < 6\,\text{TeV}$ or so.

In Table 4.4 the relevant constraints extracted in this section are listed.

Limits from flavour changing kaon decays are summarized in Fig. 4.4.

If the limit on $K_S \to \mu^+\mu^-$ from Ref. [147] is used, the result is that the product $|V_{L4d}V^*_{L4s}|$ cannot exceed the limit:

$$|V_{L4d}V^*_{L4s}| \lesssim 4.9 \cdot 10^{-6} \tag{4.211}$$

which is obtained for a relative phase $\sin\delta^d_{L21} \approx 0.45$. This means

$$|V_{L4s}| \approx s^d_{L2} < 1.4 \cdot 10^{-4} \tag{4.212}$$

with $|V_{L4d}| = 0.035$.

By using the result $\mathrm{Br}(K_\mathrm{S} \to \mu^+\mu^-) < 1.0 \cdot 10^{-9}$ at 95% CL [148] adopted by PDG [111],

$$|V_{L4d}V^*_{L4s}| \lesssim 6.2 \cdot 10^{-6} \tag{4.213}$$

which is obtained for a relative phase $\delta^d_{L21} = \pi/4$, which leads

$$|V_{L4s}| \approx s^d_{L2} < 1.8 \cdot 10^{-4} \tag{4.214}$$

with $|V_{L4d}| = 0.035$.

From (4.25):

$$V_{cb'} \simeq -s^d_{L2} - \tilde{s}^d_{L1}V_{cd} \tag{4.215}$$

There is no much room to accommodate this relation without contradicting experimental constraints, since this model barely account for $|V_{L4d}| = 0.035$ without contradicting the constraint obtained on D^0 mixing and flavour changing kaon decays. In Fig. 4.5, $|V_{L4d}| = 0.035$ is set. The green region is excluded by the relation in Eq. (4.215), for any value of the phase δ^d_{L1}. A narrow allowed region (including $V_{L4s} = 0$) can be found by allowing the new contribution to the mass difference in D^0 mesons system to be higher than the SM one, but less than the experimental value of $\Delta m_D + 1.96\sigma_{\mathrm{exp}}$. In this way the extra singlet quark can be thought as a way to account for the mass difference in neutral D-mesons system. Moreover, as can be inferred from Fig. 4.5, the mass of the extra quark cannot exceed few TeV.

It is worth underline that the mixing of b' with both c-quark and t-quark should be at least four times smaller than the mixing with the u-quark $|V_{ub'}| \sim 0.04$. Moreover $|V_{ub'}| \sim 0.04$ is comparable to $|V_{cb}|$ and ten times larger than $|V_{ub}|$. Although it may seem unnatural to expect a larger mixing of the 4th state with the lighter family than with the heavier, still it cannot be excluded.

4.2 Extra up-type quark

The case of the addition of a fourth up-type vector-like isosinglet couple of quarks (u_{4L}, u_{4R}) is now examined. Anagolously to the just examined case, the additional piece of the Yukawa Lagrangian is:

$$h^u_i\tilde{\varphi}\overline{q_{Li}}u_{4R} + M_u\overline{u_{4L}}u_{4R} + \mathrm{h.c.} \tag{4.216}$$

with $i = 1, 2, 3$ and $\tilde{\varphi} = i\tau_2\varphi^*$. Then the up-type quarks mass matrix looks like:

$$\overline{u_{Li}}\mathbf{m}^{(u)}_{ij}u_{Rj} + \text{h.c.} = \tag{4.217}$$

$$=(\overline{q_{1L}}, \overline{q_{2L}}, \overline{q_{3L}}, \overline{u_{4L}}) \left(\begin{array}{ccc|c} & & & h_{u1}v_w \\ & \mathbf{m}^{(u)}_{3\times 3} & & h_{u2}v_w \\ & & & h_{u3}v_v \\ \hline 0 & 0 & 0 & M_{4u} \end{array} \right) \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}_R + \text{h.c.} \tag{4.218}$$

The mass matrix $\mathbf{m}^{(u)}$ can be diagonalized with positive eigenvalues by a biunitary transformation:

$$V_L^{(u)\dagger}\mathbf{m}^{(u)}V_R^{(u)} = \mathbf{m}^{(u)}_{\text{diag}} = \text{diag}(y_1^u v_w, y_2^u v_w, y_3^u v_w, M_u) \tag{4.219}$$

where $V^{(u)}_{L,R}$ are two unitary 4×4 matrices. $\mathbf{m}^{(u)}_{\text{diag}}$ is the diagonal matrix of mass eigenvalues $m_{u,c,t} = y^u_{1,2,3}v_w$ and $M_u \approx M_{4u}$. For the mixing in the left-handed sector we have:

$$V_L^{(u)} = \begin{pmatrix} V_{1u} & V_{1c} & V_{1t} & V_{1t'} \\ V_{2u} & V_{2c} & V_{2t} & V_{2t'} \\ V_{3u} & V_{3c} & V_{3t} & V_{3t'} \\ V_{4u} & V_{4c} & V_{4t} & V_{4t'} \end{pmatrix}_L \tag{4.220}$$

The mass eigenstates are:

$$\begin{pmatrix} u \\ c \\ t \\ t' \end{pmatrix}_L = V_L^{(u)\dagger} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}_L \tag{4.221}$$

while the three up-type quarks involved in charged weak interactions expressed in terms of mass eigenstates are:

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \tilde{V}_L^{(u)} \begin{pmatrix} u \\ c \\ t \\ t' \end{pmatrix}_L \tag{4.222}$$

where $\tilde{V}_L^{(u)}$ is the 3×4 submatrix of $V_L^{(u)}$ without the last row. The Lagrangian for the charged current interaction is:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L} \end{pmatrix} \gamma^\mu \left(\begin{array}{ccc} & \mathbf{V}_{\rm CKM} & \\ \hline V_{t'd} & V_{t's} & V_{t'b} \end{array} \right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \text{h.c.} = \tag{4.223}$$

$$= \frac{g}{\sqrt{2}} \overline{\mathbf{u}_L} \gamma^\mu \tilde{V}_{\rm CKM} \mathbf{d}_L W_\mu^+ + \text{h.c.} \tag{4.224}$$

where

$$\tilde{V}_{\rm CKM} = \tilde{V}_L^{(u)\dagger} V_L^{(d)} \tag{4.225}$$

is a 4×3 matrix, $V_L^{(d)\dagger}$ being the unitary 3×3 matrix diagonalizing the down-type quark mass matrix from the left. Again $\tilde{V}_{\rm CKM}$ is not unitary, thus going towards agreement with the violation of unitarity pointed out in Ref. [46]. In fact $\tilde{V}_L^{(u)} \tilde{V}_L^{(u)\dagger} = \mathbf{1}$ but

$$\tilde{V}_{\rm CKM} \tilde{V}_{\rm CKM}^\dagger = \tilde{V}_L^{(u)\dagger} \tilde{V}_L^{(u)} \neq \mathbf{1} \tag{4.226}$$

In particular, for the first row it holds that

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = [\tilde{V}_{\rm CKM} \tilde{V}_{\rm CKM}^\dagger]_{11} = [\tilde{V}_L^{(u)\dagger} \tilde{V}_L^{(u)}]_{11} = 1 - |V_{L4u}|^2 \tag{4.227}$$

Then with $|V_{L4u}| \simeq 0.04$ the same effect on unitarity as $|V_{ub'}| = 0.04$ in Ref. [46] is obtained. Then the elements of the fourth row of $\tilde{V}_{\rm CKM}$ determine the strenght of the violation of CKM unitarity.

The weak neutral current for up quarks lagrangian reads:

$$\mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W} \left[\frac{1}{2} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L} \end{pmatrix} \gamma^\mu \tilde{V}_L^{(u)\dagger} \tilde{V}_L^{(u)} \begin{pmatrix} u \\ c \\ t \\ t' \end{pmatrix}_L \right.$$
$$\left. - \frac{2}{3} \sin^2\theta_W \left(\overline{\mathbf{u}_L} \gamma^\mu \mathbf{u}_L + \overline{\mathbf{u}_R} \gamma^\mu \mathbf{u}_R \right) \right] Z_\mu \tag{4.228}$$

where $\mathbf{u}$ is the column vector of u, c, t. So the non-unitarity of $\tilde{V}_L^{(u)}$ is at the origin of non-digonal couplings with Z boson, which means flavor changing neutral currents (FCNC) at tree level. The strenght of the coupling of flavor changing interactions is determined by the elements of the matrix:

$$\begin{aligned} V_{\rm nc}^{(u)} &= \tilde{V}_L^{(u)\dagger}\tilde{V}_L^{(u)} \\ &= \begin{pmatrix} 1-|V_{4u}|^2 & -V_{4u}^*V_{4c} & -V_{4u}^*V_{4t} & -V_{4u}^*V_{4t'} \\ -V_{4c}^*V_{4u} & 1-|V_{4c}|^2 & -V_{4c}^*V_{4t} & -V_{4c}^*V_{4t'} \\ -V_{4t}^*V_{4u} & -V_{4t}^*V_{4c} & 1-|V_{4t}|^2 & -V_{4t}^*V_{4t'} \\ -V_{4t'}^*V_{4u} & -V_{4t'}^*V_{4c} & -V_{4t'}^*V_{4t} & 1-|V_{4t'}|^2 \end{pmatrix}_L \end{aligned} \tag{4.229}$$

The elements of $V_L^{(d)}$ and $V_L^{(u)}$ can be naturally chosen with small mixings, that is $V_{\rm CKM}$ is not obtained after large cancellations in their product. Then $\tilde{V}_{\rm CKM}$ can be parameterized as:

$$\tilde{V}_{\rm CKM} \simeq \begin{pmatrix} c_{L1}^u & 0 & 0 \\ -\tilde{s}_{L2}^{u*}\tilde{s}_{L1}^u & c_{L2}^u & 0 \\ -\tilde{s}_{L1}^u\tilde{s}_{L3}^{u*} & -\tilde{s}_{L2}^u\tilde{s}_{L3}^{u*} & c_{L3}^u \\ \tilde{s}_{L1}^u & \tilde{s}_{L2}^u & \tilde{s}_{L3}^u \end{pmatrix} \cdot \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \tag{4.230}$$

$$\simeq \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ \tilde{s}_{L1}^u & \tilde{s}_{L2}^u V_{cs} + \tilde{s}_{L1}^u V_{us} & \tilde{s}_{L3}^u \end{pmatrix} \tag{4.231}$$

with:

$$V_{L4u} \approx -\tilde{s}_{L1}^{u*}\,, \qquad V_{L4c} \approx -\tilde{s}_{L2}^{u*}\,, \qquad V_{L4t} = -\tilde{s}_{L3}^{u*} \tag{4.232}$$

$\tilde{s}_{Li}^u, c_{Li}^u$ are complex sines and cosines (one of which can be made real) of angles in the 1 4, 2 4, 3 4 family planes parameterizing the mixing of the first three families with the fourth family:

$$\tilde{s}_{Li}^u = \sin\theta_{Li4}^u e^{-i\delta_{Li}^u} = s_{Li}^u e^{-i\delta_{Li}^u} \tag{4.233}$$

where $s_{L1}^u \sim |h_{u1}|v_w/M_u$, $s_{L2}^u \sim |h_{u2}|v_w/M_u$, $s_{L3}^u \sim |h_{u3}|v_w/M_u$. In this parameterization the 3×3 submatrix of $\tilde{V}_{\rm CKM}$ contains 3 angles and one phase. The elements of the fourth row of $\tilde{V}_{\rm CKM}$ in (4.12) are then parameterized as:

$$\begin{aligned} V_{t'd} &\simeq \tilde{s}_{L1}^u \\ V_{t's} &\simeq s_{L2}^u V_{cs} + \tilde{s}_{L1}^u V_{us} \\ V_{t'b} &\simeq \tilde{s}_{L3}^u \end{aligned} \tag{4.234}$$

where the phase of $\tilde{s}_{L2}$ has been absorbed.

The LHC limit on extra t' mass $M > 1160$ GeV [111] implies that $|V_{t'd}| \simeq 0.04$ can be obtained if $|h_1^u| \gtrsim 0.25$, much larger than the Yukawa constant of the bottom quark. In turn, by taking $|V_{L4u}| > 0.03$ in $M_u = |h_{u1}|v_w/|V_{t'd}|$, and assuming (for the perturbativity) $h_{u1} \lesssim 1$, there is an upper limit on the extra quark mass, $M_u < 6$ TeV or so.

4.2.1 D decays

Mixings of the standard quarks with the additional vector-like singlet up-quark t' can be constrained by D-mesons rare decays. The additional contribution to the effective lagrangian is:

$$\mathcal{L}_{\rm new} = 4\frac{G_F}{\sqrt{2}}V_{4u}^* V_{4c}(\overline{u_L}\gamma^\mu c_L)\left[(-\frac{1}{2}+\sin^2\theta_W)(\overline{\ell_L}\gamma_\mu\ell_L)+\sin^2\theta_W(\overline{\ell_R}\gamma_\mu\ell_R)\right]+\text{h.c.} \tag{4.235}$$

which contributes to semileptonic decays of both neutral and charged D-mesons. The most stringent constraint from experimental limits on semileptonic neutral mesons decays may come from the limit ${\rm Br}(D^0 \to \pi^0 e^+e^-) < 4\cdot 10^{-6}$ at 90% C.L. [111]:

$$\begin{aligned}{\rm Br}(D^0 \to \pi^0 e^+e^-)_{\rm new} &\simeq {\rm Br}(D^0\to\pi^- e^+\nu_e)\frac{1}{2}\frac{|V_{L4u}^* V_{L4c}|^2[(-\frac{1}{2}+\sin^2\theta_W)^2+\sin^4\theta_W]}{|V_{cd}|^2} \\ &< 4\cdot 10^{-6}\end{aligned} \tag{4.236}$$

from which

$$|V_{L4u}^* V_{L4c}| < 0.03 \tag{4.237}$$

where $|V_{cd}| = 0.218\pm 0.004$ and we take advantage of the experimental branching ratio ${\rm Br}(D^0\to\pi^- e^+\nu_e) = (2.91\pm 0.04)\cdot 10^{-3}$ [111].

As regards charged mesons decays, we can consider the limit ${\rm Br}(D^+\to\pi^+\mu^+\mu^-) < 7.3\cdot 10^{-8}$ at 90% C.L. [111]:

$$\begin{aligned}{\rm Br}(D^+ \to \pi^+\mu^+\mu^-)_{\rm new} &\simeq {\rm Br}(D^+\to\pi^0 \mu^+\nu_\mu)\frac{2|V_{L4u}^* V_{L4c}|^2[(-\frac{1}{2}+\sin^2\theta_W)^2+\sin^4\theta_W]}{|V_{cd}|^2} \\ &< 7.3\cdot 10^{-8}\end{aligned} \tag{4.238}$$

from which

$$|V_{L4u}^* V_{L4c}| < 0.002 \tag{4.239}$$

where we take advantage of the experimental branching ratio $\mathrm{Br}(D^+ \to \pi^0 \mu^+ \nu_\mu) = (3.50 \pm 0.15) \cdot 10^{-3}$ [111].

The effective Lagrangian (4.235) also contributes to the decay $D^0 \to \mu^+\mu^-$. The decay rate generated by this porcess alone (without considering interference) would be:

$$\Gamma_{\mathrm{new}} = \frac{1}{16\pi} G_F^2 f_D^2 m_\mu^2 M_{D^0} \left(1 - \frac{4m_\mu^2}{M_{D^0}^2}\right)^{\frac{1}{2}} |V_{L4u}^* V_{L4c}|^2 \tag{4.240}$$

We can impose on this contribution the experimental limit $\mathrm{Br}(D^0 \to \mu^+\mu^-) < 6.2 \cdot 10^{-9}$ [111]. Making use of the decay $D^+ \to \mu^+\nu_\mu$:

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} V_{cd}^* (\bar{d}_L \gamma_\alpha c_L)(\bar{\nu}_\mu \gamma^\alpha \mu_L) \tag{4.241}$$

$$\Gamma = \frac{1}{8\pi} G_F^2 |V_{cd}|^2 f_D^2 m_\mu^2 M_{D^+} \left(1 - \frac{m_\mu^2}{M_{D^+}^2}\right)^2 \tag{4.242}$$

it is obtained that:

$$\frac{\mathrm{Br}(D^0 \to \mu^+\mu^-)_{\mathrm{new}}}{\mathrm{Br}(D^+ \to \mu^+\nu_\mu)} = \frac{1}{2} \frac{\tau(D^0) M_{D^0} \sqrt{1 - 4\frac{m_\mu^2}{M_{D^0}^2}}}{\tau(D^+) M_{D^+} \left(1 - \frac{m_\mu^2}{M_{D^+}^2}\right)^2} \frac{|V_{L4u}^* V_{L4c}|^2}{|V_{cd}|^2} \tag{4.243}$$

It is obtained that:

$$|V_{L4u}^* V_{L4c}| \approx s_{L2}^u s_{L1}^u < 0.0020 \tag{4.244}$$

$$|V_{L4c}| < 0.05 \tag{4.245}$$

using the values in Table 4.1.

4.2.2 D_s decays

The effective Lagrangian (4.235) also contributes to the decay $D_s^\pm \to K^\pm \ell^+ \ell^-$. The most stringent limit comes from the experimental limit $\mathrm{Br}(D^+ \to K^+ e^+ e^-)_{\mathrm{exp}} < 3.7 \cdot 10^{-6}$ [111] 90% C.L.. By using $\mathrm{Br}(D^+ \to K^0 e^+ \nu_e) = (3.9 \pm 0.9) \cdot 10^{-3}$ [111]:

$$\begin{aligned} &\mathrm{Br}(D^+ \to K^+ e^+ e^-)_{\mathrm{new}} \simeq \\ &\simeq \mathrm{Br}(D^+ \to K^0 e^+ \nu_e) \frac{|V_{4u}^* V_{4c}|^2 [(-\frac{1}{2} + \sin^2\theta_W)^2 + \sin^4\theta_W]}{|V_{cd}|^2} < 3.7 \cdot 10^{-6} \end{aligned} \tag{4.246}$$

from which

$$|V_{L4u}^* V_{L4c}| < 0.02 \tag{4.247}$$

4.2.3 Neutral mesons systems

D^0-$\bar{D}^0$ mixing

The experimental value of the mass difference in the D^0 mesons system is

$$\Delta m_{D\,\mathrm{exp}} = (6.25 \pm 2.6) \cdot 10^{-15}\mathrm{GeV} \tag{4.248}$$

which however seems to be dominated by long-distance contributions. The SM short-distance contribution due to the s-quark running in the box diagram is of order

$$\Delta m_{D\,\mathrm{SD}} \approx 2|M_{12}| \approx \frac{G_F^2 m_W^2}{6\pi^2} |(V_{us}V_{cs}^*)^2| S_0(x_s) f_D^2 m_D B_D \sim 10^{-16}\mathrm{GeV} \tag{4.249}$$

where $x_s = \frac{m_s^2}{m_W^2}$, $S_0(x_s)$ is the Inami-Lim function, with $f_D = 212.0 \pm 0.7$ MeV [107]. The effective Lagrangian responsible for the new contribution to the mixing of neutral mesons is:

$$\mathcal{L}_{\mathrm{new}} = -\frac{G_F}{\sqrt{2}} (V_{L4u}^* V_{L4c})^2 (\overline{u_L}\gamma^\mu c_L)^2 + \mathrm{h.c.} \tag{4.250}$$

The matrix element can be defined:

$$\langle \bar{D}^0 | (\overline{u_L}\gamma^\mu c_L)^2 | D^0 \rangle = \frac{2}{3} B_D f_D^2 m_D^2 \tag{4.251}$$

where B_D is the correction to the VIA approximation and should be of order unity. Similarly to the case of neutral kaons mixing, we have:

$$2 m_D M_{12}^* = -\langle \bar{D}^0 | \mathcal{L} | D^0 \rangle \tag{4.252}$$

that is

$$\Delta m_{D\,\mathrm{new}} \approx 2|M_{12\,\mathrm{new}}| \simeq \frac{2}{3}\frac{G_F}{\sqrt{2}} |(V_{L4u}^* V_{L4c})^2| f_D^2 m_D \tag{4.253}$$

In order to make the new contribution less than the SM short-distance contribution, it should be that:

$$|(V_{L4u}^* V_{L4c})^2| < \frac{\sqrt{2} G_F m_W^2 S_0(x_s)}{4\pi^2} \left|(V_{us}V_{cs}^*)^2\right| \tag{4.254}$$

which gives:

$$|V_{L4u}^* V_{L4c}| < 1.4 \cdot 10^{-5} \tag{4.255}$$
$$|V_{L4c}| < 3.4 \cdot 10^{-4} \tag{4.256}$$

with $|V_{L4u}| = 0.04$. In a more conservative way we can set

$$\Delta m_{D\,\mathrm{new}} < \Delta m_{D\,\mathrm{exp}} \tag{4.257}$$

leading to:

$$|V^*_{L4u} V_{L4c}| \approx s^u_{L1} s^u_{L2} < 1.2 \cdot 10^{-4} \tag{4.258}$$
$$|V_{L4c}| \approx s^u_{L2} < 0.003 \tag{4.259}$$

then $|V_{t's}| \simeq s^u_{L2} V_{cs} + \tilde{s}^u_{L1} V_{us} \sim 0.01$.

K^0-$\bar{K}^0$ mixing

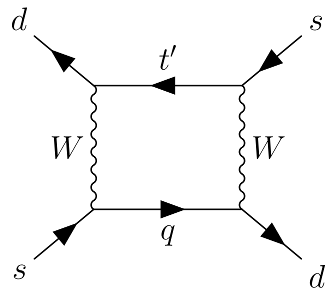

Figure 4.6: New contribution to $K^0 - \bar{K^0}$ mixing, $q = u, c, t, t'$.

The new up-quark contributes to the box diagram originating neutral kaons mixing:

$$\mathcal{L}_{\mathrm{new}} = -\frac{G_F^2 m_W^2}{4\pi^2} \left((V^*_{t's} V_{t'd})^2 S(x_{t'}) + 2(V^*_{t's} V_{t'd})(V^*_{cs} V_{cd}) S(x_c, x_{t'}) + 2(V^*_{t's} V_{t'd})(V^*_{ts} V_{td}) S(x_t, x_{t'}) \right) \tag{4.260}$$
$$\cdot \, (\overline{s_L} \gamma^\mu d_L)(\overline{s_L} \gamma^\mu d_L) + h.c.$$

where $S_0(x_a)$ are the Inami-Lim functions [149], with:

$$S_0(x_j, x_k) = x_j x_k \Bigg[\left(\frac{1}{4} - \frac{3}{2(x_j - 1)} - \frac{3}{4(x_j - 1)^2} \right) \frac{\log x_j}{x_j - x_k} + \left(\frac{1}{4} - \frac{3}{2(x_k - 1)} - \frac{3}{4(x_k - 1)^2} \right) \frac{\log x_k}{x_k - x_j} - \frac{3}{4(x_j - 1)(x_k - 1)} \Bigg] \tag{4.261}$$

Remembering that

$$\Delta m_K \approx 2|M_{12}| \tag{4.262}$$

$$M_{12}^* = -\frac{1}{2m_K}\langle \bar{K}^0|\mathcal{L}|K^0\rangle \tag{4.263}$$

if $|V_{t's}^* V_{t'd}| = 0.0004$, making the hypothesis that the QCD correction for the three terms in (4.261) is $\eta_{t'} = 0.5 \sim \eta_2$, with $M_u = 1160$ GeV, the three contributions to Δm_K coming from the Lagrangian in (4.261) are (in the same order) respectively $5.9 \cdot 10^{-16}$, $2.9 \cdot 10^{-17}$, $1.0 \cdot 10^{-16}$ GeV, which is already less than the SM contribution. In fact, choosing $M_u = 1160$ GeV, in order to make an estimation of the constraint on new mixings, the following inequality can be considered:

$$|V_{t's}^* V_{t'd}| < \sqrt{\frac{\eta_1 S(x_c)|(V_{cs}^* V_{cd})^2|}{\eta_2 S(x_{t'})}} = 0.00082 \tag{4.264}$$

$$|V_{t's}| < 0.02 \tag{4.265}$$

with $V_{t'd}| = 0.04$, under the hypothesis that the value of η_2 is similar to the magnitude of QCD corrections in the case with t running in the loop.

In the Standard Model the CP violation is parameterized by ϵ_K:

$$\epsilon_K \approx \frac{\mathrm{Im} M_{12}}{\sqrt{2}\Delta m_K} e^{-i\frac{\pi}{4}} \tag{4.266}$$

Mixing with t' also gives a CP-violating contribution:

$$\begin{aligned}\mathrm{Im}M_{12} \simeq \frac{G_F^2 m_W^2}{6\pi^2} f_K^2 m_K B_K \cdot \\ \cdot \left[\mathrm{Re}(V_{t's}V_{t'd}^*)S_0(x_{t'}) + \mathrm{Re}(V_{cs}V_{cd}^*)S_0(x_c,x_{t'}) + \mathrm{Re}(V_{ts}V_{td}^*)S_0(x_t,x_{t'})\right] \mathrm{Im}(V_{t's}V_{t'd}^*) + \\ + \mathrm{Re}(V_{t's}V_{t'd}^*)S_0(x_t,x_{t'})\mathrm{Im}(V_{ts}V_{td}^*)\end{aligned} \tag{4.267}$$

Since the sign of each contribution is unknown, in order to obtain constraints on the elements in the fourth column of V_{CKM}, each term of (4.267) can be required to be smaller than the SM short-distance contribution. Defining:

$$\mathcal{C}_{\mathrm{SM}} = \left[\mathrm{Re}(V_{ts}^* V_{td})\, S_0(x_t) + \mathrm{Re}(V_{cs}^* V_{cd})\, S_0(x_c,x_t)\right] \mathrm{Im}(V_{ts}^* V_{td}) \tag{4.268}$$

then

$$|\mathrm{Im}(V_{t's}V_{t'd}^*) \cdot \mathrm{Re}(V_{ts}V_{td}^*)S_0(x_t,x_{t'})| < |\mathcal{C}_{\mathrm{SM}}| \tag{4.269}$$

$$|\mathrm{Im}(V_{t's}V_{t'd}^*) \cdot \mathrm{Re}(V_{cs}V_{cd}^*)S_0(x_c,x_{t'})| < |\mathcal{C}_{\mathrm{SM}}| \tag{4.270}$$

$$|\mathrm{Re}(V_{t's}V_{t'd}^*) \cdot \mathrm{Im}(V_{ts}V_{td}^*)S_0(x_t,x_{t'})| < |\mathcal{C}_{\mathrm{SM}}| \tag{4.271}$$

$$|\mathrm{Im}(V_{t's}V_{t'd}^*) \cdot \mathrm{Re}(V_{t's}V_{t'd}^*)S_0(x_{t'})| < |\mathcal{C}_{\mathrm{SM}}| \tag{4.272}$$

from which:

$$|\mathrm{Im}(V_{t's}V^*_{t'd})| < 2.3 \cdot 10^{-5} \tag{4.273}$$
$$|\mathrm{Im}(V_{t's}V^*_{t'd})| < 8.4 \cdot 10^{-5} \tag{4.274}$$
$$|\mathrm{Re}(V_{t's}V^*_{t'd})| < 5.4 \cdot 10^{-5} \tag{4.275}$$
$$|\mathrm{Im}(V_{t's}V^*_{t'd})\mathrm{Re}(V_{t's}V^*_{t'd})| < 1.1 \cdot 10^{-9} \tag{4.276}$$

with $M_u = 1$ TeV.

$B^0_{s,d}$-$\bar{B}^0_{s,d}$ mixing

The introduction of t' also affects neutral B-mesons systems:

$$-\frac{G_F^2 m_W^2}{4\pi^2}\left((V^*_{t'b}V_{t'd})^2 S(x_{t'}) + 2(V^*_{t'b}V_{t'd})(V^*_{cb}V_{cd})S(x_c, x_{t'}) + 2(V^*_{t'b}V_{t'd})(V^*_{tb}V_{td})S(x_t, x_{t'})\right) \cdot \tag{4.277}$$
$$\cdot\, (\overline{b_L}\gamma^\mu d_L)(\overline{b_L}\gamma^\mu d_L) + h.c.$$

where $S(x_a)$ are the complete Inami-Lim functions [149] and for B_s it's the same with the substitution $d \to s$. The c-quark contribution is negligible, the other two contributions in principle can be of the same order of magnitude. Choosing $M_u = 1160$ GeV and comparing with the SM short-distance contribution in order to make an estimation, the following inequalities can be considered:

$$|V^*_{t'b}V_{t'd}| < \sqrt{\frac{S(x_t)|(V^*_{tb}V_{td})^2|}{S(x_{t'})}} \sim 0.002 \tag{4.278}$$
$$|V^*_{t'b}V_{t's}| < \sqrt{\frac{S(x_t)|(V^*_{tb}V_{ts})^2|}{S(x_{t'})}} \sim 0.009 \tag{4.279}$$

4.2.4 Z-boson physics

Analogously to the case of the down-type isosinglet, the presence of the additional up-isosinglet quark affects both off-diagonal couplings and diagonal couplings of Z-boson with quarks, changing the prediction of many observables related to the Z boson physics.

The variation of the diagonal couplings of weak neutral-current interaction also modifies interactions of quarks with leptons, modifying the expectation of low energy ($Q^2 << M_Z^2$) electroweak precision observables.

Experimental values and SM predictions for Z pole quantities are given in averages and global fit results of Particle Data Group [111], as reported in Table 4.3.

$Z \to c\bar{c}$

By using the values in [111] it is obtained that:

$$\Gamma(Z \to c\bar{c})_{\rm SM} = R_{c,\,\rm SM}\,\Gamma(\text{had})_{\rm SM} = 0.29984 \pm 0.00015\,\text{GeV} \tag{4.280}$$

$$\Gamma(Z \to c\bar{c})_{\rm exp} = \text{Br}(Z \to c\bar{c})_{\rm exp}\Gamma(Z)_{\rm exp} = 0.3002 \pm 0.0052\,\text{GeV} \tag{4.281}$$

The SM decay rate $\Gamma(Z \to c\bar{c})$ at tree level is given by:

$$\Gamma(Z \to c\bar{c})_{0,\rm SM} = \frac{G_F M_Z^3}{\sqrt{2}\pi}\left[\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)^2 + \left(\frac{2}{3}\sin^2\theta_W\right)^2\right] \tag{4.282}$$

By inserting a fourth up-quark, the decay rate changes in:

$$\Gamma(Z \to c\bar{c})_{0,\rm new} = \frac{G_F M_Z^3}{\sqrt{2}\pi}\left[\left(\frac{1}{2}(1-|V_{L4c}|^2) - \frac{2}{3}\sin^2\theta_W\right)^2 + \left(\frac{2}{3}\sin^2\theta_W\right)^2\right] \tag{4.283}$$

so the prediction for the decay rate is lowered. Then a limit can be set on $|V_{L4c}|$ by constraining the difference between the SM contribution (4.282) and the new contribution (4.283) to be less than the experimental error:

$$|\Gamma(Z \to c\bar{c})_{\rm new} - \Gamma(Z \to c\bar{c})_{\rm SM}| = \frac{G_F M_Z^3}{\sqrt{2}\pi}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)|V_{L4c}|^2 < 0.0052\,\text{GeV} \tag{4.284}$$

It is obtained that:

$$|V_{L4c}| \approx s^u_{L2} < 0.09 \tag{4.285}$$

or at 95% C.L. $|V_{L4c}| < 0.12$.

Z decay rate

In this model the Z decay rate would be changed with respect to the SM expectation by:

$$
\begin{aligned}
&|\Gamma(Z)_{\rm new}-\Gamma(Z)_{\rm SM}| = |\Gamma({\rm had})_{\rm new}-\Gamma({\rm had})_{\rm SM}| = \\
=&\frac{G_F M_Z^3}{\sqrt{2}\pi}\left|\sum_{q,q'=u,c}\left|\frac{1}{2}\sum_{k=1}^{3}V_{Lkq}^{(u)*}V_{Lkq'}^{(u)}-\frac{2}{3}\sin^2\theta_W\delta_{qq'}\right|^2-2\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)^2\right| = \\
=&\frac{G_F M_Z^3}{\sqrt{2}\pi}\left|\sum_{q=u,c}\left(\frac{1}{2}(1-|V_{L4q}^{(u)}|^2)-\frac{2}{3}\sin^2\theta_W\right)^2+\frac{1}{2}|V_{L4u}^*V_{L4c}|^2-2\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)^2\right| \simeq \\
\simeq&\frac{G_F M_Z^3}{\sqrt{2}\pi}\left(\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)\left(|V_{L4u}|^2+|V_{L4c}|^2\right)+\frac{1}{2}|V_{L4u}^*V_{L4c}|^2\right) \qquad (4.286)
\end{aligned}
$$

As done in the previous case, using the data in Table 4.3, We can impose that this deviation is less than the experimental error, then:

$$
\frac{G_F M_Z^3}{\sqrt{2}\pi}\left(\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)\left(|V_{L4u}|^2+|V_{L4c}|^2\right)+\frac{1}{2}|V_{L4u}^*V_{L4c}|^2\right) < 0.0020\,\mathrm{GeV} \qquad (4.287)
$$

The constraints in Table 4.5 imply that $|V_{L4u}^*V_{L4c}|$ is not giving the relevant contribution, that is

$$
|V_{L4u}|^2+|V_{L4c}|^2 < 0.003 \qquad (4.288)
$$

or

$$
|V_{L4u}|^2+|V_{L4c}|^2 < 0.006 \qquad (4.289)
$$

to stay within $1.96\sigma_{\rm exp}$. which is satisfied if both $|V_{L4u,c}| \lesssim 0.04$. With $V_{L4c}=0$, the constraint (4.287) implies $|V_{L4u}|<0.054$, or it should be that $|V_{L4u}|<0.075$ at 95% C.L..

Similarly a limit can be exctracted by the results on the decays of Z into up-quarks:

$$
\begin{aligned}
\Gamma(Z\to(u\bar{u}+c\bar{c})/2)_{\rm SM} &= 0.29987\pm0.00019\,\mathrm{GeV} \qquad (4.290)\\
\Gamma(Z\to(u\bar{u}+c\bar{c})/2)_{\rm exp} &= 0.289\pm0.015\,\mathrm{GeV} \qquad (4.291)
\end{aligned}
$$

where $\Gamma(Z\to u\bar{u})_{\rm SM} = 0.29991\pm0.00018$ GeV and ${\rm Br}(Z\to(u\bar{u}+c\bar{c})/2)_{\rm exp} = 0.116\pm0.006$ [111] have been used. Then also in this case the experimental error may be used to set the limit on the discrepancy between the SM prediction and the expected width after the new contribution is included:

$$\frac{1}{2}\frac{G_F M_Z^3}{\sqrt{2}\pi}\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)\left(|V_{L4u}|^2+|V_{L4c}|^2\right) < 0.015\,\mathrm{GeV}$$
$$|V_{L4u}|^2+|V_{L4c}|^2 < 0.02 \qquad (4.292)$$

Z-pole asymmetries

The insertion of the extra isosinglet also changes the diagonal couplings of weak neutral-current interaction, as follows from Eqs. (4.228) and (4.229). In terms of the axial-vector and vector couplings, as already stated in the previous section, the expectation for Z-pole asymmetries are hence affected.

Regarding A_c, the experimental value $A_c = 0.670 \pm 0.027$ is in agreement with the SM prediction $A_c = 0.6677 \pm 0.0001$. By extracting A_c from $A_{FB}^{(0,c)} = 0.0707 \pm 0.0035$ using $A_e = 0.1501 \pm 0.0016$, it is obtained the result $A_c = 0.628 \pm 0.032$. The average between the two values is $A_c = 0.653 \pm 0.021$.

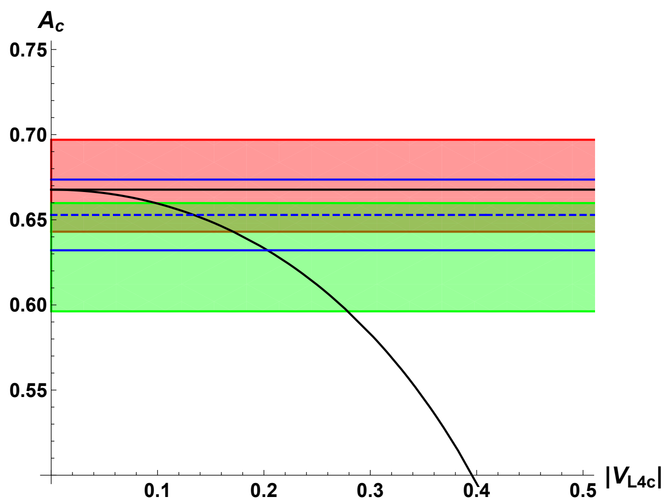

Figure 4.7: Determination of A_c obtained from Eq. (4.202) as a function of V_{L4c} (black curve). Ther red band is the experimental value $A_c = 0.670 \pm 0.027$, the green band shows the value obtained from the experimental determination $A_c = 0.628 \pm 0.032$ $A_{FB}^{(0,c)} = 0.0707 \pm 0.0035$. The blue lines correspond to the average of the two $A_c = 0.653 \pm 0.021$. The black straight line is the SM prediction $A_c = 0.6677$.

The presence of an additional up-type isosinglet changes the couplings as in Eq. (4.199), and consequently the predictions for the asymmetries according to

Eq. (4.202).

The SM prediction $A_c = 0.6677 \pm 0.0001$ reported by PDG [111] corresponds to an effective angle $\bar{s}_c^2 = 0.231465$. This value can be used in Eq. (4.202) fro $q = c$. As can be seen in Fig. 4.7, the expected value of A_c remains in the 1σ interval of the average $A_c = 0.653 \pm 0.021$ when $V_{R4c} < 0.20$, or at 95% C.L.

$$V_{R4c} < 0.25 \tag{4.293}$$

and still in one errorbar of the A_c value extracted from $A_{FB}^{(0,c)}$ for $V_{R4c} < 0.28$.

4.2.5 Low energy electroweak observables

As already shown, the dependence of the weak charge of the proton and of nuclei in terms of the couplings is changed with Eq. (4.206) by the weak isosinglets. The values of the weak charge of the proton and the nuclear weak charges are shown in Table 4.3. Again, the contribution of the new quark species to the weak charge of cesium is:

$$\Delta Q_W^{55,78}(Cs) = -2[94|V_{L4u}|^2 - 105.5|V_{L4d}|^2](1 - \frac{\alpha}{2\pi}) \tag{4.294}$$

In this second scenario $V_{L4d} = 0$ and the fourth up-type quark is not helping in filling the 1.5σ gap with the experimental result. With $|V_{L4u}| = 0.04$, $\Delta Q_W^{55,78}(Cs) = 0.3$, which is less than the experimental error. $\Delta Q_W^{55,78}(Cs)$ is less than the experimental error with $|V_{L4u}| < 0.05$. The additional contribution to $2g_{AV}^{eu} + g_{AV}^{ed}$ is $|V_{L4u}|^2 - \frac{1}{2}|V_{L4d}|^2$.

$$|V_{L4u}|^2 - \frac{1}{2}|V_{L4d}|^2 < \sigma_{\rm exp} \tag{4.295}$$

where $\sigma_{\rm exp} = 0.0023$. With $V_{L4d} = 0$, $|V_{L4u}| < 0.05$ gives a contribution less than the experimental error. In both cases if $|V_{L4u}| < 0.067$ the new mixings give a contribution less than 1.96 times experimental error.

4.2.6 Discussion

In Table 4.5 the relevant constraints on the mixing of the new up-type vector-like quark with the SM families are summarized.

Constraints from CP violation in K-mesons systems are summarized in Fig. 4.8. The result is that the product $|V_{t'd}^* V_{t's}|$ cannot exceed the limit:

$$|V_{t'd}^* V_{t's}| < 5.8 \cdot 10^{-5} \tag{4.296}$$

	Constraint	Process	$\lvert V_{L4u}\rvert = 0.04$	
$\lvert V^*_{L4u}V_{L4c}\rvert$	$< 1.4 \cdot 10^{-5}$ (th)	$D^0 - \bar{D}^0$	$\lvert V_{L4c}\rvert < 3.4 \cdot 10^{-4}$	$\lvert V_{t's}\rvert \sim 0.01$
	$< 1.2 \cdot 10^{-4}$ (exp)		$\lvert V_{L4c}\rvert < 0.003$	$\lvert V_{t's}\rvert \sim 0.01$
$\lvert V_{L4c}\rvert$	< 0.12	$Z \to c\bar{c}$		
$\lvert V_{L4u}\rvert^2 + \lvert V_{L4c}\rvert^2$	< 0.006	Z decay	$\lvert V_{L4c}\rvert < 0.036$	
$\lvert V_{L4u}\rvert$	< 0.067	EW obs.		
$\lvert V^*_{t'd}V_{t's}\rvert$	$< 8.2 \cdot 10^{-4}$	Δm_K		
$\lvert V^*_{t'd}V_{t's}\rvert\lvert\sin\delta_{ds}\rvert$	$< 2.3 \cdot 10^{-5}$	ϵ_K		$\lvert V_{t's}\rvert < 0.0014$
$\lvert V^*_{t'd}V_{t's}\rvert\lvert\cos\delta_{ds}\rvert$	$< 5.4 \cdot 10^{-5}$	ϵ_K		
$\lvert V^*_{t'd}V_{t's}\rvert\lvert\sin(2\delta_{ds})\rvert^{1/2}$	$< 4.6 \cdot 10^{-5}$	ϵ_K		
$\lvert V^*_{t'b}V_{t'd}\rvert$	< 0.002	$B^0 - \bar{B}^0$	$\lvert V_{L4t}\rvert < 0.05$	$\lvert V_{t'b}\rvert < 0.05$
$\lvert V^*_{t'b}V_{t's}\rvert$	< 0.009	$B^0_s - \bar{B}^0_s$		

Table 4.5: Limits on the mixing of the first three families with a fourth vectorial up-type family. Here δ_{ds} is the phase of the complex number $V^*_{t'd}V_{t's}$.

which is obtained for a relative phase $\sin\delta^d_{ds} \approx 0.34$. This means:

$$|V_{t's}| < 1.6 \cdot 10^{-3} \tag{4.297}$$

with $|V_{t'd}| = 0.035$. However, remembering that in the given parameterization:

$$V_{t's} = s^u_{L2}V_{cs} + \tilde{s}^u_{L1}V_{us} \tag{4.298}$$

then:

$$|V_{t's}| > -s^u_{L2} + |\tilde{s}^u_{L1}|V_{us} \tag{4.299}$$

A less stringent limit can be adopted for the new contribution to the neutral D-mesons mass difference, requiring to be less than $\Delta m_D + 1.96\sigma_{\text{exp}}$, which leads to $|V_{L4c}| < 0.0045$. Remembering that $V_{t'd} \approx \tilde{s}^u_1$ and $|V_{L4c}| \approx s^u_{L2}$

$$|V_{t's}| > 3.4 \cdot 10^{-3} \tag{4.300}$$

which in principle can account for the experimental value of D^0 mass difference, but which is twice the upper value found in (4.297) from K-mesons system, which then can be accommodated only allowing big cancellations in K-mesons systems.

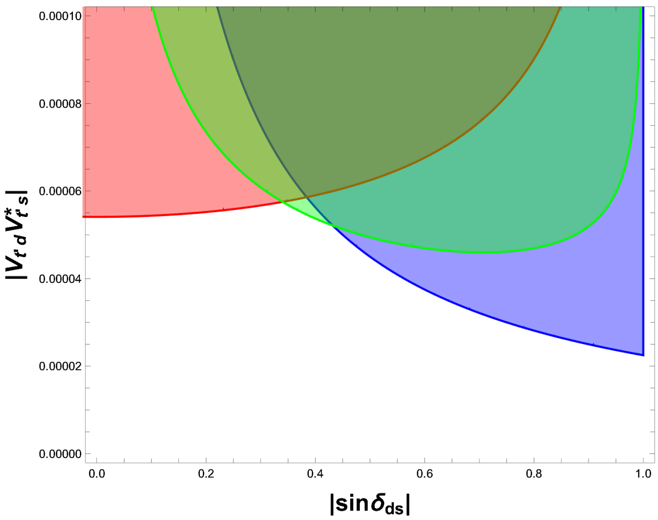

Figure 4.8: Upper limit on the product $|V^*_{t'd}V_{t's}|$ as a function of the relative phase δ_{ds}. Limits come from the new contributions to the CP violating parameter ϵ_K, from the imaginary part of $V_{t's}V^*_{t'd}$ (blue area), the real part (red area) and their product (green area), taken as described in the text.

Besides, it should be underlined that the mixing of t' with the s-quark should be at least four times smaller than the mixing with the d-quark $|V_{t'd}| \sim 0.04$, and the mixing with the third family should be of the same order of the mixing with the first. Moreover $|V_{t'd}| \sim 0.04$ is comparable to $|V_{cb}|$ and ten times larger than $|V_{ub}|$. Again, the scenario with the 4th state having a larger mixing with the lighter family than with the heavier may seem unnatural, but it cannot be excluded.

4.3 Extra isosinglets

Extra down-type and up-type quarks can also exist together, assembling a kind of fourth “family" made of isosinglets vector-like quarks. Then the Yukawa couplings in both (4.4) and (4.216) would be present. The mass matrices of up and down quarks would be the same as in (4.5) and (4.217). The mixing matrices which diagonalize mass matrices arranging left handed particles, are shown in (4.315) and (4.220). Flavour changing neutral currents arise in both sectors, exactly in the same way described in the previous two sections. Charged currents instead in this case would involve four flavours of both up and down type quarks. In fact the Lagrangian for the charged current interaction in this case is:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \left(\begin{array}{cccc} \overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L} \end{array} \right) \gamma^\mu V_L^{(u)\dagger} V_L^{(d)} \left(\begin{array}{c} d \\ s \\ b \\ b' \end{array} \right)_L W_\mu^+ + \text{h.c.} \tag{4.301}$$

and

$$\tilde{V}_{\rm CKM} = V_L^{(u)\dagger} V_L^{(d)} \tag{4.302}$$

is a 4×4 matrix, which can be described by 6 angles and 10 phases, 7 of which can be absorbed into the quark fields. In the same parameterization as in (4.323) and (4.231):

$$\tilde{V}_{\rm CKM} = \left(\begin{array}{cccc} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{array} \right) \simeq \tag{4.303}$$

$$\left(\begin{array}{cccc} & & & -\tilde{s}_1^d V_{ud} - \tilde{s}_1^{u*} \\ & V_{\rm CKM} & & -\tilde{s}_1^d V_{cd} - \tilde{s}_2^d V_{cs} - \tilde{s}_2^{u*} \\ & & & -\tilde{s}_3^d V_{tb} - \tilde{s}_3^{u*} \\ \tilde{s}_1^u V_{ud} + \tilde{s}_1^{d*} & \tilde{s}_1^u V_{us} + \tilde{s}_2^u V_{cs} + \tilde{s}_2^{d*} & \tilde{s}_3^u V_{tb} + \tilde{s}_3^{d*} & 1 \end{array} \right) \tag{4.304}$$

Then

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2 \tag{4.305}$$
$$|V_{ub'}| \approx |\tilde{s}_1^d + \tilde{s}_1^{u*}| \approx 0.04 \tag{4.306}$$
$$V_{t'd} \approx -V_{ub'}^* \tag{4.307}$$
$$V_{t'b} \approx -V_{tb'}^* \tag{4.308}$$

In Table 4.6 the relevant constraints are summarized. In this scenario there is a little more room to arrange mixing elements. Then also with $|V_{ub'}| = 0.04$ in principle both the constraints from kaon physics and at least the soft constraints from D-mesons can be managed to be satisfied. However in this case $V_{t's}$ would be one order of magnitude less than $|V_{ub'}| = 0.04$ and the mixing of b' with c-quark should be more than four times smaller than the mixing with the u-quark, which again seems unnatural but not forbidden.

Process	Constraint	$\lvert V_{ub'}\rvert = 0.04$
K decays	$\lvert V^*_{L4s}V_{L4d}\rvert < 4.8 \cdot 10^{-6}$	
$B^0 \to \mu^+\mu^-$	$\lvert V^*_{L4b}V_{L4d}\rvert < 1.3 \cdot 10^{-4}$	
$B^0_s \to \mu^+\mu^-$	$\lvert V^*_{L4b}V_{L4s}\rvert < 7.4 \cdot 10^{-5}$	
$D^0 - \bar{D}^0$	$\lvert V^*_{L4u}V_{L4c}\rvert < 1.4 \cdot 10^{-5}$ (th)	
$D^0 - \bar{D}^0$	$\lvert V^*_{L4u}V_{L4c}\rvert < 1.2 \cdot 10^{-4}$ (exp)	
$D^0 - \bar{D}^0$	$\lvert V_{ub'}V^*_{cb'}\rvert < 3 \cdot 10^{-5}$	$\lvert V_{cb'}\rvert < 0.0007$ (theo)
	$\lvert V_{ub'}V^*_{cb'}\rvert < 2 \cdot 10^{-4}$	$\lvert V_{cb'}\rvert < 0.006$ (exp)
$K^0 - \bar{K}^0$	$\lvert V^*_{t'd}V_{t's}\rvert < 5.8 \cdot 10^{-5}$	$\lvert V_{t's}\rvert < 1.44 \cdot 10^{-3}$
$B^0 - \bar{B}^0$	$\lvert V^*_{t'b}V_{t'd}\rvert < 0.002$	$\lvert V_{tb'}\rvert \approx \lvert V_{t'b}\rvert < 0.05$
$B^0_s - \bar{B}^0_s$	$\lvert V^*_{t'b}V_{t's}\rvert < 0.009$	
$Z \to b\bar{b}$	$\lvert V_{L4b}\rvert < 0.055$	
$Z \to c\bar{c}$	$\lvert V_{L4c}\rvert < 0.12$	
Z decay	$\sum_{q=d,s,b,u,c} \lvert V_{L4q}\rvert^2 \lesssim 0.006$	
	$\lvert V_{L4d,s,b}\rvert < 0.07$	
	$\lvert V_{L4u,c}\rvert < 0.075$	
$Q_W(Cs)$	$-(0.07)^2 < 1.12\lvert V_{L4d}\rvert^2 - \lvert V_{L4u}\rvert^2 < (0.09)^2$	
$Q_W(p)$	$\lvert\lvert V_{L4u}\rvert^2 - \frac{1}{2}\lvert V_{L4d}\rvert^2\rvert < (0.07)^2$	

Table 4.6: Limits on the mixing of the SM three families in presence of both down-type and up-type vector-like isosinglets. In the lowest part, limits are taken at 95% C.L.

4.4 Extra vector-like doublet

Let us suppose the existence of a fourth vectorial $SU(2)$-doublet family:

$$q_{4\,L,R} = \left(\begin{array}{c} u_4 \\ d_4 \end{array} \right)_{L,R} \tag{4.309}$$

Then new Yukawa couplings and mass terms should appear in the Lagrangian.

$$y^{u\prime}_{ij}\tilde{\varphi}\overline{q_{Li}}'u_{Rj} + y^{d\prime}_{ij}\varphi\overline{q_{Li}}'d_{Rj} + m_i\overline{q_{Li}}'q_{R4} + \text{h.c.} \tag{4.310}$$

with $i = 1, 2, 3, 4$, $j = 1, 2, 3$. As regards the mass terms $m_i\overline{q'_{Li}}q_{R4}$+h.c., a unitary transformation can be applied on the four components q'_{Li} so that $m_i = 0$ for $i = 1, 2, 3$. Then the Yukawa couplings and the mass term of the fourth doublet are:

$$\sum_{i=1}^{4}\sum_{j=1}^{3}\left[y^u_{ij}\tilde{\varphi}\overline{q_{Li}}u_{Rj} + y^d_{ij}\varphi\overline{q_{Li}}d_{Rj}\right] + M_4\overline{q_{L4}}q_{R4} + \text{h.c.} \tag{4.311}$$

The down quark mass matrix looks like:

$$\overline{d_{Li}}\mathbf{m}^{(d)}_{ij}d_{Rj} + h.c. =$$

$$= \left(\begin{array}{cccc} \overline{q_{1L}} & \overline{q_{2L}} & \overline{q_{3L}} & \overline{q_{4L}} \end{array} \right) \left(\begin{array}{cccc} & & & 0 \\ & \mathbf{y}^{(d)}_{3\times 3}v_w & & 0 \\ & & & 0 \\ y^d_{41}v_w & y^d_{42}v_w & y^d_{43}v_w & M_4 \end{array} \right) \left(\begin{array}{c} d_{R1} \\ d_{R2} \\ d_{R3} \\ q_{R4} \end{array} \right) \tag{4.312}$$

whith $v_w = 174$ GeV and $\mathbf{y}^{(d)}_{3\times 3}$ being a 3×3 mass matrix, and similarly for the up-type quarks. The mass matrices can be diagonalized with positive eigenvalues by biunitary transformations:

$$V^{(d)\dagger}_L \mathbf{m}^{(d)} V^{(d)}_R = \mathbf{m}^{(d)}_{\text{diag}} = \text{diag}(y^d_1 v_w\,, y^d_2 v_w\,, y^d_3 v_w\,, M_q) \tag{4.313}$$

$$V^{(u)\dagger}_L \mathbf{m}^{(u)} V^{(u)}_R = \mathbf{m}^{(u)}_{\text{diag}} = \text{diag}(y^u_1 v_w\,, y^u_2 v_w\,, y^u_3 v_w\,, M_q) \tag{4.314}$$

$V^{(d,u)}_{L,R}$ are unitary 4×4 matrices:

$$V^{(d)}_L = \left(\begin{array}{cccc} V_{L\,1d} & V_{L\,1s} & V_{L\,1b} & V_{L\,1b'} \\ V_{L\,2d} & V_{L\,2s} & V_{L\,2b} & V_{L\,2b'} \\ V_{L\,3d} & V_{L\,3s} & V_{L\,3b} & V_{L\,3b'} \\ V_{L\,4d} & V_{L\,4s} & V_{L\,4b} & V_{L\,4b'} \end{array} \right) , \quad V^{(u)}_L = \left(\begin{array}{cccc} V_{L\,1u} & V_{L\,1c} & V_{L\,1t} & V_{L\,1t'} \\ V_{L\,2u} & V_{L\,2c} & V_{L\,2t} & V_{L\,2t'} \\ V_{L\,3u} & V_{L\,3c} & V_{L\,3t} & V_{L\,3t'} \\ V_{L\,4u} & V_{L\,4c} & V_{L\,4t} & V_{L\,4t'} \end{array} \right) \tag{4.315}$$

and analogously for the matrices diagonalizing from the right. Flavour eigenstates in terms of mass eigenstates are:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}_{L,R} = V^{(d)}_{L,R} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_{L,R} , \qquad \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}_{L,R} = V^{(u)}_{L,R} \begin{pmatrix} u \\ c \\ t \\ t' \end{pmatrix}_{L,R} \tag{4.316}$$

The charged-current lagrangian is changed in:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \sum_{i=1}^{4} (\overline{u_{Li}} \gamma^\mu d_{Li}) W^+_\mu + \frac{g}{\sqrt{2}} \overline{u_{4R}} \gamma^\mu d_{4R} W_\mu + \text{h.c.} = \tag{4.317}$$

$$= \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L} \end{pmatrix} \gamma^\mu V_{\text{CKM,L}} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} W^+_\mu + \tag{4.318}$$

$$+ \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} \end{pmatrix} \gamma^\mu V_{\text{CKM,R}} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix} W^+_\mu + \text{h.c.} \tag{4.319}$$

where $V_{\text{CKM,L}} = V_L^{(u)\dagger} V_L^{(d)}$ is a 4×4 unitary matrix $V^\dagger_{\text{CKM,L}} V_{\text{CKM,L}} = V_{\text{CKM,L}} V^\dagger_{\text{CKM,L}} = \mathbf{1}$:

$$V_{\text{CKM,L}} = V_L^{(u)\dagger} V_L^{(d)} = \begin{pmatrix} V_{L\,ud} & V_{L\,us} & V_{L\,ub} & V_{L\,ub'} \\ V_{L\,cd} & V_{L\,cs} & V_{L\,cb} & V_{L\,cb'} \\ V_{L\,td} & V_{L\,ts} & V_{L\,tb} & V_{L\,tb'} \\ V_{L\,t'd} & V_{L\,t's} & V_{L\,t'b} & V_{L\,t'b'} \end{pmatrix} \tag{4.320}$$

with:

$$V_{L\,\alpha\beta} = \sum_{i=1}^{4} V^{(u)*}_{L\,i\alpha} V^{(d)}_{L\,i\beta} \tag{4.321}$$

But in this case, as shown in (4.402), $\mathcal{L}_{cc}$ involves also right weak charged currents and the mixing matrix $V_{\rm CKM,R}$ is not unitary:

$$\begin{aligned} V_{\rm CKM,R} &= V_R^{(u)\dagger}\mathrm{diag}(0,0,0,1)V_R^{(d)} = \\ &= \begin{pmatrix} V^*_{R\,4u}V_{R\,4d} & V^*_{R\,4u}V_{R\,4s} & V^*_{R\,4u}V_{R\,4b} & V^*_{R\,4u}V_{R\,4b'} \\ V^*_{R\,4c}V_{R\,4d} & V^*_{R\,4c}V_{R\,4s} & V^*_{R\,4c}V_{R\,4b} & V^*_{R\,4c}V_{R\,4b'} \\ V^*_{R\,4t}V_{R\,4d} & V^*_{R\,4t}V_{R\,4s} & V^*_{R\,4t}V_{R\,4b} & V^*_{R\,4t}V_{R\,4b'} \\ V^*_{R\,4t'}V_{R\,4d} & V^*_{R\,4t'}V_{R\,4s} & V^*_{R\,4t'}V_{R\,4b} & V^*_{R\,4t'}V_{R\,4b'} \end{pmatrix} = \\ &= \begin{pmatrix} V_{R\,ud} & V_{R\,us} & V_{R\,ub} & V_{R\,ub'} \\ V_{R\,cd} & V_{R\,cs} & V_{R\,cb} & V_{R\,cb'} \\ V_{R\,td} & V_{R\,ts} & V_{R\,tb} & V_{R\,tb'} \\ V_{R\,t'd} & V_{R\,t's} & V_{R\,t'b} & V_{R\,t'b'} \end{pmatrix} \end{aligned} \tag{4.322}$$

Regarding left handed sector, $V_L^{(d)}$, $V_L^{(u)}$ can be parameterized with the same parameterization as in (4.323):

$$\begin{aligned} V_L^{(d)} &= \begin{pmatrix} V_{L\,1d} & V_{L\,1s} & V_{L\,1b} & V_{L\,1b'} \\ V_{L\,2d} & V_{L\,2s} & V_{L\,2b} & V_{L\,2b'} \\ V_{L\,3d} & V_{L\,3s} & V_{L\,3b} & V_{L\,3b'} \\ V_{L\,4d} & V_{L\,4s} & V_{L\,4b} & V_{L\,4b'} \end{pmatrix} \simeq \mathbf{D}^d_{\phi L} V^{(d)}_{L\,3} L^{(d)} = \\ &= \mathbf{D}^d_{\phi L} \begin{pmatrix} & & & 0 \\ & V^{(d)}_{L\,3\times 3} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} c^d_{L1} & 0 & 0 & -\tilde{s}^d_{L1} \\ -\tilde{s}^d_{L2}\tilde{s}^{d*}_{L1} & c^d_{L2} & 0 & -\tilde{s}^d_{L2}c^d_{L1} \\ -\tilde{s}^d_{L3}\tilde{s}^{d*}_{L1}c^d_{L2} & -\tilde{s}^{d*}_{L2}\tilde{s}^d_{L3} & c^d_{L3} & -\tilde{s}^d_{L3}c^d_{L2}c^d_{L1} \\ \tilde{s}^{d*}_{L1}c^d_{L2}c^d_{L3} & \tilde{s}^{d*}_{L2}c^d_{L3} & \tilde{s}^{*d}_{L3} & c^d_{L1}c^d_{L2}c^d_{L3} \end{pmatrix} \end{aligned} \tag{4.323}$$

where $\mathbf{D}^d_{\phi L} = \mathrm{diag}(e^{i\phi^d_{L1}}, e^{i\phi^d_{L2}}, e^{i\phi^d_{L3}}, e^{i\phi^d_{L4}})$. $V^{(d)}_{L\,3\times 3}$ contains 3 angles and 3 phases. $\tilde{s}^d_{Li}, c^d_{Li}$ are complex sines and cosines of angles in the 1 4, 2 4, 3 4 family planes parameterizing the mixing of the first three families with the fourth family:

$$\tilde{s}^d_{Li} = \sin\theta^d_{Li4} e^{-i\delta^d_{Li}} = s^d_{Li} e^{-i\delta^d_{Li}} \tag{4.324}$$

and similarly for $V_L^{(u)}$. However in this scenario rotations in the left-handed sector are much smaller and indeed negligible:

$$s^{u,d}_{Li} \approx -\frac{y^{u,d}_i y^{u,d}_{4i} v_w^2}{M_q^2} \tag{4.325}$$

As regards the right-handed sector, without loss of generality, the basis can be chosen in which first the mixing between the SM three families is diagonalized:

$$V_R^{(d)} = \begin{pmatrix} V_{R\,1d} & V_{R\,1s} & V_{R\,1b} & V_{R\,1b'} \\ V_{R\,2d} & V_{R\,2s} & V_{R\,2b} & V_{R\,2b'} \\ V_{R\,3d} & V_{R\,3s} & V_{R\,3b} & V_{R\,3b'} \\ V_{R\,4d} & V_{R\,4s} & V_{R\,4b} & V_{R\,4b'} \end{pmatrix} =$$

$$= \mathbf{D}^d_{\phi R} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c^d_{R3} & -\tilde{s}^d_{R3} \\ 0 & 0 & \tilde{s}^{d*}_{R3} & c^d_{R3} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c^d_{R2} & 0 & -\tilde{s}^d_{R2} \\ 0 & 0 & 1 & 0 \\ 0 & \tilde{s}^{d*}_{R2} & 0 & c^d_{R2} \end{pmatrix} \cdot \begin{pmatrix} c^d_{R1} & 0 & 0 & -\tilde{s}^d_{R1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \tilde{s}^{d*}_{R1} & 0 & 0 & c^d_{R1} \end{pmatrix} =$$

$$= \mathbf{D}^d_{\phi R} \cdot \begin{pmatrix} c^d_{R1} & 0 & 0 & -\tilde{s}^d_{R1} \\ -\tilde{s}^d_{R2}\tilde{s}^{d*}_{R1} & c^d_{R2} & 0 & -\tilde{s}^d_{R2}c^d_{R1} \\ -\tilde{s}^d_{R3}\tilde{s}^{d*}_{R1}c^d_{R2} & -\tilde{s}^{d*}_{R2}\tilde{s}^d_{R3} & c^d_{R3} & -\tilde{s}^d_{R3}c^d_{R2}c^d_{R1} \\ \tilde{s}^{d*}_{R1}c^d_{R2}c^d_{R3} & \tilde{s}^{d*}_{R2}c^d_{R3} & \tilde{s}^{*d}_{R3} & c^d_{R1}c^d_{R2}c^d_{R3} \end{pmatrix} \tag{4.326}$$

where $\mathbf{D}^d_{\phi R} = \mathrm{diag}(e^{i\phi^d_{R1}}, e^{i\phi^d_{R2}}, e^{i\phi^d_{R3}}, e^{i\phi^d_{R4}})$. $\tilde{s}^d_{Ri}, c^d_{Ri}$ are complex sines and cosines of angles in the 1 4, 2 4, 3 4 family planes parameterizing the mixing of the first three families with the fourth family:

$$\tilde{s}^d_{Ri} = \sin\theta^d_{Ri4} e^{-i\delta^d_{Ri}} = s^d_{Ri} e^{-i\delta^d_{Ri}} \tag{4.327}$$

and similarly for $V_R^{(u)}$. Because of the mass matrices as in (4.312), in this case the angles parameterizing the mixing of the first three families with the fourth family in the right-handed sector would be equal in magnitude to the ones concerning left-handed sector in the previous sections:

$$s^{u,d}_{Ri} \simeq -\frac{y^{u,d}_{4i} v_w}{M_q} + O(|y^{u,d}_{4i}|(y^{u,d}_i)^2 \frac{v_w^3}{M_q^3}) \tag{4.328}$$

$$\tan\delta^{u,d}_{Ri} \simeq \frac{\mathrm{Im}(y^{u,d}_{4i})}{\mathrm{Re}(y^{u,d}_{4i})} \tag{4.329}$$

The piece of $\mathcal{L}_{cc}$ determining the couplings of u-quark with down type quarks (which in the SM would correspond with the determination of the first row of

SM CKM matrix) becomes:

$$\frac{g}{\sqrt{2}}(\overline{u_L}\gamma^\mu V_{L\,ud} d_L + \overline{u_R} V_{R\,ud}\gamma^\mu d_R)\, W_\mu +$$

$$+\frac{g}{\sqrt{2}}(\overline{u_L}\gamma^\mu V_{L\,us} s_L + \overline{u_R} V_{R\,us}\gamma^\mu s_R)\, W_\mu + \text{h.c.} = \tag{4.330}$$

$$=\frac{1}{2}\frac{g}{\sqrt{2}}\hat{V}_{ud}\,\overline{u}\gamma^\mu\left(1-\gamma^5 k_A^{ud}\right) d\, W_\mu + \frac{1}{2}\frac{g}{\sqrt{2}}\hat{V}_{us}\,\overline{u}\gamma^\mu\left(1-\gamma^5 k_A^{us}\right) s\, W_\mu \tag{4.331}$$

and similarly for the mixing with the bottom quark, where the vector and axial coupling are respectively:

$$\hat{V}_{u\alpha} = V_{L\,u\alpha} + V_{R\,u\alpha}\,, \qquad k_A^{u\alpha}\hat{V}_{u\alpha} = V_{L\,u\alpha} - V_{R\,u\alpha} \tag{4.332}$$

The most precise determination of the SM V_{ud} comes from superallowed beta decays. Superallowed 0^+ - 0^+ beta decays are Fermi transitions, that is they uniquely depend on the vector part of the hadronic weak interaction $G_V = G_F V_{ud}$. This means that the determination of the weak coupling in superallowed beta decays gives $|\hat{V}_{ud}|$ appearing in (4.331). Regarding $|V_{us}|$, it is determined both from semileptonic kaon decays $K\ell3$ and from leptonic kaon decay $K\mu2$. It is assumed that only the vector current contributes to semileptonic kaon decays $K\ell3$ ($\hat{V}_{us}$ in (4.331)). Leptonic decays instead depend on the axial coupling ($k_A^{us}\hat{V}_{us}$ in (4.331)). The SM ratio $|V_{us}/V_{ud}|$ from kaon and pion leptonic decays corresponds to the ratio of axial couplings in (4.331). Then, in this scenario, the observables are:

$$\begin{aligned}
A\ :&\qquad |\hat{V}_{us}| = |V_{L\,us} + V_{R\,us}| = 0.22326(55)\\
A'\ :&\qquad |\hat{V}_{us} k_A^{us}| = |V_{L\,us} - V_{R\,us}| = 0.22567(42)\\
B\ :&\qquad \frac{|\hat{V}_{us} k_A^{us}|}{|\hat{V}_{ud} k_A^{ud}|} = \frac{|V_{L\,us} - V_{R\,us}|}{|V_{L\,ud} - V_{R\,ud}|} = 0.23130(50)\\
C\ :&\qquad |\hat{V}_{ud}| = |V_{L\,ud} + V_{R\,ud}| = 0.97370(14)
\end{aligned} \tag{4.333}$$

$V_{\mathrm{CKM,L}}$ can be parameterized as:

$$V_{\mathrm{CKM,L}} \approx L^{(u)\dagger}\cdot\begin{pmatrix} & & & 0\\ & V_{\mathrm{CKM}\,3\times3} & & 0\\ & & & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot L^{(d)} \tag{4.334}$$

with $L^{(d)}$ (and similarly $L^{(u)}$) defined in Eq. (4.323). In this parameterization the 3×3 submatrix $V_{\mathrm{CKM}\,3\times3}$ contains 3 angles and one phase. Also the phases in

$\tilde{s}^{u,d}_{L1}$ and $\tilde{s}^{u,d}_{L2}$ can be absorbed in quark fields. In principle three real parameters can be enough to fit the dataset (4.333). In fact, for the first row of $V_{\rm CKM,L}$ it holds that:

$$|V_{L\,ud}|^2 + |V_{L\,us}|^2 + |V_{L\,ub}|^2 = 1 - |V_{L\,ub'}|^2 \tag{4.335}$$
$$V_{L\,ub'} = V^*_{L\,1u}V_{L\,1b'} + V^*_{L\,2u}V_{L\,2b'} + V^*_{L\,3u}V_{L\,3b'} + V^*_{L\,4u}V_{L\,4b'} \tag{4.336}$$

$|V_{L\,ud}|$ and $|V_{L\,us}|$ can be made real. $|V_{L\,ub}|$ has little influence and the value for the SM $|V_{ub}|$ can be used. $|V_{L\,ub'}| \sim O(y_1^{u,d} y_4^{u,d} v_w^2/M_q^2)$ is totally negligible. So $|V_{L\,ud}|$ can be determined from $|V_{L\,us}|$ using (4.335). As shown in Fig. 3.1, deter-

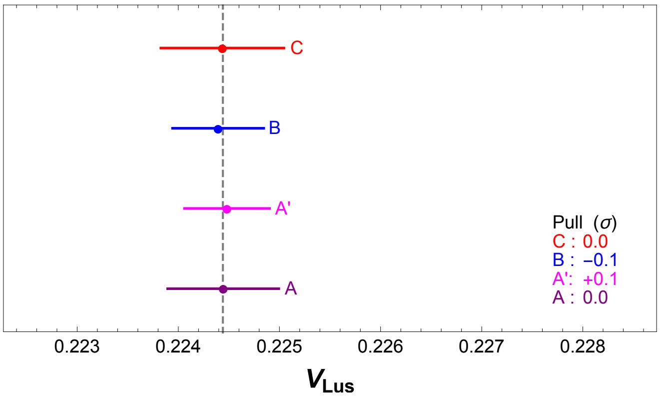

Figure 4.9: Values of V_{Lus} obtained from the different determinations under the hypothesis of the existence of weak right-handed currents, with $V_{Rus} = -0.0012$ and$V_{Rud} = -0.0008$. Values from Bazavov et al [106], Aoki et al. [107], Di Carlo et al. [109], Seng et al. [108] are used.

minations A' and B are basically in agreement, and the shift of determination B will be dominated by V_{Rus} rather than V_{Rud}. Then a fit with real $V_{R\,us}$ and $V_{R\,ud}$ can be performed. The following solution is found for the dataset (4.333) ($\chi^2_{\rm tot} = 0.02$):

$$\begin{aligned} V_{L\,us} &= 0.22444 & V_{L\,ud} &= 0.97370 \\ V_{R\,us} &= -0.0012 & V_{R\,ud} &= -0.0008 \end{aligned} \tag{4.337}$$

or, if the determination C is exctracted from the result in Ref. [117] ($\chi^2_{\rm tot} = 0.04$):

$$\begin{aligned} V_{L\,us} &= 0.22443 & V_{L\,ud} &= 0.97389 \\ V_{R\,us} &= -0.0012 & V_{R\,ud} &= -0.0006 \end{aligned} \tag{4.338}$$

By assuming the values of V_{Rus} andV_{Rud} in (4.337), then the values of V_{Lus} obtained from the different determinations (4.333) are shown in Fig. 4.9.

However also in this scenario flavour changing neutral currents appear at tree level.

The neutral current interactions are described by the Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm nc} =& \frac{g}{\cos\theta_W} Z^\mu \left[\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) \overline{\mathbf{u}_L}\gamma_\mu \mathbf{u}_L + \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right) \overline{\mathbf{d}_L}\gamma_\mu \mathbf{d}_L + \right.\\ & \left. -\frac{2}{3}\sin^2\theta_W \overline{\mathbf{u}_R}\gamma_\mu \mathbf{u}_R + \frac{1}{3}\sin^2\theta_W \overline{\mathbf{d}_R}\gamma_\mu \mathbf{d}_R + \frac{1}{2}\overline{u_{R4}}\gamma_\mu u_{R4} - \frac{1}{2}\overline{d_{R4}}\gamma_\mu d_{R4}\right]\end{aligned} \tag{4.339}$$

where $\mathbf{u}$ and $\mathbf{d}$ are the vectors whose components are $u_{L/Ri}$, $d_{L/Ri}$ with $i = 1, ..., 4$. Then, in term of the mass eigenstates, the neutral currents are:

$$\mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W} Z^\mu \left(g_L^\alpha \overline{q_{\alpha L}}\gamma_\mu q_{\alpha L} + g_R^{\alpha\beta} \overline{q_{\alpha R}}\gamma_\mu q_{\beta R}\right) \tag{4.340}$$

with $q_\alpha = u, c, t, t'$ or $q_\alpha = d, s, b, b'$ and:

$$\begin{aligned} g_L^\alpha &= t_3^a - Q_\alpha \sin^2\theta_W \\ g_R^{\alpha\beta} &= t_3^\alpha V_{R4\alpha}^* V_{R4\beta} - Q_\alpha \sin^2\theta_W \delta_{\alpha\beta}\end{aligned} \tag{4.341}$$

where T_3 refers to the isospin. Clearly, FCNC arise from:

$$\mathcal{L}_{\rm fcnc} = \frac{g}{\cos\theta_W} Z^\mu t_3^\alpha V_{R4\alpha}^* V_{R4\beta} \overline{q_{\alpha R}}\gamma_\mu q_{\beta R} = \tag{4.342}$$

$$= \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu (\overline{u_{R4}}\gamma_\mu u_{R4} - \overline{d_{R4}}\gamma_\mu d_{R4}) \tag{4.343}$$

or, explicitly:

$$\begin{aligned}\mathcal{L}_{\rm fcnc} =& \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R}\end{pmatrix} \gamma^\mu V_R^{(u)\dagger} {\rm diag}(0,0,0,1) V_R^{(u)} \begin{pmatrix} u_R \\ c_R \\ t_R \\ t'_R\end{pmatrix} + \\ & - \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu \begin{pmatrix} \overline{d_R} & \overline{s_R} & \overline{b_R} & \overline{b'_R}\end{pmatrix} \gamma^\mu V_R^{(d)\dagger} {\rm diag}(0,0,0,1) V_R^{(d)} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R\end{pmatrix}\end{aligned} \tag{4.344}$$

Process	Constraint
K decays	$\lvert V^*_{R4s} V_{R4d}\rvert < 4.8 \cdot 10^{-6}$
$B^0 \to \mu^+\mu^-$	$\lvert V^*_{R4b} V_{R4d}\rvert < 1.3 \cdot 10^{-4}$
$B^0_s \to \mu^+\mu^-$	$\lvert V^*_{R4b} V_{R4s}\rvert < 7.4 \cdot 10^{-5}$
$D^0 - \bar{D}^0$	$\lvert V^*_{R4u} V_{R4c}\rvert < 1.4 \cdot 10^{-5}$ (th)
$D^0 - \bar{D}^0$	$\lvert V^*_{R4u} V_{R4c}\rvert < 1.2 \cdot 10^{-4}$ (exp)

Table 4.7: Limits on the mixing of the first three families with an extra vector-like isodoublet from FCNC.

where

$$V_R^{(u)\dagger}\mathrm{diag}(0,0,0,1)V_R^{(u)} = \begin{pmatrix} |V_{R\,4u}|^2 & V^*_{R\,4u}V_{R\,4c} & V^*_{R\,4u}V_{R\,4t} & V^*_{R\,4u}V_{R\,4t'} \\ V^*_{R\,4c}V_{R\,4u} & |V_{R\,4c}|^2 & V^*_{R\,4c}V_{R\,4t} & V^*_{R\,4c}V_{R\,4t'} \\ V^*_{R\,4t}V_{R\,4u} & V^*_{R\,4t}V_{R\,4c} & |V_{R\,4t}|^2 & V^*_{R\,4t}V_{R\,4t'} \\ V^*_{R\,4t'}V_{R\,4u} & V^*_{R\,4t'}V_{R\,4c} & V^*_{R\,4t'}V_{R\,4t} & |V_{R\,4t'}|^2 \end{pmatrix} \tag{4.345}$$

$$V_R^{(d)\dagger}\mathrm{diag}(0,0,0,1)V_R^{(d)} = \begin{pmatrix} |V_{R\,4d}|^2 & V^*_{R\,4d}V_{R\,4s} & V^*_{R\,4d}V_{R\,4b} & V^*_{R\,4d}V_{R\,4b'} \\ V^*_{R\,4s}V_{R\,4d} & |V_{R\,4s}|^2 & V^*_{R\,4s}V_{R\,4b} & V^*_{R\,4s}V_{R\,4b'} \\ V^*_{R\,4b}V_{R\,4d} & V^*_{R\,4b}V_{R\,4s} & |V_{R\,4b}|^2 & V^*_{R\,4b}V_{R\,4b'} \\ V^*_{R\,4b'}V_{R\,4d} & V^*_{R\,4b'}V_{R\,4s} & V^*_{R\,4b'}V_{R\,4b} & |V_{R\,4b'}|^2 \end{pmatrix} \tag{4.346}$$

4.4.1 FCNC

In the previous sections constraints on mixing matrices of the left-handed sector were found from contributions to FCNC. Now the Lagrangian in (4.344) and the matrices in (4.345), (4.346) can be compared with Lagrangians in (4.14), (4.228) and matrices in (4.16), (4.229). It follows that the constraints from FCNC listed in Tables 4.4, 4.5 can be directly translated for this case with the substitution $L \to R$. The relevant constraints from FCNC are listed in Table 4.7.

4.4.2 Low energy electroweak observables

The insertion of the extra isodoublet also changes the diagonal couplings of weak neutral-current interaction, as follows from Eqs. (4.340) and (4.341). In

Process	Constraint
$Z \to b\bar{b}$	$\|V_{R4b}\| < 0.13$
$Z \to c\bar{c}$	$\|V_{R4c}\| < 0.18$
Z decay	$\|V_{R4u}\|^2 + \|V_{R4c}\|^2 + \frac{1}{2}\left(\|V_{R4d}\|^2 + \|V_{R4s}\|^2 + \|V_{R4b}\|^2\right) < 0.013$
$Q_W(Cs)$	$-0.0045 < \|V_{R4u}\|^2 - 1.12\|V_{R4d}\|^2 < 0.0077$
$Q_W(p)$	$-0.0048 < \|V_{R4u}\|^2 - \frac{1}{2}\|V_{R4d}\|^2 < 0.0040$
$g_{AV}^{eu} + 2g_{AV}^{ed}$	$-0.0025 < \frac{1}{2}\|V_{R4u}\|^2 - \|V_{R4d}\|^2 < 0.0097$
$2g_{AV}^{eu} - g_{AV}^{ed}$	$\|V_{R4u}\|^2 + \frac{1}{2}\|V_{R4d}\|^2 < 0.0087$
A_c	$\|V_{R4c}\| < 0.12$
A_b	$\|V_{R4b}\| < 0.2$

Table 4.8: Limits on the mixing of the first three families with an extra vector-like isodoublet from low energy electroweak observables and Z physics. Limits are taken at 95% C.L.

terms of the axial-vector and vector couplings, the diagonal Lagrangian is written as:

$$\frac{g}{2\cos\theta_W} Z^\mu \left(g_V^q \bar{q}\gamma_\mu q - g_A^q \bar{q}\gamma_\mu\gamma_5 q\right) \tag{4.347}$$

where $q = u, d, c, s, t, b$ and:

$$\begin{aligned} g_V^q &= t_3^q(1 + |V_{R4q}|^2) - 2Q_q \sin^2\theta_W \\ g_A^q &= t_3^q(1 - |V_{R4q}|^2) \end{aligned} \tag{4.348}$$

SM couplings are obtained when $|V_{R4q}| = 0$. As a consequence, also interactions of quarks with leptons are affected. Then constraints can arise from low energy ($Q^2 << M_Z^2$) electroweak precision observables. The low-energy four-fermion Lagrangian corresponding to ν-hadron and e-hadron processes with Z-boson exchange is written as [111]:

$$\begin{aligned} \mathcal{L} = &-\frac{G_F}{\sqrt{2}}\bar{\nu}\gamma_\mu(1-\gamma^5)\nu \sum_q \left[g_{LL}^{\nu q}\bar{q}\gamma^\mu(1-\gamma^5)q + g_{LR}^{\nu q}\bar{q}\gamma^\mu(1+\gamma^5)q\right] + \\ &+ \frac{G_F}{\sqrt{2}}\sum_q \left[g_{AV}^{eq}\bar{e}\gamma_\mu\gamma^5 e\bar{q}\gamma^\mu q + g_{VA}^{eq}\bar{e}\gamma_\mu e\bar{q}\gamma^\mu\gamma^5 q\right] \end{aligned} \tag{4.349}$$

From the relations (4.341) we obtain:

$$g_{LR}^{\nu q} = -Q_q \sin^2\theta_W + t_3^q |V_{R4q}|^2 \tag{4.350}$$

and from the relations (4.348):

$$g^{eu}_{AV} = -\frac{1}{2}(1+|V_{R4u}|^2) + \frac{4}{3}\sin^2\theta_W \qquad g^{ed}_{AV} = \frac{1}{2}(1+|V_{R4d}|^2) - \frac{2}{3}\sin^2\theta_W \tag{4.351}$$

$$g^{eu}_{VA} = (-\frac{1}{2} + 2\sin^2\theta_W)(1-|V_{R4u}|^2) \quad g^{ed}_{VA} = -(-\frac{1}{2} + 2\sin^2\theta_W)(1-|V_{R4d}|^2) \tag{4.352}$$

The weak charge of the proton and of nuclei are measurable quantities which can be expressed in terms of the couplings in (4.351).

The weak charge of the proton is proportional to $2g^{eu}_{AV} + g^{ed}_{AV}$, it was measured by Qweak at Jefferson Lab, from which it is obtained [111], [167]:

$$2g^{eu}_{AV} + g^{ed}_{AV} = 0.0356 \pm 0.0023 \tag{4.353}$$

After considering the existence of an extra doublet, the expected weak charge changes. The additional contribution to $2g^{eu}_{AV} + g^{ed}_{AV}$ is $-|V_{4u}|^2 + \frac{1}{2}|V_{4d}|^2$. By imposing to stay within the experimental error it is obtained that:

$$-(0.052)^2 < |V_{R4u}|^2 - \frac{1}{2}|V_{R4d}|^2 < (0.043)^2 \tag{4.354}$$

or at 95% C.L. :

$$-(0.069)^2 < |V_{R4u}|^2 - \frac{1}{2}|V_{R4d}|^2 < (0.063)^2 \tag{4.355}$$

Nuclear weak charges $Q^{Z,N}_W$ can be exctracted from measurements of parity violating amplitude. From PDG [111]:

$$Q^{Z,N}_W = -2[Z(g^{ep}_{AV} + 0.00005) + N(g^{en}_{AV} + 0.00006)](1 - \frac{\alpha}{2\pi}) \tag{4.356}$$

where Z and N are the numbers of protons and neutrons in the nucleus, $g^{ep}_{AV} = 2g^{eu}_{AV} + g^{ed}_{AV}$, $g^{en}_{AV} = g^{eu}_{AV} + 2g^{ed}_{AV}$. The most precise measurement of atomic parity violation is in Cesium, $Q_W(Cs) = -72.62 \pm 0.43$, while the SM prediction is $Q_W(Cs) = -73.23 \pm 0.01$, so with 1.5σ discrepancy. The contribution of the new quark to the weak charge of Cesium is:

$$\Delta Q^{55,78}_W(Cs) = Q^{55,78}_W(Cs)_{\text{new}} - Q^{55,78}_W(Cs)_{\text{SM}} = -2[-94|V_{R4u}|^2 + 105.5|V_{R4d}|^2](1 - \frac{\alpha}{2\pi}) \tag{4.357}$$

When the SM prediction is increased, then the new contribution is helping in filling the discrepancy with the experimental result, and the additional charge can be used to fill the gap and constrained to stay within $1\sigma_{\rm exp}$ (or $1.96\sigma_{\rm exp}$) from the experimental result. When the expected value is decreased with respect to the SM prediction, than the new contribution is not helping in filling the gap with the experimental result. Then the additional contribution can be constrained to be less than the experimental error (or $1.96\sigma_{\rm exp}$):

$$-(0.048)^2 < |V_{R4u}|^2 - 1.12|V_{R4d}|^2 < (0.074)^2 \tag{4.358}$$

or at 95% C.L.:

$$-(0.067)^2 < |V_{R4u}|^2 - 1.12|V_{R4d}|^2 < (0.088)^2 \tag{4.359}$$

The following results are also reported in PDG [111]:

$$(g^{eu}_{AV} + 2g^{ed}_{AV})_{\rm exp} = 0.4914 \pm 0.0031 \tag{4.360}$$

$$(2g^{eu}_{AV} - g^{ed}_{AV})_{\rm exp} = -0.7148 \pm 0.0068 \tag{4.361}$$

to be compared with the SM expectations:

$$g^{eu}_{AV} + 2g^{ed}_{AV} = 0.4950 \tag{4.362}$$

$$2g^{eu}_{AV} - g^{ed}_{AV} = -0.7194 \pm 0.0068 \tag{4.363}$$

Regarding the quantity $g^{eu}_{AV} + 2g^{ed}_{AV}$, the mixing with the extra doublet bring an extra contriubution $-\frac{1}{2}|V_{R4u}|^2 + |V_{R4d}|^2$. The experimental value is 1.16σ below the SM prediction, then it is obtained that for $-0.0067 < -\frac{1}{2}|V_{R4u}|^2 + |V_{R4d}|^2 < 0$ the expectation is within the experimental error, or at 95% C.L.

$$-0.0097 < -\frac{1}{2}|V_{R4u}|^2 + |V_{R4d}|^2 < 0.0025 \tag{4.364}$$

which means $|V_{R4u}| < 0.098$ if $|V_{R4d}| \ll |V_{R4u}|$. As regards the quantity $2g^{eu}_{AV} - g^{ed}_{AV}$, the prediction is lowered with the extra doublet. In order to remain within the experimental errorbar we have:

$$|V_{R4u}|^2 + \frac{1}{2}|V_{R4d}|^2 < (0.047)^2 \tag{4.365}$$

or within 1.96σ:

$$|V_{R4u}|^2 + \frac{1}{2}|V_{R4d}|^2 < (0.093)^2 \tag{4.366}$$

In PDG 2016 [169] results on ν-hadron scattering are also discussed. In particular it is reported the experimental value $g_{LR}^{\nu u} = -0.179 \pm 0.013$. The SM prediction is $g_{LR}^{\nu_\mu u} = -0.1552$ [111]. In our case this coupling is changed at tree level in $g_{LR}^{\nu u} = -\frac{2}{3}\sin^2\theta_W + \frac{1}{2}|V_{R4u}|^2$. Then $|V_{R4u}| = 0.22$ would recover the gap, then every lower value would still be acceptable.

Constraints from low energy electroweak observables are summarized in Table 4.8.

4.4.3 Z-boson physics

The SM predictions for the Z decay rate and partial decay rate into hadrons are $\Gamma(Z)_{\mathrm{SM}} = 2.4942 \pm 0.0008$ GeV, $\Gamma(Z \to \mathrm{hadr})_{\mathrm{SM}} = 1.7411 \pm 0.0008$ GeV, to be compared with the experimental results $\Gamma(Z)_{\mathrm{exp}} = 2.4952 \pm 0.0023$ GeV, $\Gamma(Z \to \mathrm{hadr})_{\mathrm{exp}} = 1.7444 \pm 0.0020$ GeV, showing a 0.4σ pull of the predicted width with respect to the experimental value and agreement as regards the partial decay rate into hadrons [111].

In this model the deviation of the Z decay rate from the SM prediction is:

$$\begin{aligned}
&|\Gamma(Z)_{\mathrm{new}} - \Gamma(Z)_{\mathrm{SM}}| = |\Gamma(Z\to\mathrm{had})_{\mathrm{new}} - \Gamma(Z\to\mathrm{had})_{\mathrm{SM}}| = \\
=&\frac{G_F M_Z^3}{\sqrt{2}\pi}\left|\sum_{q=u,c}\left(-\frac{2}{3}\sin^2\theta_W + \frac{1}{2}|V_{R4q}|^2\right)^2 - 2\left(-\frac{2}{3}\sin^2\theta_W\right)^2 + \right.\\
&+\sum_{q=d,s,b}\left(\frac{1}{3}\sin^2\theta_W - \frac{1}{2}|V_{R4q}|^2\right)^2 - 3\left(\frac{1}{3}\sin^2\theta_W\right)^2 + \\
&\left.+\frac{1}{2}\left(|V_{R4u}^* V_{R4c}|^2 + |V_{R4d}^* V_{R4s}|^2 + |V_{R4d}^* V_{R4b}|^2 + |V_{R4s}^* V_{R4b}|^2\right)\right| \qquad (4.367)
\end{aligned}$$

We can impose that this deviation is less than the experimental error:

$$|\Gamma(Z\to\mathrm{had})_{\mathrm{new}} - \Gamma(Z\to\mathrm{had})_{\mathrm{SM}}| < 0.0020\,\mathrm{GeV} \qquad (4.368)$$

The constraints in Table 4.4 imply that non-diagonal terms in (4.190) do not give any noticeable contribution. Then:

$$\begin{aligned}
&\frac{G_F M_Z^3}{\sqrt{2}\pi}\left|-\frac{2}{3}\sin^2\theta_W\left(|V_{R4u}|^2 + |V_{R4c}|^2\right) - \frac{1}{3}\sin^2\theta_W\left(|V_{R4d}|^2 + |V_{R4s}|^2 + |V_{R4b}|^2\right)\right| \\
&< 0.0020\,\mathrm{GeV} \qquad (4.369)
\end{aligned}$$

that is

$$|V_{R4u}|^2 + |V_{R4c}|^2 + \frac{1}{2}\left(|V_{R4d}|^2 + |V_{R4s}|^2 + |V_{R4b}|^2\right) < 0.0065 \qquad (4.370)$$

which implies for $|V_{R4q}| = 0$, with $q = c, d, s, b$, that $|V_{R4u}| < 0.08$. In the case $|V_{R4q}| = 0$ for $q = u, c, s, b$, then $|V_{R4d}| < 0.114$. Or, in order to be within $1.96\sigma_{\rm exp}$:

$$|V_{R4u}|^2 + |V_{R4c}|^2 + \frac{1}{2}\left(|V_{R4d}|^2 + |V_{R4s}|^2 + |V_{R4b}|^2\right) < 0.013 \tag{4.371}$$

If $|V_{R4q}| = 0$ for $q = c, d, s, b$, then $|V_{R4u}| < 0.11$, or, with $|V_{R4q}| = 0$ for $q = u, c, s, b$, $|V_{R4d}| < 0.16$.

Similarly, for the decays $Z \to b\bar{b}$ and $Z \to c\bar{c}$, the additional contribution is:

$$|\Gamma(Z \to b\bar{b})_{\rm new} - \Gamma(Z \to b\bar{b})_{\rm SM}| = \frac{1}{3}\sin^2\theta_W \frac{G_F M_Z^3}{\sqrt{2}\pi}|V_{R4b}|^2 \tag{4.372}$$

$$|\Gamma(Z \to c\bar{c})_{\rm new} - \Gamma(Z \to c\bar{c})_{\rm SM}| = \frac{2}{3}\sin^2\theta_W \frac{G_F M_Z^3}{\sqrt{2}\pi}|V_{R4c}|^2 \tag{4.373}$$

which, as in previous sections, can be required to be less than the experimental error (or 1.96 times experimental error), so it is obtained:

$$|V_{R4b}| < 0.09 \tag{4.374}$$

$$|V_{R4c}| < 0.13 \tag{4.375}$$

or at 95% C.L.:

$$|V_{R4b}| < 0.13 \tag{4.376}$$

$$|V_{R4c}| < 0.18 \tag{4.377}$$

Constraints are expected also from Z-pole asymmetry analyses, from $e^+e^- \to ff$ processes around Z resonance. In particular, left-right asymmetries A_{LR}, forward-backward asymmetries A_{FB}, and left-right forward-backward asymmetries A_{LRFB} are measured [111], [163]:

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \tag{4.378}$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \tag{4.379}$$

$$A_{LRFB} = \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R} \tag{4.380}$$

where L, R are the incident electron helicities. Cross sections for Z-boson exchange are usually written in terms of the asymmetry parameters A_f, $f =$

e, μ, τ, b, c, s, q, containing final-state couplings:

$$A_f = \frac{\bar{g}_L^{f2} - \bar{g}_R^{f2}}{\bar{g}_L^{f2} + \bar{g}_R^{f2}} = \frac{2\bar{g}_V^f \bar{g}_A^f}{\bar{g}_V^{f2} + \bar{g}_A^{f2}} = \frac{1 - 4|Q_f|\bar{s}_f^2}{1 - 4|Q_f|\bar{s}_f^2 + 8(|Q_f|\bar{s}_f^2)^2} \tag{4.381}$$

where in the SM at tree level $g_L^f = t_{3L}^f - Q_f \sin^2\theta_W$, $g_R^f = -Q_f \sin^2\theta_W$. The bar in the couplings in (4.381) indicates that EW radiative corrections must be taken into account, so effective couplings are defined and for convenience the effective angles $\bar{s}_f^2$ are used [111], [164] [165], [166]. The parameters A_f can also be exctracted from the measured asymmetries. For example, they are related as $A_{LR} = A_e P_e$ where P_e is the initial electron polarization, $A_{FB}^{(0,f)} = \frac{3}{4} A_e A_f$ for $P_e = 0$, $A_{LRFB} = \frac{3}{4} A_f$ for $P_e = 1$.

As regards b-quark final state, the measured value $A_{FB}^{(0,b)} = 0.0992 \pm 0.0016$ exhibits a -2.3σ pull with respect the SM prediction $A_{FB}^{(0,b)} = 0.1030 \pm 0.0002$. On the other hand, the value $A_b = 0.923 \pm 0.020$ is obtained from $A_{LRFB}(b)$ at SLD, basically in agreement with the SM prediction $A_b = 0.9347$ (-0.6σ pull). However it can also be inferred from $A_{FB}^{(0,b)}$ using $A_e = 0.1501 \pm 0.0016$ (as suggested by PDG [111]) obtaining $A_b = 0.881 \pm 0.017$. The average between the two can be considered: $A_b = 0.899 \pm 0.013$, which is 2.8σ below the SM prediction.

Regarding A_c, the experimental value $A_c = 0.670 \pm 0.027$ is in agreement with the SM prediction $A_c = 0.6677 \pm 0.0001$. By extracting A_c from $A_{FB}^{(0,c)} = 0.0707 \pm 0.0035$ using $A_e = 0.1501 \pm 0.0016$, it is obtained the result $A_c = 0.628 \pm 0.032$. The average between the two values is $A_c = 0.653 \pm 0.021$.

The presence of an additional doublet changes the couplings as in (4.348), and consequently the predictions for the asymmetries are also changed:

$$A_b = \frac{1 - \frac{4}{3}\bar{s}_b^2(1 - |V_{R4b}|^2) - |V_{R4b}|^4}{1 - \frac{4}{3}\bar{s}_b^2(1 + |V_{R4b}|^2) + \frac{8}{9}\bar{s}_b^4 + |V_{R4b}|^4} \tag{4.382}$$

$$A_c = \frac{1 - \frac{8}{3}\bar{s}_c^2(1 - |V_{R4c}|^2) - |V_{R4c}|^4}{1 - \frac{8}{3}\bar{s}_c^2(1 + |V_{R4c}|^2) + \frac{32}{9}\bar{s}_c^4 + |V_{R4c}|^4} \tag{4.383}$$

The SM prediction $A_c = 0.6677 \pm 0.0001$ reported by PDG [111] corresponds to an effective angle $\bar{s}_c^2 = 0.231465$. By using this value in Eq. (4.383), it is obtained that the expected value of A_c remains in the 1σ interval of the average $A_c = 0.653 \pm 0.021$ when $V_{R4c} < 0.057$, or at 95% C.L.

$$V_{R4c} < 0.12 \tag{4.384}$$

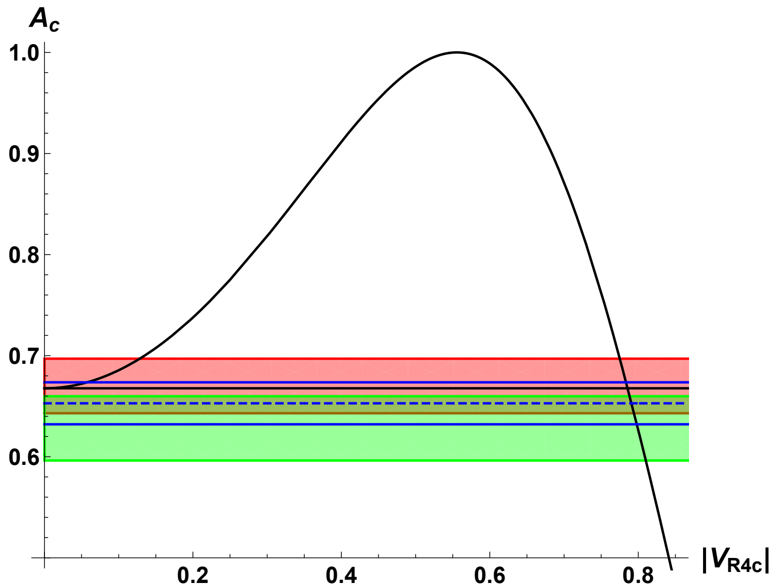

Figure 4.10: Determination of A_c obtained from Eq. (4.383) as a function of V_{R4c} (black curve). Ther red band is the experimental value $A_c = 0.670 \pm 0.027$, the green band shows the value obtained from the experimental determination $A_c = 0.628 \pm 0.032$ $A_{FB}^{(0,c)} = 0.0707 \pm 0.0035$. The blue lines correspond to the average of the two $A_c = 0.653 \pm 0.021$. The black straight line is the SM prediction $A_c = 0.6677$.

as can be seen in Fig. 4.10. Regarding the asymmetry for b-quark, if the discrepancy of the measured asymmetry $A_{FB}^{(0,b)}$ with respect to the SM prediction belonged to A_b, then the presence of the extra-doublet would not help the prediction to approach the experimental value, at least for $V_{R4b} < 0.55$, so the discrepancy would be still there. Considering the value of A_b obtained from $A_{LRFB}(b)$ at SLD, it can be noticed that the expected value of A_b remains in the 1σ interval when $V_{R4b} < 0.10$, or in 1.96σ interval when

$$V_{R4b} < 0.20 \tag{4.385}$$

as can be seen in Fig. 4.11.

Constraints from Z-boson physics are summarized in Table 4.8.

4.4.4 Discussion

Although the extra weak isodoublet can address the whole problem of CKM unitarity, it seems not possible to obtain an acceptable solution satisfying both

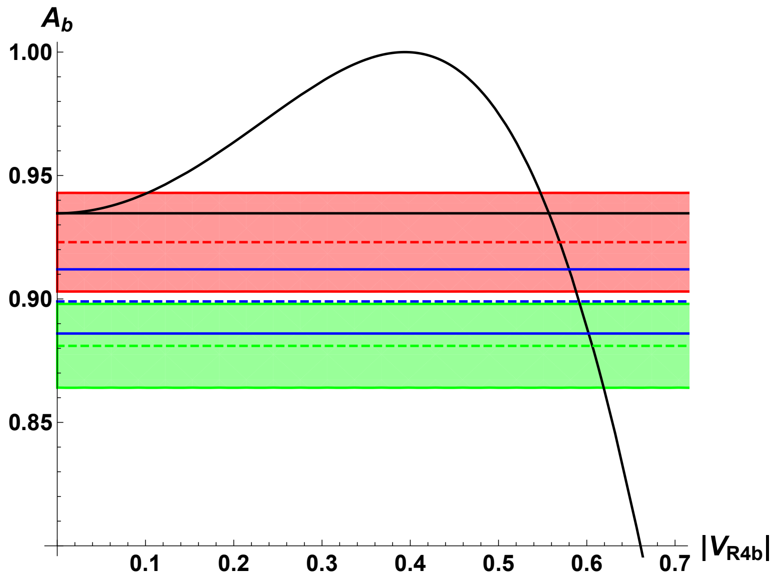

Figure 4.11: Determination of A_b obtained from Eq. (4.382) as a function of V_{R4b} (black curve). The 1σ intervals of the experimental values $A_b = 0.923 \pm 0.020$ (red band) and $A_b = 0.881 \pm 0.017$ obtained from $A_{FB}^{(0,b)}$ (green band) are shown. The blue lines correspond to the average of the two $A_b = 0.899 \pm 0.013$. The black straight line is the SM prediction $A_b = 0.9347$.

Eqs. (4.337) and the experimental constraints in Tables 4.7, 4.8 by adding one vector-like isodublet of quarks.

In fact, since the case of interest is described by the mixings in (4.337), we should focus on the elements V_{R4u}, V_{R4d}, V_{R4s} (for our concern the other mixing elements can also be zero). Then we should consider the strong restrictions coming from flavor changing kaon decays. In particular the limits on the branching ratios of $K^+ \to \pi^+ \nu \bar{\nu}$, $K_L \to \mu^+ \mu^-$ and $K_L \to \mu^+ \mu^-$ are summarized in Fig. 4.4 (with the exchange $L \leftrightarrow R$). The result is that the product $|V_{R4d} V^*_{R4s}|$ cannot exceed the limit $|V_{R4d} V^*_{R4s}| \lesssim 5 \cdot 10^{-6}$, which is obtained if for the relative phase it is chosen the value $\sin \delta^d_{R21} \approx 0.4$, with $\delta^d_{R21} = \delta^d_{R2} - \delta^d_{R1}$.

Constraints on the moduli of V_{R4u}, V_{R4d} from Z-pole physics and low energy electroweak observables are summarized in Fig. 4.12. The solution in (4.337) is also shown as the blue curve. As an example, we can use the maximum allowed value of V_{R4u} on the blue curve $(|V_{R4d}|, |V_{R4u}|) = (0.013, 0.064)$. The red curve in Fig. 4.13 shows the constraint on V_{R4d}, V_{R4s} from flavor changing kaon decays. The black cross stands for the values of V_{R4d} and V_{R4s} needed to obtain the solution (4.337) corresponding to the value of V_{R4u} marked in Fig. 4.12. It is

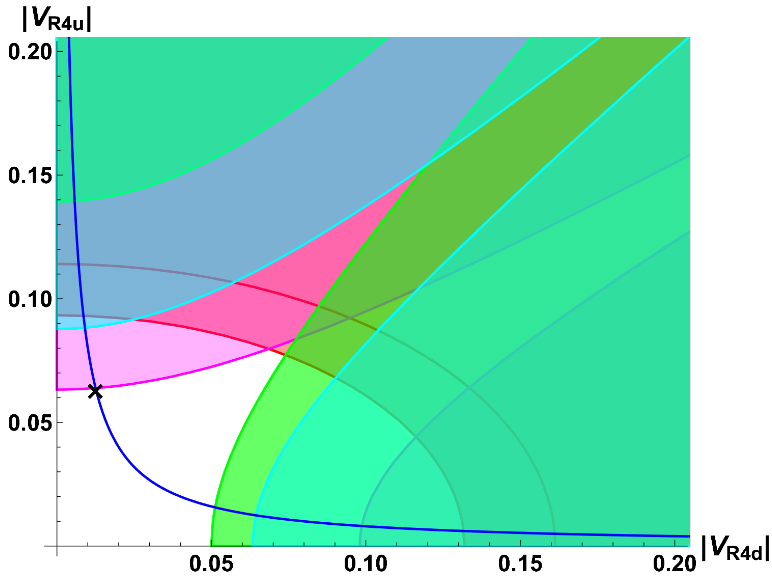

Figure 4.12: Excluded values of $|V_{R4d}|$ and $|V_{R4u}|$ from low energy electroweak quantities and Z physics: $Q_W(Cs)$ (cyan), $Q_W(p)$ (magenta), the couplings $g_{AV}^{eu} + 2g_{AV}^{ed}$ (green) and $2g_{AV}^{eu} - g_{AV}^{ed}$ (red), Z hadronic decay rate with $|V_{R4d}|, |V_{R4u}| \neq 0$ (orange). The blue curve stands fro the relation $|V_{R4u}^* V_{R4d}| = 0.0008$. The black cross stands for the minimum value of $|V_{R4d}|$ satisfying all the constraints and relations: $(|V_{R4d}|, |V_{R4u}|) = (0.0125, 0.064)$.

clear that the values needed as solutions are unachievable without contradiction with flavor changing experimental limits.

Namely, by performing a fit of the values in (4.333) with real parameters V_{Lus}, V_{R4d}, V_{R4s}, V_{R4u}, but constraining them with experimental results, the best fit point is obtained with $\chi^2_{\text{dof}} = 6.5$:

$$V_{R4u} = -0.064 \tag{4.386}$$

$$V_{R4s} = 0.0005 \tag{4.387}$$

$$V_{R4d} = 0.001 \tag{4.388}$$

and consequently

$$V_{Rus} = V_{R4u}^* V_{R4s} = -0.000032 \qquad V_{Rud} = V_{R4u}^* V_{R4d} = -0.00062 \tag{4.389}$$

and

$$V_{Lus} = 0.22503 \pm 0.00026 \tag{4.390}$$

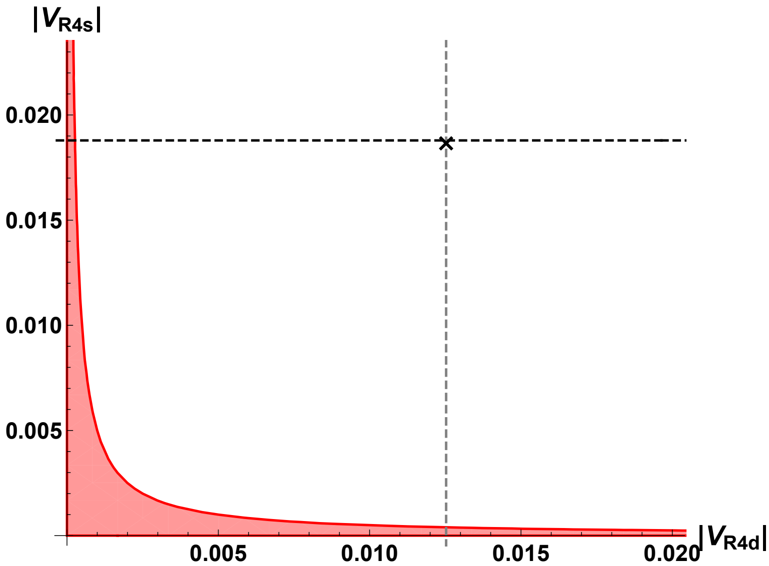

Figure 4.13: The red area is the region of values allowed by flavor changing K decays $|V_{R4d}V^*_{R4s}| < 5 \cdot 10^{-6}$. The black cross corresponds to the value of $|V_{R4s}|$ needed in order to obtain the products in (4.337) with $(|V_{R4d}|, |V_{R4u}|) = (0.013, 0.064)$, that is $(|V_{R4d}|, |V_{R4s}|) = (0.013, 0.019)$, $|V_{R4s}V^*_{R4u}| = 0.0012$.

Fig. 4.14 shows the result of the fit for V_{Lus}. Clearly the determinations are still not compatible.

Moreover, the LHC limit on extra b' mass $M > 1530$ GeV [111] implies that $|V_{R4u}| \simeq 0.06$ can be obtained if $y^u_{41} \sim 0.5$, much larger than the Yukawa constant of the bottom quark.

The problem is not much softened if determination C is inferred from the result in [117]. It is obtained that in the minimum $\chi^2_{\rm dof} = 6.3$ with:

$$V_{Lus} = 0.22477 \qquad V_{R4s} = 0.0124 \tag{4.391}$$
$$V_{R4u} = -0.07 \qquad V_{R4d} = 0.0011 \tag{4.392}$$

from which

$$V_{Rus} = V^*_{R4u}V_{R4s} = -0.00087 \qquad V_{Rud} = V^*_{R4u}V_{R4d} = -0.000079 \tag{4.393}$$

In any case, only one of the two discrepancies between the determinations of the elements of CKM from vector couplings can be filled.

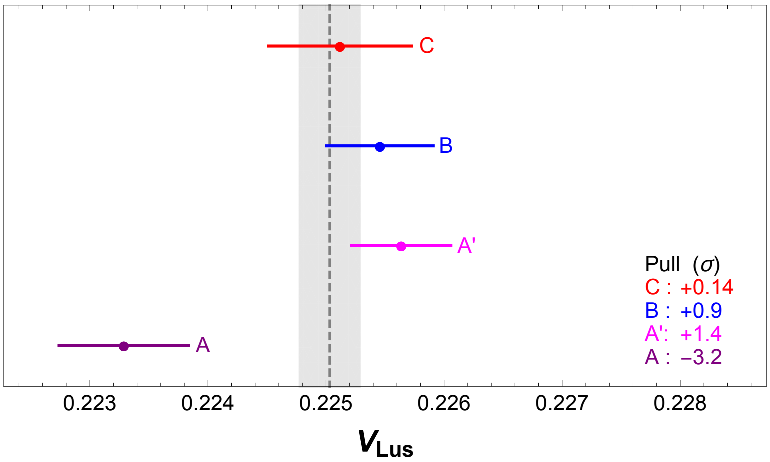

Figure 4.14: Values of V_{Lus} obtained from the fit of different determinations under the hypothesis of the existence of an extra vector-like doublet mixing with SM families, with $V_{R4u} = -0.0636$, $V_{R4s} = 0.00051$, $V_{R4d} = 0.0098$. Values from Bazavov et al [106], Aoki et al. [107], Di Carlo et al. [109], Seng et al. [108] are used. The grey region shows the 1σ interval of the average.

4.5 Possible solutions

Then a different hypothesis should be made. For example there can be two or more vector-like doublets

$$q_{4\,L,R} = \begin{pmatrix} u_4 \\ d_4 \end{pmatrix}_{L,R} \qquad q_{5\,L,R} = \begin{pmatrix} u_5 \\ d_5 \end{pmatrix}_{L,R} \tag{4.394}$$

Then the Yukawa couplings and the mass term of the fourth doublet are:

$$\sum_{i=1}^{5}\sum_{j=1}^{3}\left[y_{ij}^{u}\tilde{\varphi}\overline{q_{Li}}u_{Rj} + y_{ij}^{d}\varphi\overline{q_{Li}}d_{Rj}\right] + M_4\overline{q_{L4}}q_{R4} + M_5\overline{q_{L5}}q_{R5} + \text{h.c.} \tag{4.395}$$

Then in order to avoid flavour changing effects, not all the couplings $y_{ij}^{u,d}$ should be non-zero. In particular, let us suppose that the basis can be chosen in which the first doublet has couplings $y_{41}^{d} \neq 0$, $y_{42}^{d} = 0$, $y_{41}^{u} \neq 0$, while for the second doublet $y_{52}^{d} \neq 0$, y_{51}^{d}, $y_{51}^{u} \neq 0$. For simplicity, let us set also $y_{43}^{d}, y_{53}^{d} = 0$. The mass matrices for down and up quarks are then:

$$\sum_{i=1}^{4}\sum_{j=1}^{3}\overline{q_{Li}}\mathbf{m}_{ij}^{(d)}d_{Rj}+\overline{q_{L4}}q_{R4}M_4+\overline{q_{L5}}q_{R5}M_5+h.c.=$$
$$=\begin{pmatrix}\overline{q_{1L}} & \overline{q_{2L}} & \overline{q_{3L}} & \overline{q_{4L}} & \overline{q_{5L}}\end{pmatrix}\begin{pmatrix} & & & 0 & 0\\ & \mathbf{y}_{3\times 3}^{(d)}v_w & & 0 & 0\\ & & & 0 & 0\\ y_{41}^{d}v_w & 0 & 0 & M_4 & 0\\ 0 & y_{52}^{d}v_w & 0 & 0 & M_5\end{pmatrix}\begin{pmatrix}d_{R1}\\ d_{R2}\\ d_{R3}\\ q_{R4}\\ q_{R5}\end{pmatrix} \tag{4.396}$$

$$\sum_{i=1}^{4}\sum_{j=1}^{3}\overline{q_{Li}}\mathbf{m}_{ij}^{(u)}u_{Rj}+\overline{q_{L4}}q_{R4}M_4+\overline{q_{L5}}q_{R5}M_5+h.c.=$$
$$=\begin{pmatrix}\overline{q_{1L}} & \overline{q_{2L}} & \overline{q_{3L}} & \overline{q_{4L}} & \overline{q_{5L}}\end{pmatrix}\begin{pmatrix} & & & 0 & 0\\ & \mathbf{y}_{3\times 3}^{(u)}v_w & & 0 & 0\\ & & & 0 & 0\\ y_{41}^{u}v_w & y_{42}^{u}v_w & y_{43}^{u}v_w & M_4 & 0\\ y_{51}^{u}v_w & y_{52}^{u}v_w & y_{53}^{u}v_w & 0 & M_5\end{pmatrix}\begin{pmatrix}u_{R1}\\ u_{R2}\\ u_{R3}\\ q_{R4}\\ q_{R5}\end{pmatrix} \tag{4.397}$$

Then $V_{L,R}^{(d,u)}$ diagonalizing the mass matrices are unitary 5×5 matrices. Flavour eigenstates in terms of mass eigenstates are:

$$\begin{pmatrix}d_1\\ d_2\\ d_3\\ d_4\\ d_5\end{pmatrix}_{L,R}=V_{L,R}^{(d)}\begin{pmatrix}d\\ s\\ b\\ b'\\ b''\end{pmatrix}_{L,R}\quad ,\quad \begin{pmatrix}u_1\\ u_2\\ u_3\\ u_4\\ u_5\end{pmatrix}_{L,R}=V_{L,R}^{(u)}\begin{pmatrix}u\\ c\\ t\\ t'\\ t''\end{pmatrix}_{L,R} \tag{4.398}$$

However in this case $V_R^{(d)}$ can be parameterized as:

$$V_R^{(d)} = \begin{pmatrix} V_{R1d} & V_{R1s} & V_{R1b} & V_{R1b'} \\ V_{R2d} & V_{R2s} & V_{R2b} & V_{R2b'} \\ V_{R3d} & V_{R3s} & V_{R3b} & V_{R3b'} \\ V_{R4d} & V_{R4s} & V_{R4b} & V_{R4b'} \end{pmatrix} =$$

$$= \mathbf{D}_{\phi R}^d \cdot \begin{pmatrix} c_{R14}^d & 0 & 0 & -\tilde{s}_{R14}^d & 0 \\ 0 & c_{R25}^d & 0 & 0 & -\tilde{s}_{R25}^d \\ 0 & 0 & 1 & 0 & 0 \\ \tilde{s}_{R14}^{d*} & 0 & 0 & c_{R14}^d & 0 \\ 0 & \tilde{s}_{R25}^{d*} & 0 & 0 & c_{R25}^d \end{pmatrix} \tag{4.399}$$

where $\mathbf{D}_{\phi R}^d = \mathrm{diag}(e^{i\phi_{R1}^d}, e^{i\phi_{R2}^d}, e^{i\phi_{R3}^d}, e^{i\phi_{R4}^d})$. $\tilde{s}_{Ri}^d, c_{Ri}^d$ are complex sines and cosines as in (4.327) and (4.328).

The charged-current lagrangian is changed in:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \sum_{i=1}^{5} (\overline{u_{Li}}\gamma^\mu d_{Li}) W_\mu^+ + \frac{g}{\sqrt{2}} \overline{u_{4R}}\gamma^\mu d_{4R} W_\mu + \frac{g}{\sqrt{2}} \overline{u_{5R}}\gamma^\mu d_{5R} W_\mu + \text{h.c.} = \tag{4.400}$$

$$= \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L} & \overline{t''_L} \end{pmatrix} \gamma^\mu V_{\mathrm{CKM,L}} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \\ b''_L \end{pmatrix} W_\mu^+ + \tag{4.401}$$

$$+ \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} & \overline{t''_R} \end{pmatrix} \gamma^\mu V_{\mathrm{CKM,R}} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \\ b''_R \end{pmatrix} W_\mu^+ + \text{h.c.} \tag{4.402}$$

where in this case $V_{\mathrm{CKM,R}}$ is:

$$V_{\mathrm{CKM,R}} = V_R^{(u)\dagger} \mathrm{diag}(0,0,0,1,0) V_R^{(d)} + V_R^{(u)\dagger} \mathrm{diag}(0,0,0,0,1) V_R^{(d)} =$$

$$= \begin{pmatrix} V_{R\,4u}^* V_{R\,4d} & V_{R\,5u}^* V_{R\,5s} & 0 & V_{R\,4u}^* V_{R\,4b'} & V_{R\,5u}^* V_{R\,5b'} \\ V_{R\,4c}^* V_{R\,4d} & V_{R\,5c}^* V_{R\,5s} & 0 & V_{R\,4c}^* V_{R\,4b'} & V_{R\,5c}^* V_{R\,5b'} \\ V_{R\,4t}^* V_{R\,4d} & V_{R\,5t}^* V_{R\,5s} & 0 & V_{R\,4t}^* V_{R\,4b'} & V_{R\,5t}^* V_{R\,5b'} \\ V_{R\,4t'}^* V_{R\,4d} & V_{R\,5t'}^* V_{R\,5s} & 0 & V_{R\,4t'}^* V_{R\,4b'} & V_{R\,5t'}^* V_{R\,5b'} \end{pmatrix} = \tag{4.403}$$

Then (4.337) is requiring:

$$V^*_{R\,5u}V_{R\,5s} = -0.0012 \qquad\qquad V^*_{R\,4u}V_{R\,4d} = -0.0008 \tag{4.404}$$

However in this scenario the additional terms in the neutral current lagrangian are:

$$\begin{aligned}
\mathcal{L}_{\rm fcnc} =& \frac{1}{2}\frac{g}{\cos\theta_W}Z^\mu(\overline{u_{R4}}\gamma_\mu u_{R4} - \overline{d_{R4}}\gamma_\mu d_{R4} + \overline{u_{R5}}\gamma_\mu u_{R5} - \overline{d_{R5}}\gamma_\mu d_{R5}) = \\
=& \frac{1}{2}\frac{g}{\cos\theta_W}Z^\mu \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} & \overline{t''_R} \end{pmatrix} \gamma^\mu V^{(u)}_{\rm nc} \begin{pmatrix} u_R \\ c_R \\ t_R \\ t'_R \\ t''_R \end{pmatrix} + \\
& - \frac{1}{2}\frac{g}{\cos\theta_W}Z^\mu \begin{pmatrix} \overline{d_R} & \overline{s_R} & \overline{b_R} & \overline{b'_R} & \overline{b''_R} \end{pmatrix} \gamma^\mu V^{(d)}_{\rm nc} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \\ b''_R \end{pmatrix}
\end{aligned} \tag{4.405}$$

where in this case the matrix $V^{(d)}_{\rm nc}$ is:

$$V^{(d)}_{\rm nc} = V^{(d)\dagger}_R {\rm diag}(0,0,0,1,1) V^{(d)}_R = \begin{pmatrix} |V_{R\,4d}|^2 & 0 & 0 & V^*_{R\,4d}V_{R\,4b'} & 0 \\ 0 & |V_{R\,4s}|^2 & 0 & 0 & V^*_{R\,5d}V_{R\,5b''} \\ 0 & 0 & 0 & 0 & 0 \\ V^*_{R\,4b'}V_{R\,4d} & 0 & 0 & |V_{R\,4b'}|^2 & 0 \\ 0 & V^*_{R\,5b''}V_{R\,5s} & 0 & 0 & |V_{R\,5b''}|^2 \end{pmatrix} \tag{4.406}$$

so there are no FCNC at tree level between the three SM families. Then the solution in Eq. (4.337), and equivalently in (4.404), can be obtained. Again, the LHC limit on extra b' mass $M > 1530$ GeV [111] implies that $|V_{R\,4d}|, |V_{R\,5s}|, |V_{R\,4,5u}| \simeq 0.035$ can be obtained if $\mathrm{Re}(y^{u,d}_{ij}) \simeq 0.3\cos\delta_{ij}$, or, in order to have (for the perturbativity) $|y^{u,d}_{ij}| < 1$, the mass of the extra quarks should be $M_{4,5} \lesssim 5/\cos\delta_{ij}$ TeV or so.

Alternatively it can be imagined that there exist a vector-like isodoublet and a down-type or up-type isosinglet, with some zero-couplings:

$$\overline{q_{Li}}\mathbf{m}^{(d)}_{ij} d_{Rj} + h.c. =$$

$$= \left(\overline{q_{1L}} \quad \overline{q_{2L}} \quad \overline{q_{3L}} \quad \overline{q_{4L}} \quad \overline{d_{5L}} \right) \left(\begin{array}{cccc} & & 0 & y^d_{15} \\ & \mathbf{y}^{(d)}_{3\times3} v_w & 0 & 0 \\ & & 0 & 0 \\ 0 \quad y^d_{42} v_w & 0 & M_4 & 0 \\ 0 \quad 0 & 0 & 0 & M^d_5 \end{array} \right) \left(\begin{array}{c} d_{R1} \\ d_{R2} \\ d_{R3} \\ q_{R4} \\ d_{R5} \end{array} \right) \tag{4.407}$$

$$\overline{q_{Li}}\mathbf{m}^{(u)}_{ij} u_{Rj} + h.c. =$$

$$= \left(\overline{q_{1L}} \quad \overline{q_{2L}} \quad \overline{q_{3L}} \quad \overline{q_{4L}} \right) \left(\begin{array}{cccc} & & & 0 \\ & \mathbf{y}^{(u)}_{3\times3} v_w & & 0 \\ & & & 0 \\ y^u_{41} v_w & 0 & 0 & M_4 \end{array} \right) \left(\begin{array}{c} u_{R1} \\ u_{R2} \\ u_{R3} \\ q_{R4} \end{array} \right) \tag{4.408}$$

or:

$$\overline{q_{Li}}\mathbf{m}^{(u)}_{ij} u_{Rj} + h.c. =$$

$$= \left(\overline{q_{1L}} \quad \overline{q_{2L}} \quad \overline{q_{3L}} \quad \overline{q_{4L}} \quad \overline{u_{5L}} \right) \left(\begin{array}{ccccc} & & & 0 & y^u_{15} \\ & \mathbf{y}^{(u)}_{3\times3} v_w & & 0 & 0 \\ & & & 0 & 0 \\ y^u_{41} v_w & 0 & 0 & M_4 & 0 \\ 0 & 0 & 0 & 0 & M^u_5 \end{array} \right) \left(\begin{array}{c} d_{R1} \\ d_{R2} \\ d_{R3} \\ q_{R4} \\ d_{R5} \end{array} \right) \tag{4.409}$$

In this way the non-zero couplings y^d_{42}, y^u_{41} can cancel the discrepancy among the determinations obtained from leptonic and semileptonic kaon decays without introducing flavor changing. Then a down-type or up-type singlet can fix the discrepancy between the determination obtained from superallowed beta decays with the average of the other determinations from kaon decays with ($\chi^2_{\text{tot}} = 0.5$):

$$V_{Lus} = 0.22436 \qquad |V_{L4d}| = 0.039 \tag{4.410}$$

$$V_{R4s} = -0.0011 \tag{4.411}$$

or $V_{L4u} = 0.039$ in the case of up-type singlet, where the value of V_{ud} from [108] has been used. Using instead the value from [117] ($\chi^2_{\text{tot}} = 0.4$):

$$V_{Lus} = 0.22437 \qquad |V_{L4d}| = 0.034 \tag{4.412}$$

$$V_{R4s} = -0.0011 \tag{4.413}$$

or $V_{L4u} = 0.034$ in the case of up-type singlet.

Chapter 5

Conclusions

This thesis is based on Refs. [44], [45], [46].

Chapter 2 is based on our works [44], [45]. $SU(3)_q \times SU(3)_u \times SU(3)_d \times SU(3)_\ell \times SU(3)_e$ symmetry between families was considered as gauge symmetry extending the SM.

It was shown that the inter-family symmetry $SU(3)_\ell \times SU(3)_e$ acting on left-handed and right-handed leptons can give a natural explanation to the origin of mass hierarchy among charged leptons and large mixing of neutrinos as a consequence of spontaneous breaking pattern of this symmetry. In fact, the lepton mass hierarchy $m_\tau \gg m_\mu \gg m_e$ can be related to the hierarchy of the scales $v_3 \gg v_2 \gg v_1$ of $SU(3)_e$ breaking.

First horizontal gauge symmetry $SU(3)_e$ acting between three families of right-handed leptons was analyzed. It was obtained the remarkable result that masses of flavour gauge bosons can be as light as TeV [45]. In fact it was shown that LFV effects induced by flavor changing gauge bosons are strongly suppressed due to custodial properties of $SU(2)_e \subset SU(3)_e$ symmetry and the respective scale can be as small as $v_2 = 2$ TeV, which is in fact a limit set by the compositeness limits on the flavor-conserving operators. Taken into account that the gauge coupling constant g of horizontal $SU(3)_e$ can be less than 1, then masses of the $SU(2)_e$ gauge bosons $M_{1,2,3} \simeq (g/\sqrt{2})v_2$ can be as small as 1 TeV or even smaller.

Then the symmetry of the lepton sector $SU(3)_\ell \times SU(3)_e$ acting on right-handed and left-handed leptons was discussed. It was shown that also the breaking scale of $SU(3)_\ell$ family symmetry between left-handed leptons can be low (few TeV) [46]. An important consequence is that flavour gauge bosons mediated interactions can then be a solution for the 4σ discrepancy from unitarity of the first row of CKM matrix illustrated in the third chapter (and Ref. [46]).

As far as quark sector is regarded, the origin of quark mass hierarchy can be related to the pattern of gauge $SU(3)_q \times SU(3)_u \times SU(3)_d$ symmetry breaking, and the respective flavor gauge bosons also can be at few TeV scale, without contradicting the limits from flavor changing processes [44].

Then gauge bosons acting in the quark sector can be in the reach of LHC for direct detection. Lepton gauge bosons cannot be easily ruled out by LHC but they could be accessible at new electron positron machines for direct detection in the TeV range. Nevertheless, some of these LFV processes, as e.g. $\tau \to 3\mu$, can have widths close to present experimental limits and can be within the reach of future high precision experiments.

In supersymmetric extension of the SM, the chiral gauge symmetries $SU(3)_\ell \times SU(3)_e$ for leptons and $SU(3)_Q \times SU(3)_u \times SU(3)_d$ for quarks can be also motivated as a natural tool for realizing the minimal flavor violation scenario [9, 10, 21].

There is an interesting possibility for anomaly cancellation between the ordinary and mirror fermions [21]. Then flavor gauge symmetries can be common symmetries between the particles of ordinary and mirror sectors and flavor gauge bosons would be messengers between the two sectors and mediate new flavor violating phenomena such as muonium–mirror muonium, kaon–mirror kaon oscillations, etc. with implications for the invisible decay channels of neutral mesons. Moreover, as far as the presence of mirror sector is concerned, mirror matters is a viable candidate for light dark matter (see e.g. reviews [170, 171, 172]). The flavor gauge bosons interacting with both ordinary and mirror fermions appear as messengers between two sectors and can give an interesting portal for mirror matter direct detection.

Chapter 3 of this thesis is based on our work Ref. [46], in which we raised the question that there may be a signal for a violation of CKM unitarity, after analyzing the tension between independent determinations of the elements in the first row of CKM matrix. In fact, the present experimental and theoretical accuracy in the determination of V_{us} and V_{ud} allows to test unitarity of the first row of the CKM matrix. A deviation of about 4σ from SM unitarity is found [46].

As a possibility for restoring unitarity we considered a new effective operator in positive interference with the SM muon decay as the one generated by flavour changing gauge bosons. Then the Fermi constant would be different from the muon decay constant, $G_F = G_\mu/(1 + \delta_\mu)$, where $\delta_\mu \simeq 7 \times 10^{-4}$ would suffice for unitarizing the CKM matrix. Since the values of V_{us} and V_{ud} are normally extracted by assuming $G_F = G_\mu$, in this scenario they are shifted by a factor $1 + \delta_\mu$ while their ratio is not affected. The gauge horizontal symmetry $SU(3)_\ell$

should be broken at the scale of few TeV in order to obtain the needed effective operator mediated by a $SU(3)_\ell$ flavor changing boson, which was previously (second chapter) shown to be possible without contradicting experimental constraints. Then new flavour changing phenomena at the experimental sensitivity border are predicted.

Hence we showed that beyond any prejudice the muon decay constant G_μ can be different from the Fermi constant G_F without contradicting experimental data, rather providing a testable and falsifiable solution for the CKM unitarity problem.

Also the neutron lifetime problem, that is about 4σ discrepancy between the neutron lifetimes measured in beam and trap experiments, was analyzed in the light of the these determinations of the CKM matrix elements. It is shown in Ref. [46] that the present redetermination of V_{ud} has no influence on the determination of the neutron lifetime.

Alternative, and perhaps more immediate, possibility to solve the CKM unitarity problem can be the mixing of ordinary quarks with vector-like quarks. This scenario is analyzed in chapter 4, based on our works [46], [173]. A quite large mixing with SM fermions is needed in order to recover unitarity. For example, the mixing of an extra down-type vectorlike quark with the first family should be $|V_{ub'}| \approx 0.04$, which is comparable to $|V_{cb}|$ and ten times larger than $|V_{ub}|$.

The consequences of so large mixing need to be investigated. In fact, the presence of an extra weak doublet or isosinglet quark species in the mixing generates FCNC at tree level. Also predictions of many observables related to Z-boson physics are affected.

The presence of weak isosinglets (up-type or down-type) is aimed at recovering the gap between the determination of V_{ud} from super-allowed beta decays and the average of the determinations of V_{us} obtained from kaon physics. However the tension among the results from leptonic and semileptonic kaon decays themselves is significant too. An extra vector-like weak isodoublet coupling with the SM families can in principle address the whole problem.

The existence of two or more vector-like doublets or a vector-like isodoublet with down-type or up-type isosinglet can yield an acceptable solution solving the problem of CKM unitarity without contradicting experimental constraints [173]. Assuming the existence of more vectorlike quarks in fact flavour changing can be avoided by setting to zero some couplings of extra species with SM families.

In order to obtain a large mixing of vectorlike quarks with ordinary quarks, the mass scale of these particles should be of few TeV. Anyway, with such large mixing, these new particles cannot be heavier than few TeV without contradicting experimental data on neutral mesons systems. Then further search at LHC can falsify or discover the vectorlike quarks. Moreover, data on flavour changing processes set very stringent constraints, then new predicted flavour changing phenomena would be at the experimental sensitivity border.

Concluding, in order to explain fermion masses and mixing, as motivated in this thesis in different scenarios, in the TeV range there may exist a new physics related to the fermion flavour which can be revealed in future experiments at the energy and precision frontiers.

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